

Universals as Individuals: Reply to Levine

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Contents

1	Introduction	I
2	Unpublished Writings	2
2.1	1903 Manuscripts	3
2.2	“The Paradox of the Liar”	5
2.3	Manuscripts between <i>PL</i> and <i>PM</i>	7
3	Published Writings	14
3.1	“The Existential Import of Propositions”	15
3.2	“On the Relations of Universals and Particulars”	16
4	What is an Individual?	21
4.1	The One and The Many	21
4.2	Definitions of Individuals in the <i>PM</i> Period	26
4.3	The Actual World and the World of Logic	30
5	The <i>Theory of Knowledge</i> Manuscript	38
6	Propositional Functions	42

I Introduction

James Levine and I both believe that in the early parts of his career¹, Russell believed that universals (relations-in-intension, and what Russell calls “concepts” or “predicates”) count as individuals, and occur as logical subjects in mind-independent propositions in exactly the same way that particulars do, so that a particular could replace a universal with nothing else changing, resulting in another well-formed proposition. We also both believe that at least one point in his career, Russell maintained the belief that universals were

¹Though perhaps not the *very* early parts, since there were times during his idealist phase in which Russell would have denied this.

capable of occurring in a complex as logical subject or relatum of a relation, while denying that it would ever be possible in such cases for a particular to replace them. Our difference then is mainly one of how long and during what period Russell maintained the latter view, and perhaps, whether or not it occurs in or influenced any of Russell's major works published in his lifetime. Given that I also agree with his claim that the metaphysics of universals is not an explicit theme of the major work of the disputed period, *Principia Mathematica* (*PM*), I think it is important to make sure we don't exaggerate our differences. Moreover, if there is a type difference in Russell's mind between individuals and universals of various sorts at the time of *PM*, we seem to agree that *that* is not the celebrated "Theory of Types" central to the work.

But there do seem to be disagreements between us that may be very relevant to evaluating *PM*, such as whether Russell would have been prepared to grant any kind of being to propositional functions at the time of *PM*, but since Levine places less stress on this, so shall I. Nonetheless, the issue cannot be completely skirted, since it bears on Russell's understanding of the notion of a "logical type" generally, and how Russell's thinking about such issues drove his philosophical development. This seemingly small issue turns out to be connected in many ways to a large number of aspects of Russell's core philosophy. With that in mind, I remain convinced that the reading I gave in [7] that in 1910 Russell continued to regard universals as individuals is the most plausible one.

2 Unpublished Writings

Levine makes use of a number of sources of evidence to cast doubt on the hypothesis that Russell was still committed to the explicit view of *PoM* that universals² are individuals, and they occur as logical subjects in precisely the same positions that particulars or "things" do. Levine points to manuscripts from the period between *PoM* and *PM* in which Russell seems to have adopted a contrary use of the word "individual".

All of these surviving manuscripts are fascinating and quite revealing about Russell's philosophical method and interests; they have in many ways profoundly influenced my own reading of Russell. But I'm sure Levine would agree that they must be approached cautiously, and cannot be given the same

²I am here assuming, uncontroversially, I believe, that the "predicates", "class concepts" and "relations" of *PoM* are to be understood as universals, even though Russell does not use the latter term in his early work.

kind of weight as Russell's published works. This is especially so as many of them seem to consist in something like experiments in which Russell was simply exploring what strengths a given position or tack would have in solving the paradoxes plaguing the continuation of his work in mathematical logic. But by Russell's own lights, nearly all these experiments failed.

In fairness to Levine, it should be noted that his discussion of these is mainly limited to his appendices, and his principal goal is the very minor one of establishing that Russell's use of the word "individual" has some looseness to it: that he didn't always use it as an all-embracing category, and was sometimes willing to exclude universals from the category of individuals. Levine is, I must admit, successful in establishing that goal. However, it hardly seems as if establishing this does much to bolster his case about how to read *PM*, unless there is some continuity between the views explored in these manuscripts and the position Russell held then. The two manuscripts which Levine principally draws upon are the 1903 paper "On the Meaning and Denotation of Phrases" (*OMDP*), and the 1906 piece "The Paradox of the Liar" (*PL*), I think Levine does convincingly show that in them Russell takes up consideration of a view that is in some ways unlike both his earlier view, and the view which Landini, Stevens and I attribute to Russell at the time of *PM*. However, I think he fails to make the case that the positions explored in these manuscripts were anything but failed experiments, involving positions adopted only temporarily for the sake of exploration. A closer look at these manuscripts show that even when Russell was willing to exclude universals from the category of particulars, the view he adopted was very unlike the one Levine reads into *PM*. A closer examination of manuscripts coming after them also show, I believe, that Russell's reasons for rejecting these positions are not compatible with the view Levine reads into *PM* as being a likely successor.

2.1 1903 Manuscripts

Somewhat naturally given that it comes well before the period that is his main interest, Levine puts less stress on what he finds in *OMDP*. In that piece, Russell restricts individuals to entities which can *only* be *denoted*, not *meant*, where it seems clear that for something to be meant by a phrase is for that phrase to indicate it occurring in a proposition in a way that makes it not a logical subject of that proposition [16, p. 287]. This contrasts with functions and concepts, and during this period it seems clear that Russell does identify what he would later call "universals" with functions. This was during the period in which Russell seems to have been most influenced by Frege, and

he was exploring a quasi-Fregean understanding of a function as something *unsaturated*, which comes together to form a complex with its argument. The unity provided by the completion of a function by its argument, Russell wrote in the manuscript “Functions” of this period, was thought to provide “the logical genesis of all complexes” ([15, p. 50]). On this view, there is little reason for him to resist identifying those constituents of complexes he had hitherto regarded as providing the unity of complexes, viz., relations and, perhaps, other universals, with functions, and so he does. During this period, Russell also hoped, by adapting Frege’s notation for the Wertverläufe, or courses-of-values, of functions as a notation for a function itself, to make functions do the work of classes in mathematical logic, having written to Frege in May 1903 that “I have discovered that classes are entirely superfluous” ([3, p. 158]). It is, I think, because Russell hoped to make functions do the work of classes that he regarded it as inappropriate to use the word “individual” for them.³ I shall explore more in sec. 4.1 below exactly why Russell saw fit to contrast classes with individuals.

These views did not last long. Although he then used the word “entity” rather than “individual” for the category, Russell did believe that there was a broader category to which both individuals and functions belonged during this period [15, p. 51], and also thought that it was in general possible for a function to take itself as argument, which left him susceptible to the propositional functions version of the paradox, and in letters both to Frege in 1904 [3, p. 166] and Jourdain in 1906 [4, p. 78] he cites this as having led him to rethink this approach. Some of his doubts involved those complex functions which seem to require taking a complex first and replacing multiple occurrences of the same constituent with a variable, as in $\dot{x}(x \text{ loves } x)$, suggesting that it is not clear in these cases that one can regard what is common between the values as any sort of part of those values, and certainly not a thing which is more primitive than those values. But this led to vacillation on Russell’s part on the question of whether functions or complexes are more primitive, and through most of 1904 he seemed to gravitate more and more toward the latter view, denying that functions are constituents of their values [18, 19, 21]. But if functions are not constituents of their values, then Russell could no longer identify universals with functions, and universals, as entities distinct from functions, were recruited once again to provide the explanation for the unity of complexes.

³I have written about Russell’s views during this period (mid-1903 until perhaps early 1904) at greater length elsewhere [7, 6, 8].

Once Russell is again committed to distinguishing functions from predicates and relations, he was left without a reason not to apply the word “individual” in a way that would include predicates and relations. And indeed, during the heyday of the substitutional theory of late 1905 and early 1906—which might be described as an even more radical version of the suggestion that complexes are logically more primitive than the mere *façons de parler* that propositional functions have been reduced to—Russell is very explicit that the word “individual” can be used in a way that subsumes all genuine entities, writing that “there really is nothing that is not an individual” [22, p. 206]. In eschewing functions and classes as genuine entities during this period, he is certainly not eschewing universals, which he regards as “essential to complexity” [23, p. 174].

This part of the story Levine seems to accept, or at least not deny. But then, does it really help Levine’s case for his reading of *PM* that, in 1903, Russell very briefly used the word “individual” in a way that precluded universals, if this was a relic of a failed experiment that he soon scrapped? In fairness to Levine, two things should be mentioned. Russell’s substitutional theory too can be thought of as a “failed experiment”, and this does not stop Landini, Stevens and I from taking it to shed quite a lot of light on Russell’s later views. And secondly, Levine is fairly clear that he does not put much importance on this early terminology. But I think it may be worth noting in passing that I think there are very good reasons to allow the substitutional theory to color our reading of *PM* to a greater extent than the 1903 views should be allowed to. After all, Russell was sufficiently confident to *publish* two works endorsing the substitutional theory, and submitted a third [23] for publication before withdrawing it. The 1903 view never made it past the stage of manuscripts and letters.

2.2 “The Paradox of the Liar”

Let us turn, then, to the manuscript “The Paradox of the Liar” from 1906. Should this too, be regarded as merely a failed experiment, or should we take it as indicating a true change in direction for Russell’s understanding of metaphysical and logical categories?

In this manuscript, Russell *considers* a view according to which only particulars are individuals, and predicates and other universals are placed in a wholly distinct logical type, but like nearly everything else in the manuscript, he does not endorse the theory with any definiteness. The *PL* manuscript as a whole does not come across as the work of a person whose mind is very

settled. For example, at least a half dozen times in the manuscript, Russell goes back and forth with himself over the issue of whether propositions and propositional functions can be made into “apparent variables” (i.e., quantified over), trying to reconcile the apparent need to do so for mathematics with the dangers of doing so in light of the paradoxes, and what this means as regards their “being”. Later in the manuscript, Russell considers a wholly different kind of metaphysics focused on property instances, which Russell calls “qualities”, taken as more fundamental. He seems to have in mind what we could now call “tropes”, but Russell clearly is not endorsing that theory over his more usual metaphysical fare or the kinds of views explored earlier in the paper. The tone of the entire piece is one of exploration.

The passages in which Russell considers a theory adopting genuine metaphysical types of entities, are no exception. The discussion of this view begins with the words “The question is this:”, and many of the sentences that follow are prefaced with “In this view ...”, making it seem as if Russell is asking himself what to make of a certain position, rather than actually setting it forth as his own. To be sure, such qualifications become less pronounced as the exploration goes more in depth, but the provisional nature of the discussion is never missing. Indeed, a few pages into the discussion, Russell stops himself and explicitly calls into question the very conclusions that seem most important for Levine’s case: that the theory involves mutually exclusive non-overlapping types, and the conclusion that there is no kind of subject-predicate construction which admits of any kind of entity whatever in its subject position. In particular, Russell, claims it may be too harsh a conclusion to deny that there are certain predicates which apply across the board, the most important being *being* itself:

But the demand that no statement should be significant for more than one type seems excessive. ...

Thus as regards the kind of being, in particular, it may be possible the types do not differ. The hierarchy of propositions may perhaps therefore be more simply constructed. [24]

Russell goes on to rehearse what benefits he saw along these lines in the substitutional theory, and speaks in favor of trying to find a theory according to which all first-order propositions are alike treated as expressing things about individuals. This view of course would either require that universals be individuals, or else that they can be subjects only in higher-order propositions. (I’ll discuss this second suggestion in the next section.)

Now in fairness Levine does not need to claim that Russell had at this point *definitely* made up his mind in favor of a typed metaphysics. But it is again hard to see how these manuscripts help Levine's case for reading Russell as being amenable to a typed metaphysics in *PM* unless there is some continuity between these explorations and the views Russell held in *PM*. But then the natural place to look would be the intervening manuscripts (and publications) to what trace can be found of these experimental thoughts, and what Russell's attitude toward them seems to have been then. It seems to me, however, that if we pursue this, we see not only that this exploration of a typed metaphysics doesn't last, but that his reasons for giving it up are incompatible with seeing the trajectory of Russell's thoughts as headed toward the kinds of views Levine reads Russell as having in *PM*; indeed, his reasons for abandoning this experiment seem to have been doubts about typed-metaphysics generally. To see this, we must take a closer look at manuscripts which follow.

2.3 Manuscripts between *PL* and *PM*

One difficulty with pursuing this line of inquiry is that it is difficult to date some of the manuscripts from this period, and so hard to get a good sense of the chronology. I shall focus my attention on three manuscripts, "Individuals", "Types" and "Fundamentals". Internal evidence strongly suggest these stem from roughly the same time (although I do not know precisely when that was), and later than *PL*, since that manuscript is referred to in one of them [43]. The first two of these seem to be early drafts of *PM*. Since the point at issue between Levine and myself is the nature of individuals, it makes sense to begin with the "Individuals" MS, which begins with a definition:

Such objects as constitute the real world as opposed to the world of logic. They may be defined as whatever can be the subject of any proposition not containing any apparent variable. [40]

On this definition, *if* predicates can be subjects of elementary propositions, *then* they would have to be individuals. But it would be hasty (and probably incorrect) to conclude from this that Russell is already back to thinking of predicates and relations as individuals. At this point, it seems instead that he would tollens rather than ponens this conditional. In other words, he concludes that predicates must always occur in a predicating position. This possibility is already mentioned in *PL*:

It is open to us to say that a predicate can only be predicated, and that propositions in which there is an appearance to the contrary

are concerned with words or ideas, not with their objects. If so, the question in what sense predicates have being cannot be raised, since it puts them in a position in which they are not predicated.

[24]

This makes Russell's work at the time more closely approximate his views in *PLA* than those in *ToK*, as Levine himself notes.

In the early parts of the "Types" manuscript, this line of thinking is pursued, and takes a rather strange twist. His conclusion is *not* that because predicates and relations must occur in a predicating or relating way that there is *no* sense in which they can be subjects of *any* propositions, but rather that the propositions of which they can be regarded as subjects must be higher order. This is then compatible with their not being individuals according to the above definition, given the restriction to elementary propositions. Here, a comparison to Frege is very helpful. For Frege, a concept or other function is essentially predicative, and symbols for them cannot stand alone in a non-functional position. Function expressions are "incomplete" and thus not meaningful in isolation. But this does not mean that there is no sense in which they can be subjects of propositions or have concepts apply to them. It is just that when they are, the concepts that apply to them are higher-order, and the expression for these higher-order concepts must make use of a *bound variable* which can occupy the argument position of the lower-order concept expression, so that the lower order concept, despite being the *argument* to the higher-order expression, is nevertheless still occurring predicatively. Thus " $\vdash^a \text{---} F(a)$ " can be understood as claiming *of* the concept $F()$ that it "falls within" the second-order concept represented by the quantifier, but it does this without putting $F()$ into a *pure* subject position; its predicative nature is respected.

Something similar could be said of any proposition of the form " $\vdash \text{---} M_\beta(F(\beta))$ " in Frege's logic.

In "Types", Russell seems again to be experimenting with a quasi-Fregean perspective regarding predication and functionality. If predicates and relations always occur in predicating or relating positions, then he seems willing to identify them with the constant part of the various values of a predicative propositional function, the ϕ part of $\phi!\hat{x}$. He writes:

A function must be an incomplete symbol. This seems to follow from the fact that $\phi!(\phi!\hat{z})$ is nonsense. The whole difficulty lies in reconciling this with the fact that a function can be an apparent

variable. It would seem the only *complete* symbols are individuals and asserted propositions.

We may, if we choose, say that in “ $\phi!x$ ”, the ϕ is a predicate. Then we shall have to say that the ways in which ϕ can occur significantly differ from those in which x can, and there are *no* occurrences possible for both. This seems reasonable. What meaning can we give to $\phi!\hat{x}$? This is only wanted as argument, and primarily in $f!(\phi!\hat{x})$. We may regard this as again a subject-predicate proposition. An *individual* is what can't occur to the left of a shriek.

...

...

How about double functions? It would seem here again that we want a new Π , that of a relation in intension. Then $\phi!(x,y)$ is a proposition stating the intensional relation ϕ between x and y .

[43]

Russell agrees with the Fregean perspective that a predicate or relation expression is not “complete”, though it is not perhaps clear that Russell thinks of this in quite the same way as Frege thinks of functions as *ungesättigt*. In Russell's mouth, the label “incomplete symbol” makes it sound as if function expressions are to be regarded as not having their own semantic values at all, and hence, that while it might be appropriate in some sense to regard the propositions which are values of second-order functions for a first-order function as argument as having “subject-predicate” form, the appearance of an *entity* that is the “subject” is misleading.

There is something about this experiment with a quasi-Fregean approach which I think did survive into the period of *PM*, and I shall try to explain what that is in sec. 6 below. But what did not survive intact was the *more-than-just-quasi-Fregean* idea that predicates and relations can basically be thought of as functions. As I and others have argued elsewhere, and Levine seems to accept, there is plenty of internal evidence in *PM* and other writings of 1910–1912, that Russell in no way equated predicates or relations in that period with anything functional, and explicitly claimed that predicates and relations were capable of occurring in complexes but not in a predicating or relating way.

And indeed, it is evident even these manuscripts that Russell's old doubts about the Fregean object/concept hierarchy have not been forgotten, and represented some of the driving forces which eventually pushed Russell away from this position. Recall in particular that Russell had called Frege's doctrine self-stultifying since one could not claim that something was not an object

without making that something into an object, i.e., a logical subject [3, p. 134]. More generally, it seems to be a requirement of any theory of logical types that it be expressible, and in a non-trivial way. If entities fall into a given type, a proposition asserting that they fall into this type must be significant and true. If entities do not fall into a given type, it must be significant and true to say that they do not, and hence significant and false to say that they do.

In discussing how his own interpretation of Russell's views during the time of *PM* fare with regard to such problems, Levine stresses that because, on his view, all types of entities are capable of occurring as subject, it avoids the "apparent self-contradiction" that would be involved in the necessity of countenancing propositions that assert *about* something that *it* cannot be a logical subject (i.e., that no propositions can be *about* it). Since every entity can be a subject, one can form a proposition asserting that this entity has the type it has. However, as he notes, this gives us at most half of what we need. The claim about the type of something becomes pleonastic—it must be true, if it is significant. One cannot claim of an entity in one type that it is *not* a member of some other type, at least not while meaning by it the same thing as one would mean in a positive type-theoretic claim made about an entity actually in that other type. There is then something deeply unsatisfying at leaving things like this, and there is very good reason to think it would not have satisfied Russell, and it quite explicitly did not satisfy him during this early period.⁴

Indeed, Russell is explicit during this period that he believes that it must always be *somehow* significant to *deny* that something is in a distinct type from the one that it is in, and hence also *somehow* significant, albeit false, to affirm that it is. E.g., of the claim that something is not a class, Russell writes:

It seems preposterous to maintain that " $x \sim \epsilon Cls$ " is meaningless. On the contrary, it seems that the assertion that an entity is of this or that type *is* significant under all circumstances, though most other assertions are not. The sort of assertion which remains significant is an assertion as to significance. Thus " $x \epsilon \alpha$ " is significant" will be significant for *all* values of x and α . [43]

Now this line of thought seems plainly at odds with his earlier conclusion in the same manuscript that there are "*no* occurrences possible for both" x and ϕ .

⁴Part of Levine's willingness to believe that it *did* satisfy (or at least placate) Russell at the time of *PM*, however, comes by way of comparing things to the attitudes expressed in *63 and *102 of *PM*. This argument deserves an answer. However, the issue is complicated, and I cannot do it justice, especially as it requires saying a lot more than I can here about the syntactic conventions of *PM*, and the device of typical ambiguity in particular.

The line of response Russell suggests, reminiscent of much later writings (e.g. [30, 268–69]) involves insisting that a claim about what is significant or not is first and foremost a claim about the meaningfulness of symbols, rather than about what they symbolize. The above passage continues:

But when $x \in \alpha$ occurs in the ordinary way, it is the proposition, not the symbol, that is meant; whereas when we say “ $x \in \alpha$ is significant”, it is the symbol, not the proposition, that is meant. We need, therefore, as regards types, a new kind of proposition, namely one concerned with *significance*. [43]

Now at least with regard to a *metaphysical* theory of types, whereupon the reason that certain symbols are not significantly interchangeable within a statement is because the entities they represent are not interchangeable within metaphysical complexes of which *they* are parts, this response too is not very satisfying. There seems to be some metaphysical feature of reality that is not adequately expressed by only saying something about symbols.

Hints that Russell is aware of this problem are evident in the very same manuscript. Recall that on the view initially explored there, predicates can only occur predicatively in first-order propositions. This would seem to make “humanity is human” nonsense. Regarding this, Russell writes:

We want a clearer theory of *significance*.

Suppose we say “‘humanity is human’ is not significant”. In this case it is the *phrase* that is not significant; so far so good. But although primarily the phrase alone is concerned, what is said does indicate some genuine proposition about humanity, though it is hard to see what this proposition is. The fact seems to be that *humanity*, as opposed to *human*, is a mere word, and that *human* can only occur significantly as a predicate. This statement can be made well enough about the *word*, but not about the *thing*, for in making it we use *human* otherwise than as a predicate. The proper statement is:

“All propositions in whose verbal expressions the word *human* occurs are either about the word itself or have the word in a position appropriate to predicates.”

If this is true, the proposition “humanity is a predicate” is incapable of any except a grammatical meaning.

Thus if we say “ α can only occur in the form $x \in \alpha$ ”, what we mean is: “What is signified by the word or symbol α can only occur in the form $x \in \alpha$ ”. . . . [43]

Russell seems aware of the puzzle here, and does not seem entirely happy with the view he feels forced into. He seems to think there is something about humanity itself, and not the word, which the claim about significance connects with, even though it is “primarily” about the phrase. But this seems impossible: by his own lights, nothing can be *about* humanity itself as a *pure* logical subject, only the *word* “humanity” can be a logical subject in this pure sense. What could this “hard to see” proposition possibly be?

A related problem arises for the second example Russell gives here. Russell gives a linguistic analysis of what it means to say that α can only occur in certain places in a form. Notice that the analysans here, although it is about the symbol, it is so only in virtue of being about *what is signified by the symbol*. But then the analysis is no improvement. How are the words “what is signified by the word or symbol α ” meaningful here? These words either directly represent, or, more likely, serve as a definite description for some entity, and the rest of the analysans seems to say something about that entity which it could not say if it were true. Suppose it means:

$$(\exists \beta)[(\gamma)(\alpha \text{ signifies } \gamma \equiv \gamma = \beta) \ \& \ \beta \text{ can only occur in the form } x \in \beta]$$

This too is self-stultifying. Even if ‘ α ’ signifies something uniquely, it is entirely unclear how something could satisfy the final conjunct non-paradoxically. Holding that type-theoretical language is concerned with symbols would seem to be an adequate response to the difficulties if it could be interpreted as being *only* about symbols, and not about what symbols mean.

In the manuscript “Fundamentals” on this period, Russell acknowledges the problem again, but this time he also suggests a way of possibly overcoming it.

But take say “ $x \in \alpha$ is significant”. This must be significant when false, *i.e.*, when $x \in \alpha$ is not significant. Hence it must be *always* significant; unless “ $x \in \alpha$ is significant” has itself some range of significance falling short of everything, but exceeding that of $x \in \alpha$.

Significance is of course primarily a property of the symbols, not of what they symbolise. But it is hard to believe that there is no corresponding property of the things symbolised; unless, as in the no-classes theory there *are* no things symbolised at all in the cases where significance fails. This is the strong point of the no-classes theory; its weak point is having to take functions as apparent variables. [41]

This begins much as the previous statement of the problem. Statements about types, i.e., about what may occur significantly where, are “primarily” statements about symbols. If those symbols are *genuine* symbols, it is hard to escape the conclusion that something about the things themselves must correspond, but this does not seem possible. The puzzle is avoided if the symbols are not actually symbols *for* anything on their own, i.e., if they are incomplete symbols, as class terms are in the no classes theory. If there aren’t *really* such things as classes, then there is no “deep metaphysical facts” about these things, classes, which “corresponds” somehow to the semantic fact that terms for them cannot meaningfully be placed in a position where the name of an individual ought to go, or vice-versa.

What Russell here calls the “strong point” of the no classes theory seems to be that it renders the “symbolic” interpretation of type theory unobjectionable. Since there is no metaphysical hierarchy underlying the symbolic hierarchy, by insisting that statements of significance are only about symbols, one is not left with the unshakeable feeling that there is some “hard to see” or ineffable fact about what is symbolized which by the very nature of the theory itself, cannot be expressed. This seems to be the message underlying the exclamation made by Russell later in [41], viz., “Types won’t work without no-classes. Don’t forget this.”

But as the longer quotation above also shows, Russell is aware that having a no classes theory cannot by itself solve the problem. The no classes theory, as usually formulated, requires quantifying over propositional functions, and, as was clear from *PL*, Russell at this point does seem moved by the consideration what what can be quantified over must have some kind of “being”. Hence, the “weak point” of the no class theory is that it seems to make it difficult for a symbolic interpretation of type theory for functions to be unobjectionable as well. This sheds light on the opening passage of “Types”. Russell feels that because functions are type-restrictive, functions (or really function expressions) must be “incomplete symbols”—and sets for himself the task of trying to reconcile this with the need for quantifying over them.

So at this point, Russell has only made clearer what his task is, without yet completing it. Much of the remainder of these manuscripts, like *PL* before them, are concerned with further tentative proposals and “logical experiments”, most of which are not recognizable in Russell’s later work. (One rather striking one, in [41], even involves denying the law of the excluded middle.) The substitutional theory, no doubt still attractive in its prospects of eliminating the need for apparent variables for functions, is also reconsid-

ered, though there the issue becomes how to make sense of quantification over propositions while maintaining that they are not all of the same type or “order”. Russell has not yet settled his mind on exactly how it is possible to have apparent variables for propositions or propositional functions without granting them some kind of being, but nonetheless the lesson seems to have been that type-theories are philosophically preferable when they can be interpreted symbolically rather than metaphysically. The natural move here is to understand propositions too as “incomplete symbols”, so that a symbolic interpretation of types of propositions can be sanguine, and one finds an early version of the multiple relations theory of judgment offered in “Types” as well. Once Russell begins down this road, the quasi-Fregean view of predicates which Russell had begun to explore in *PL* whereupon they are not individuals is dead in the water. Obviously, and as Levine himself argues, no sense at all can be made of the multiple relations theory of judgment unless predicates and relations are thought of as having a two-fold nature, and can occur in complexes even in non-predicating and non-relating ways. At least while maintaining the definition of individual in the “Individuals” manuscript, Russell must then switch his tollens back to a ponens: by it, universals are once again individuals.

In conclusion then, while these manuscripts do show some “looseness” to Russell’s conception of individuals in that he was willing to consider a view whereupon universals were not individuals, it is not the kind of looseness that would prefigure Levine’s reading of *PM*: it is only because he temporarily did not take universals to be possible subjects in non-quantified propositions that he was willing to consider that possibility. Moreover, many of the misgivings Russell held with the approach show a hostility to any kind of typed metaphysics, and he seemed to gravitate toward an understanding of type theory according to which it is only unobjectionable if symbols of types other than symbols for individuals don’t have their own semantic values. The trajectory of Russell’s thought seems to have been headed in a different direction from Levine’s reading of *PM*.

3 Published Writings

I cannot speak for Landini or Stevens, but when I claim that Russell’s inclination towards a type-free metaphysics in many ways drove the development of his thought prior to the influence of Wittgenstein, I do not mean to suggest that Russell never even considered anything else seriously. It is rather that,

when he did consider typed metaphysical views, he found them wanting, and so gravitated back in the other direction. If this is right, then it would make sense that these flirtations would be less pronounced in his published writings, where it would seem that Russell would only have endorsed something if he had a reasonable amount of confidence in it. Elsewhere [10, pp. 31–32], I have admitted that Russell flirts with at least certain kinds of metaphysical type distinctions, including in “Mathematical Logic As Based on the Theory of Types” and elsewhere. But Levine sees evidence against the general line of interpretation I favor in places I do not.

3.1 “The Existential Import of Propositions”

The first published work we seem to interpret rather differently is 1905’s “The Existential Import of Propositions” (*EIP*). For what it’s worth, Levine does not interpret that piece as endorsing a strongly typed metaphysics, but he does again see in it evidence that Russell did not at that time regard the category of “individual” as all-embracing. In *EIP*, Russell distinguishes two senses of existence: sense (a) characterizes items of the concrete, spatial-temporal realm; sense (b) characterizes those classes which have members or are non-empty. Levine, I think rightly, believes that it was Russell’s view at the time that all and only particulars exist in sense (a). But he also connects this with “individuals”, holding Russell as having suggested that all individuals are candidates for existence, thereby suggesting that universals are not individuals. But as near as I can tell, there is not much in *EIP* to support such a reading. The closest is perhaps the opening of the discussion of sense (a) existence.

(a) The meaning of *existence* which occurs in philosophy and in daily life is the meaning which can be predicated of an individual. ... The entities dealt with in mathematics do not exist in this sense: the number 2, or the principle of the syllogism, or multiplication are objects which mathematics considers, but which certainly form no part of the world of existent things. [20, p. 98]

But surely to say that this meaning of existence *can* be predicated of an individual is not the same as to say that *all* propositions which predicate existence to individuals are true! This passage is completely consistent with holding that some individuals exist in sense (a), and some do not. Russell contrasts individuals and classes in *EIP*, but not individuals and non-existent entities. The reason I believe Russell uses the word “individual” here is to underscore the difference between sense (a) and sense (b). Sense (b) is only ever truly

predicated of a class, whereas sense (a) is *sometimes* truly predicated of an individual. But this does not mean that it *always* is, just like sense (b) of existence is sometimes (but not always) truly predicated of a class. Russell does claim that there are no unreal individuals, but he is also explicit that “real things” comprise both things that exist in sense (a) and things that do not [20, p. 99]. So I see no basis for thinking that Russell’s terminology in *EIP* excludes universals from the category of individuals.

Let me note in passing a odd result that Levine’s interpretation of *EIP* leads to, but mine avoids. Russell is explicit that he does not think that the notion of existence in sense (a) has any particular logical importance. “This sense of existence lies wholly outside Symbolic Logic, which does not care a pin whether its entities exist in this sense or not” (p. 98). As far as Logic is concerned, existence is just one predicate among many. But on Levine’s reading, the category of existents is coextensive with the category of individuals, and surely Russell must hold there to be some *a priori* reason to believe this. Then, either individuality and existence are the same thing, or there is some other kind of conceptual connection between them. But then it would seem that the notion of individual could not be an important logical notion either. But if the notion of individual is not one especially interesting to the logician, it is hard to see what could be. This result is awkward. Thankfully, I think there is no compelling reason to read *EIP* this way.

3.2 “On the Relations of Universals and Particulars”

Another published work—clearly more central to Levine’s argument—which we interpret differently is “On the Relations of Universals and Particulars” (*RUP*) (written in 1911). As Levine reminds us, both there and in some other works of the period, Russell considers two views. On one view, it is possible for a complex to consist solely of two entities, one being the subject, the other being a predicate, where the predicate itself occurs “as a verb”, i.e., what nowadays we might call, with some abuse of ordinary language, a monadic relation. On the other view, even the simplest kind of proposition will always involve a relation and multiple relata, and even the proposition “this is white”, where “this” names some simple particular, is to be analyzed as involving a relation of predication meant by the copula. Levine finds in certain things Russell claims about this view evidence for the conclusion that “predicates and particulars are entities of different logical types—so that no particular can occupy a ‘position’ that predicates can occupy and vice-versa”. The primary basis for this conclusion seems to be the passage where Russell writes:

Predication is a relation involving a fundamental logical difference between its two terms. Predicates may themselves have predicates, but the predicates of predicates will be radically different from the predicates of substances. [28, p. 181]

But it does not seem right to me that this passage can be read as evidence in favor of Levine's conclusion. We may test this in the following way: does this passage conflict with the account given of the copula in *PoM*, where it is *clear* (and Levine agrees) that Russell explicitly rejects the conclusion that no particular ("thing" in *PoM*) can ever occupy the same position as a predicate? Let us take up the first sentence. In arguing that predicates are individuals for Russell, of course, I have not and would not deny that there is a very important logical difference between predicates and other individuals. This fundamental difference does involve what positions in complexes predicates, but not other entities, can occupy. Predicates and only predicates can occur, in the language of *PoM*, "as predicate", in subject-predicate propositions. Compare *PoM*:

In "Socrates is human," the notion expressed by *human* occurs in a different way from that in which it occurs when it is called *humanity*, the difference being that in the latter case, but not in the former, the proposition is *about* this notion. . . . It is a characteristic of the terms of a proposition that any one of them may be replaced by any other entity without our ceasing to have a proposition. Thus we shall say that "Socrates is human" is a proposition having only one term; of the remaining components of the proposition, one of which is the verb, the other is a *predicate*. With the sense which *is* has in this proposition, we no longer have a proposition at all if we replace *human* by something other than a predicate. [17, §48]

Notice two things here, (1) Russell does not think that the predicate is the verb of the proposition or that it occurs "as verb", and indeed, the verb is a separate constituent, and (2) *nonetheless*, the predicate is not a term of the complex and can only be replaced by another predicate. The verb in "Socrates is human", Russell tells us, is the copula. About the copula, the view of *PoM* holds:

We may perhaps say that it is a relation, although it is distinguished from other relations in that it does not permit itself to be regarded as an assertion concerning either of its terms indifferently, but only as an assertion concerning the referent.

The important difference for Russell is not, as Levine sometimes seems to suggest, the difference between things which occur "as verb" (i.e., as a relation)

versus those which occur as term, but rather between those which occur as term, and those which occur in some kind of predicative *or* relating way. When the relation in a complex is the one indicated by “is”, which we can safely take to be a relation of predication,⁵ the two halves of the relation still occur in that complex in logically different ways; the subject occurs as term, the predicate occurs as predicate, and only the subject occurs in a way that would make it replaceable by any other individual. With regard to this very unique relation, there is a logical difference between its two relata positions. It is perfectly possible to read *RUP*’s “fundamental difference between the two terms” of the relation of predication as nothing other than the “non-indifference” between referent and relatum already acknowledged in *PoM*. Returning to the issue at hand, if Levine reads *RUP* merely as saying that a name of a particular cannot replace “white” meaningfully in “this is white”, then he is on solid ground. If he reads it instead as some indication that “whiteness” cannot meaningfully replace “this”, then his conclusion seems unwarranted. Whiteness is not white, so there is no such *complex* as whiteness being white, but that does not mean that the proposition cannot be formulated.

But perhaps Levine would here turn to the *next* sentence in the passage from *RUP*, to the effect that the predicates of predicates are “radically different” from the predicates of substances/particulars. There are two obviously different interpretations of this sentence, which we can call the “truth reading” and the “significance reading”. On the truth reading, it means that the predicates which can be *truly* predicated of predicates are distinct from those that can be *truly* predicated of particulars. This is my reading. On the significance reading, it means that the predicates which can be significantly or meaningfully predicated of predicates are distinct from those that can be meaningfully

⁵Levine rightly points out that in addition to the copula indicated by “is”, in *PoM*, Russell also speaks of a relation indicated by “has” or “is-a”, which takes as its second argument a predicate occurring as term, so that it may be replaced by any other term significantly, and usually seems to have in mind this latter relation when speaking of the “relation of predication” in *PoM*. Levine seems to think this relation is closer to the one under discussion in *RUP*, rather than the unusual relation indicated by “is”. Since the primary difference between the two, however, seems to involve whether it is logically possible for a thing to be substituted for the predicate which occurs as the second relatum to the relation, and on Levine’s view, this is never possible on Russell’s later views, it is hard to see how the difference hasn’t been obliterated. In any case, it does not seem inappropriate to use the phrase “relation of predication” for either one, and it appears to be the standard copula Russell is discussing in *RUP*. While considering (p. 182) the view that there is no fundamental predication relation, as an aside Russell writes that it contrasts with one that takes ‘This is white’ to involve a relation of a particular to whiteness, making it clear that his discussion was meant to be relevant to the copula “is”.

predicated of particulars. Advocates of the significance interpretation would presumably generalize the point, so that the relations which can meaningfully be said to hold between particulars would be distinguished from those which can meaningfully be said to hold between predicates and other universals, and so on. This, I take it, is Levine's reading since it is the one that leads to something like a type-hierarchy for particulars and universals.

In fairness, I don't think the exact wording of the sentence is sufficient to make clear which of these two interpretations is right. Indeed, I don't think that the *immediate* context surrounding it makes it plain either. But when we consider the *completely disastrous results* the significance reading would have for not only the conclusions of *RUP* but the very intellectual project it undertakes, the matter becomes clear. What are these? Russell summarized them in a letter to Lady Ottoline Morrell (quoted at [44, p. 164]) as follows:

I have been engaged in debating whether there is any sense in which it is true that a thing can't be in two places at once, and I believe I have at last found a sense. The question of particulars and universals, which is the one I am concerned with, turns upon it: space is *the* particularizer. ... There are three possible views (a) there are only particulars—this is held by Berkeley and Hume, and is demonstrably false; (b) there are only universals—this is held by the American realists ... (c) there are both, which is my view and Moore's ... My problem has been to find arguments for (c) against (b), and also to state what is the difference between particulars and universals, which is no easy matter.

Now let us assume that Levine were right that Russell accepts a kind of difference in type that makes it impossible or meaningless to assert the same kinds of things about universals as one can meaningfully assert about particulars. Then it seems as if Russell is already committed on grounds of logical grammar alone to the inverse of nearly all these views. Consider the empiricist view that there are only particulars. The "there are" here would be interpreted as involving a variable which must be internally limited in its range to particulars, so in the sense in which it is meaningful to say that there are only particulars, it's true, so Russell ought not to argue against (a). On similar grounds, Russell also ought not to argue against (b).⁶ And far from trying to find arguments in

⁶Perhaps Levine would resist treating (a) and (b) in complete parallel, since, as I discuss later, he thinks that universals resist "symbolic treatment", and hence, perhaps universals cannot be quantified over. But it still holds that Russell ought not to argue against it—how can you argue against a proposition you cannot formulate?

favor of (c), Russell ought to be pointing out that no interpretation of “there are” could range over entities of distinct logical types, and so (c) is nothing but a meaningless pseudo-proposition. Similarly, rather than attempting to “state what the difference is between particulars and universals”, he should be pointing out the impossibility or futility of that task, since nothing which is true of universals is even meaningful of particulars, and vice-versa, so no one feature one has but the other lacks could be the distinguishing characteristic.

I am probably being unfair. Levine is not the first, and will not be the last, to suggest that Russell was committed to certain philosophical and metaphysical theses which would be difficult to analyze if his views about types were really correct. Perhaps Russell, like Frege, would ask for our indulgence or a pinch of salt, so that through misusing words we could get a kind of understanding which is not felicitously represented by the misleading logical structure which ordinary language wants to force upon it. However, one must consider what the specific theses of *RUP* are: what *does* Russell think the most important and interesting difference is between universals and particulars, and is it plausible to suppose that it ought to be difficult to express this difference in a felicitous way? In *RUP*, Russell identifies four key differences between universals and particulars, but makes it plain that the one he thinks is “really the most important one” (his exact words to Ottoline in another letter [44, p. 165]) is that universals can be in multiple or no places at once, while particulars always occupy one and only one position.

We may now return to the question of particulars and universals with a better hope of being able to state precisely the nature of the opposition between them. ... in the course of our discussion a[n] ... opposition developed itself, namely ... that between entities can be in one place, but not in more than one, at a given time, and entities which either cannot be anywhere or can be in several places at one time. What makes a particular patch of white particular, whereas whiteness is a universal, is the fact that whiteness, if it exists at all, exists wherever there are white things. [28, p. 180]

Russell makes it clear in the final paragraph of the essay that he regards the two halves of this opposition as at least co-extensional with the more logically formulated opposition between substantives and verbs/predicates. If the latter division is really the kind of theory of types Levine imagines, consider what we are being asked to swallow. Since it is one of their defining features, it must be meaningful and true to say of particulars that they exist in exactly one place. Since it is one of their defining features, it must be meaningful and

true to say of universals that they exist in zero or more than one place. But now, on Levine's reading, the positions that can be occupied in significant propositions by (expressions for) particulars never overlap those which can be occupied by (expressions for) universals. One might have naturally supposed that if it is meaningful and true to say of some particular *A* that it exists at *only one* place, then it must logically follow that it is false that it exists at zero or multiple places. However, in fact, there is no such entailment, since the very hypothesis that it exists at zero or multiple places is nonsense, since that's the kind of thing that can only be said of universals. Similarly, although it is true to say that whiteness exists at *more than one* location, it is not false, but meaningless, to say that it exists at *only one* location, since that's the kind of thing we could say about a particular. I can't help but feel that the principle of charity alone makes this interpretation rather problematic.

4 What is an Individual?

4.1 The One and The Many

Levine correctly recognizes that the issue as to whether, in the disputed period, Russell understood universals as individuals requires getting clearer about what Russell's conception of an individual is. But it seems to me that Levine is rather too selective about what claims about what individuals are he is willing to take at face value, especially those claims made during the period about which we disagree. However, let us start further back.

In §47 of *PoM*, Russell claims that the notion of "term" has an absolute unrestricted extension, and further claims that it is synonymous with "unit", "individual", and "entity". If "individual" truly is synonymous with these others, then attempting to ascertain whether or not Russell's metaphysics countenanced entities other than individuals is like asking whether the room contains any bachelors who aren't unmarried men. But this is a tad unfair. One cannot help suspect that "synonymous" may be too strong there—the important thing is that the notion of concept *individual*, like that of term, has an extension under which everything falls. The particular intensional significance of "individual"—as is suggested by its etymology—seems to be largely contained in the insight that an individual can be individuated apart from every other thing except itself: it is "what can be counted as one" rather than many. The word whose meaning Russell most steadily and frequently contrasts with that of "individual" is "class", which should be no surprise since

for Russell, both early and late,⁷ understanding the nature of plurality, or the “many”, is taken as bound up in understanding the nature of classes.

In *PoM* (§70), Russell makes a distinction between “classes as one” (or classes as wholes) and “classes as many”. I think Russell’s terminology here is apt to make his views seem more complicated than they really are. The thought seems to be that if we think of some things, plural, if we predicate something of them plurally, one way to describe the predication is that we have ascribed something to a “class as many”, whereas if we take them all together to make up one thing, what we have predicated something of is a “class as one”. It is clear that the view Russell *wanted* to have in *PoM* is that for any somethings, plural, that is, for every class as many, there is a corresponding class as one. The class as one, as its name implies, is an individual. A class as many however is not a different kind of thing from its members at all. It simply is its members, or better, its members are all there are for it to be. The paradox of classes, however, led Russell to the conclusion that this view was untenable and for certain somethings, they make up only a they, not an it, i.e., some classes as many are irredeemably plural (§101). Russell’s first theory of types, from Appendix B of *PoM*, is an elaboration on this idea: propositions can either be about individuals singularly, or instead about them plurally, or plurally plurally, and so on. If one were to reconstruct this theory in a modern logical calculus, it would be more appropriate to treat it as a “plural logic” of the likes endorsed by philosophers such as Boolos [1] or McKay [13], rather than as the kind of type-stratified higher-order logic familiar from the works of Church et al. Now on this view, a specific predicate or relation is still just one thing, so it is an individual as opposed to a class—indeed, Russell claims that “predicates are individuals” explicitly in Appendix B itself (§499).

Now this theory of types is already pitched as something that is still at some level compatible with the intuition that each entity is on its own one thing and no more, and thus an individual. A class as many is not an individual, but that is because it is not something, it is somethings (or rather, *they* are some things). As Russell put it at the time, “[f]or although whatever is, is one, it is also equally true that whatever are, are many” (§127). Russell summarized what his views had been during this period in a letter to Jourdain in 1906 thusly:

You will see that in my book [*PoM*] (p. 104, art. 104) I suggest that certain functions do not determine a *class as one*. This is practically

⁷Compare, e.g., [17, §497] with the much later [32, p. 181].

the same doctrine as that they do not determine a class, for a *class as many* is not an entity. [4, p. 78]

From this perspective even this theory represents a kind of attempt to have types without types of entities. Russell however was not completely satisfied with it, and by the time *PoM* appeared in print Russell was already experimenting with various kinds of replacement.⁸

From 1903, many, though not all, of these experiments proceed from the assumption that the key for understanding classes involves understanding their relationship to their defining propositional functions. In mid-1903, this involved simply eschewing classes in favor of functions. Later, as in much of 1904, this involved attempting to get a better grip about how propositional functions determine, or really, how denoting complexes logically built from propositional functions denote, classes (whence Russell's interest in the denoting relation during these years). After classes, the word Russell is next most likely to contrast with "individual" is "function". This is in a way stranger, given that if a function is to be thought of as any kind of metaphysically real entity, it would seem to be one entity, not many, just like a predicate. Nonetheless, Russell's interest in functions is itself thoroughly tied to his attempt to explain how it is talk about classes, i.e., talk about the many, is to be explained. This is true both early and late. Some of Russell's first protracted attempts to provide a symbolic treatment of the nature of functions, as we have seen, seems to have been inspired by Frege's notion of Wertverläufe, or "ranges" of functions,⁹ which, when they were propositional, Frege identifies with extensions of concepts (see my [6]). In *PM* itself, a large philosophical battalion is mustered as a complicated type-regimented hierarchy of functions, only to have their explicit use almost completely disappear once the contextual definition of classes in terms of them is in place.

Russell seems to have thought that the problem of the one and the many could only be solved by a philosophy that did justice both to the seeming truism that each thing, each being, is only one being, or that nothing is more than one, and to the obvious fact that plurality and number (the subject matter of mathematics itself) are not in light of this impossible. This topic is a key place of overlap between Russell's logical work on the foundations of mathematics and his protracted metaphysical turf war with the idealist monists. Briefly,

⁸This is of course already clear from the last minute footnote added to the final page of Appendix A.

⁹The usual translations are "value-range" and "course-of-values"; "range" is Russell's translation.

during the honeymoon period of the substitutional theory, Russell had concluded that he had solved it by taking only propositions and their constituents as real, and constructing both classes and functions out of them.

Of the philosophical consequences of the theory I will say nothing, beyond pointing out that it affords what at least seems to be a complete solution of all the hoary difficulties about the one and the many; for, while allowing that there are many entities, it adheres with drastic pedantry to the old maxim that, 'whatever is, is one'.
[23, 189]

When the issue is framed in terms of these hoary problems, Russell's resistance to any theory that did not take the category of individuals as all-encompassing is understandable, even though he did certainly flirt with and explore alternative hypotheses. The claim that something is not an individual is roughly the hypothesis that the old maxim is false, i.e., that there is something which is not *one* thing, and then it is hard to see how it could be *a* thing or any kind of being at all.

We can then test whether or not Russell still held the theory that the category of individuals is all-embracing by examining his attitude towards whether or not he thought the philosophy underwriting his work in mathematical logic was consistent with this old maxim. Here, the debates between Russell and Bradley are instructive. Bradley published a piece critical of Russell's views in *PoM* in a paper in *Mind* from 1910 [2]. Among Bradley's worries about *PoM*'s metaphysics was the apparent contradiction in taking a class somehow to be both one and many. Russell first responded to Bradley in a letter, from April 1910 this way:

With regard to the conception of "class", I admitted in my *Principles* that I had not yet found a satisfactory theory, but I believe now that I have found a satisfactory theory, enabling me to interpret propositions verbally concerned with classes without assuming that there are classes. A class is many, and therefore not one; but I accept the principle "ens et unum convertuntur". By wholly denying the reality of classes, the dilemma is avoided. ... It appears to be implied in any consistent pluralism, that, though there are are many things, there is nothing which is many. These two statements now appear to me to be reconcilable, and I therefore accept both. [44, p. 350]

So in 1910, Russell continued to believe that each entity is one thing; classes must be treated as merely ways of speaking (in French, *façons de parler*), and their reality “wholly denied” in order to reconcile pluralism with the old maxim that each thing is just its own individual thing. Very little seems to have changed since the closing epigram of [23].

Russell’s more “official” reply was published in the same journal in the next issue. There Russell expresses more or less the same points as above, but also goes on to connect them explicitly with issues concerning types and restrictions on meaningfulness:

As regards what Mr. Bradley says about the idea of “class”, I find myself very largely in agreement with him. The theory of classes which I set forth in my *Principles* was avowedly unsatisfactory. I did not, at that time, see any way of stating the elementary propositions of Arithmetic without employing the notion of “class”. I have, however, since that time discovered that it is possible to give an interpretation to all propositions which verbally employ classes, without assuming that there really are such things as classes at all. Apart from other contradictions, the fact that a class, if there is such a thing, must be both one and many constitutes a difficulty. That it is meaningless (as Mr. Bradley contends) to regard a class as being or not being a member of itself, must be assumed for the avoidance of a more mathematical contradiction; but I cannot see that this could be meaningless if there were such things as classes. The theory that there are no such things as classes avoids at once the difficulties raised by Mr. Bradley and the difficulties with which I endeavour to contend in the *Principles*. The general contention that classes are a mere *façon de parler* has, of course, been often advanced, but it has not been accompanied by an exact account of what this manner of speaking really means, or by an interpretation of arithmetic in accordance with this contention; and such an accompaniment was essential before a philosophy of mathematics could dispense with classes.

I interpret this passage as simply a more prolix version of the “Fundamentals” manuscript’s aphorism, “Types won’t work without no-classes.” If classes were in *any sense* genuine things, Russell would not be able to make sense of how it is that the same things that could be said meaningfully about any other individual could not also be said about classes. This is a ringing endorsement

of the principle that there is an unequivocal notion of *genuine* logical subject which is coextensive with that of individual.

4.2 Definitions of Individuals in the *PM* Period

As we have seen, for Russell, classes do not pose a counterexample to the principle that the domain of individuals is all-embracing because classes aren't actually there; discourse about classes is just a linguistic convenience. But what of universals? They too are involved in, as Bradley put it, "the old problem of the universal, and of the one in the many, and the dilemmas which everywhere arise, change their particular shape but not their radical essence" [2, p. 391]. Levine and I, and presumably any other reader of Russell's works from this period, would agree that Russell's solution to the "problem of the universal" is not to think that universals are nothing more than linguistic conveniences. Universals are there all right. But are they individuals for the Russell of this period?

Here are a number of passages from this period which are instructive in understanding what the possibilities could be. Most of these Levine quotes either in the body of the paper, or in the footnotes, but I'll throw in a couple more, and add letters and dates to refer back to them later:

(A) (*ML*: 1908) In an elementary proposition we can distinguish one or more *terms* from one or more *concepts*; the *terms* are whatever can be regarded as the *subject* of the proposition, while the concepts are the predicates or relations asserted of these terms. The terms of elementary propositions we will call *individuals*; these form the first or lowest type. [26, p. 76]

(B) (*ML*: 1908) By applying the process of generalization to individuals occurring in elementary propositions, we obtain new propositions. The legitimacy of this process requires only that no individuals should be propositions. That this is so, is to be secured by the meaning we give to the word *individual*. We may define an individual as something destitute of complexity; it is then obviously not a proposition, since propositions are essentially complex.

(C) (*PM*: 1910) By applying the process of generalization to individuals occurring in elementary propositions, we obtain new propositions. The legitimacy of this process requires only that no individuals should be propositions. That this is so, is to be secured

by the meaning we give to the word *individual*. We may explain an individual as something which exists on its own account; it is then obviously not a proposition, since propositions, as explained in Chapter II of the Introduction (p. 43), are incomplete symbols, having no meaning except in use. [46, p. 162]

(D) (PM: 1910) For this purpose, we will use such letters as a , b , x , y , z , w , to denote objects which are neither propositions nor functions. Such objects we shall call *individuals*. Such objects will be constituents of propositions or functions, and will be *genuine* constituents, in the sense that they do not disappear on analysis, as (for example) classes do, or phrases of the form “the so-and-so.” [46, p. 51]

(E) (PM: 1910) Thus, giving the name “individual” to whatever there is that is neither a proposition, nor a function, the proposition “every individual is identical with itself” or the proposition “there are individuals” will be a proposition belonging to logic. [46, p. 93]

(F) (PM: 1910) *Primitive Idea: Individual*. We say that x is an “individual” if x is neither a proposition nor a function (cf. p. 51).

(G) (AIT: 1911) *The axiom of infinity* is formulated as follows: *If n is any finite cardinal number, there is a set consisting of n individuals*. Here the word *individual* contrasts with class, function, proposition, etc. In other words, *an individual is a being in the actual world, as opposed to the beings in the world of logic.*¹⁰

If we are to take these passages at all literally or at face-value, then it seems that in light of (B), (D), (E), (F), if a universal α has any place in Russell’s philosophy, at least one of the following must be true of it: (1) α is an individual, (2) α is a (propositional) function, (3) α is a class, (4) α is a proposition, (5) α is something else complex.

Nothing we have seen rules out the possibility that some universals are complex. These definitions are compatible with *those* universals not being individuals. Indeed, quotation (B) would seem to *preclude* their being individuals. However, that is the earliest of these statements, and there is little question that Russell’s precise understanding of the relationship between

¹⁰I have revised the translation to match the opening of [40]. Levine, following Grattan-Guinness, also revises it. Our revisions are more similar to each other than either one is to the original, so hopefully this will not be a bone of contention.

simples and complexes had changed between 1907 and 1911.¹¹ But in any case, not *all* universals can be complex. Complexes depend on simples, and the simplest kind of complexes consist of terms and a relating relation or predicate. This is something Russell held unwaveringly throughout his post-Idealist career. Levine himself quotes passages from [24] where Russell explicitly calls predicates simple. Let us for the moment put aside complex universals and focus just on simple ones. What is Levine's view of them? Alternatives (5) and (6) clearly cannot be right for them. Levine seems to agree with arguments that I and others have made that (2) and (3) are not right either. But that leaves only (1), which it is the chief purpose of Levine's paper to call into question.

What prevents these passages from constituting an outright refutation of Levine's theses is Levine's excuse that Russell believed that universals were not "amenable to symbolic treatment" [23, p. 175]. Since many of the above passages are clearly given in the context of explaining the types of expressions and variables to occur in his formal language, and Levine interprets Russell as shying away from including any formal devices in his mathematical language for expressing propositions about universals, Levine concludes that Russell is off the hook when it comes to explaining where universals fall into the type system. But this excuse is entirely too convenient, and especially in the context of Levine's own interpretation, it proves entirely too much. I have a shallow worry here, and a deeper worry. The shallow worry is that, on Levine's own account of the subject matter of logic, which we'll discuss further in the next section, logic is about "the world of universals", "the world of being" (or subsistence) rather than "the world of (concrete) existents". But if logic is about universals, and universals do not admit of symbolic treatment, then it begins to look as if Poincaré was right: the very project of a symbolic logic is an absurdity. It is hard to swallow reading the greatest champion of symbolic logic in the history of philosophy as committed to *that*. In fairness, this worry is perhaps not very precisely stated, and I would not rest much weight on it.

A deeper problem is that on Levine's interpretation of the ways Russell's views had changed in the period after the 1906 remark about what is and is not amenable to symbolic treatment, they take away the only reason Russell gave for the attitude to begin with. In early 1906, under the auspices of the substitutional theory, Russell was still committed to the idea that the variables of logic must be completely unrestricted. With an unrestricted variable, the ban on

¹¹That much is evident in the obvious revision of (B) to become (C); for further explanation of the differences between Russell's views in [26], as compared to those in *PM*, see Landini [12, chap. 9] and my [10, p. 32].

replacing an expression for a relation in a relating position or for a predicate in a predicating position with a variable makes sense. Doing otherwise would run the risk of having propositional functions without values for certain assignments to the variable (those entities that cannot occupy the necessary positions). If the variable is unrestricted, it is impermissible to exclude those entities as legitimate assignments to the variable, but then one is left without an explanation of what the value of the function is in those instances.¹² If Levine read *PM* more similarly to the way Landini does, whereupon the doctrine of the unrestricted variable is in a sense still somehow enshrined, perhaps he could use this as an explanation for why there are no variables for universals introduced there. But Levine doesn't read *PM* this way. Instead, Levine reads *PM* as having a typed metaphysics according to which there is *no* kind of position in a complex which admits all possible substituends. If Levine's explanation for why Russell does not quantify over universals in *PM* is the threat of what would happen if the variable involved were to be instantiated to an expression for a particular, shouldn't Levine see just as large a problem for individual variables, which on his view, would lead to difficulties if instantiated to expressions for universals? His interpretation cannot explain why Russell is not similarly of the mind that individuals or particulars are not amenable to symbolic treatment.

Once again, Russell's own writings tell a different story. During the period immediately following "The Paradox of the Liar", in which, I have admitted, Russell did explore a typed metaphysics, Russell realizes that his earlier misgivings about having variables for predicates occurring in a predicative position would, on the view being explored, have to be given up. Thus in "Fundamentals" he considers possibly not having variables and quantifiers for propositional functions, but instead having them for predicates, but where the variables would be restricted to predicate position:

We may decline to vary a *general* ϕ , but introduce a restricted (*predicative*) ϕ , of the form " $x \epsilon \alpha$ " where α is a *predicate*. Then α can be varied. Then *no-classes* becomes "predicates can only be predicated"; and *reducibility* becomes "any propositional function of x is equivalent to the ascription of some predicate". [41]

This is not to say that Russell was happy with this approach. Immediately after writing the above, he begins to doubt it, calling into question whether it can

¹²As near as I can tell, this accords with Levine's own explanation of why Russell was chary of introducing variables for predicates and relations occurring predicatively or relationally.

accommodate those truths about predicates that don't seem to rely entirely on what instantiates them, such as the truth that *featherless biped* \neq *human*. And this leads into the very worries about significance that, in sec. 2.3, I suggested led to the abandonment of the approach. So the objections Russell has to having variables specifically for predicates are just the very doubts he has about a typed metaphysics generally. It seems clear that if Russell *were* willing, as Levine seems to think, to put aside his doubts about a type hierarchy of entities, he would also put aside his doubts about special quantifiers for these types.¹³

4.3 The Actual World and the World of Logic

Since Levine does not think that the category of individual is all-encompassing for Russell during this period, and specifically, that it excludes predicates and relations, he provides his own account of what is distinctive about individuals as opposed to other kinds of beings, basing it largely on how he interprets quotation (G), taken from a 1911 lecture Russell gave in Paris. According to Levine, Russell's distinction there between a "being in the actual world" and a "being of logic" is supposed to be the same as the distinction between that which *exists* in the philosophical sense, and that which merely has being, or subsists. Levine bases this on what he sees as a consistent theme in Russell's 1911 lectures given in Paris, in which Russell also contrasts "the world of existents" and "the world of particulars" from "the world of essences" and "the world of universals".

I am in full agreement with Levine that Russell does make a distinction between existence and mere being during this period, and this does seem to be an often neglected fact about the development of Russell's views. I also agree with him that according to this distinction, particulars exist, but universals merely have being or subsist. I am even willing to grant that *sometimes* when Russell speaks of "the actual world", he means the world of particulars. But I do not agree with him that this maps on to the distinction Russell is making in passage (G), and certainly cannot be what Russell intended in his other

¹³It might be added that the view that universals somehow resist being values of variables (and thus resist being arguments to propositional functions) is also hard to reconcile with later writings too. Consider the brief discussion of the relationship between necessity and propositional functions in "On the Notion of Cause", written in 1912, where Russell writes:

For example, 'if Socrates is a man, Socrates is mortal', is necessary if Socrates is chosen as argument, but not if *man* or *mortal* is chosen. [31, p. 134]

characterizations of individuals.

There are three general reasons to doubt whether the distinction Russell is drawing in (G) between the beings of the actual world and the beings of logic is the same as that between existent particulars and merely subsistent universals. Firstly, let us consider the ramifications reading the second distinction to be a distinction in logical type would have on its ability to play the philosophical role Russell intended this distinction to play. As Russell made plain when he introduced the distinction between being and existence in *PoM*, he thought it was acceptable for it to be a truism to say of something that it is, or that it has being. One need only mention something, that one has already shown that it is. But it was not acceptable in his mind to think of existence as a pleonastic predicate:

Existence, on the contrary, is the prerogative of some only amongst beings. To exist is to have a specific relation to existence—a relation, by the way, which existence itself does not have. This shows, incidentally, the weakness of the existential theory of judgment—the theory, that is, that every proposition is concerned with something that exists. For if this theory were true, it would still be true that existence itself is an entity, and it must be admitted that existence does not exist. ... For what does not exist must be something, or it would be meaningless to deny its existence ... [17, §427]

The divide between the existent and the non-existent is a *true divide*: some things of which existence can be meaningfully predicated exist, and some things do not. But on Levine's interpretation of Russell's *PM*-period logical grammar, there can be no non-pleonastic notion of existence. In the sense in which particulars can truly be said to exist, one cannot also say that universals do not. As Levine sees it, "x exists" is typically ambiguous, and on each of the disambiguations, the propositional function is either necessary or impossible (in Russell's sense of these words). To mention a particular, then *is* to show that it exists, merely in virtue of the logical category of the expression used. It is hard to see what could possibly be philosophically useful about the concept involved in any of these disambiguations, unless it is the *same* use that Russell things that *being* has, i.e., that "it marks dissent from various philosophies" [17, §48]. But Russell seems to want it to do more than that, especially in his epistemology, as Levine seems to acknowledge.

In fact, there is evidence that Russell continued to believe that some individuals existed and some individuals did not (in the same sense of existence), and indeed, that one of the individuals that did not exist was the predicate

(universal) of existence itself. This entails, if predication is a relation, that it is meaningful, albeit false, to claim that existence has this relation to itself, just as Russell had said in *PoM*. This issue also arises in the interchange with Bradley mentioned in the previous section. No doubt affronted by even the logical possibility of something being related to itself, as against the idealist doctrine that relations imply diversity, Bradley had written:

And as we have seen, he [Russell] advocates the doctrine that a term can be related to itself—a view which for the same reason I am forced to reject. In every instance adduced, such, for example, as “predicability is predicable,” I find (I would repeat) a distinction and difference, or else I find nothing. The reader will permit me perhaps to illustrate and explain this sentiment by the instance of “being”. I do not reject as meaningless such a judgment as “being is” or “is is”. I only insist that, in order to have a meaning, I must introduce distinction and diversity. I might, for instance, mean by such an assertion that only or merely being is and that anything else must be denied. I might wish to convey that after all, or whatever else it is, being still is. I might in the end mean that in “being” itself is the distinction and diversity of “what” and “that”, and might imply that either of these thus “is,” and yet that each of them is so different from “being” that our assertion “is is” may be significant. [2, p. 390]

And in a later footnote in which he is defending his own view that self-predication is impossible, Bradley adds:

There is an objection, raised by Mr. Russell (*Principles of Math.*, p. 450), that on this view you cannot say that “Reality is real” or that “Existence exists”. . . . With regard to “Existence exists,” once more, until I know exactly what it means, I can hardly reply. What I can say is this, that to place “Existence” itself within the sphere of existence would be clearly indefensible.[44, p. 351]

In an ironic twist, according to Levine’s interpretation of Russell’s views in the *PM* period, he ought more or less have agreed with Bradley. On his reading, one can make sense of “Existence exists”, but only if the notions of existence involved are disambiguated. The existence which is said to exist must be a distinct type, and thus not the same as, the existence which is predicated of it, and even once this is done, the claim is indefensible (i.e., false).

Russell's response to this was brief, and he seems to have interpreted Bradley in the footnote as having interpreted him as having said that existence in fact exists. In his 1910 letter, Russell's only reply is to attempt to correct this misapprehension:

I don't think I can have ever meant to say that existence exists (I have not got my book here). What I wanted to urge was that there are true judgments about existence, and that these are not existential, because existence does not exist. (I should no longer be prepared to lay much stress upon this argument as it stands.) [44, p. 350]

Strictly speaking, Russell is not telling Bradley here what his current views are, he's merely trying to clarify what he had meant in *PoM*. Moreover, he's explicitly less than fully confident that those views will withstand scrutiny. But the fact that he does not simply recant is also important. Indeed, despite his misgivings, Russell's view is *still* that the supposition that *existence does not exist* is both meaningful and true, and thus the proposition that *existence does exist* (or *existence exists*) is meaningful and false, even with the subject and predicate both being the selfsame universal. Levine's interpretation is incompatible with this reading of this reply. If Levine's reading is right, one is forced to wonder why Russell didn't respond by conceding to Bradley that "existence does not exist" needs disambiguation, or that there are many senses of existence, or that, because the predication relation involves a "fundamental logical difference" between its terms, true self-predication is indeed impossible.

A second reason for not reading the division in passage (G) between beings in the actual world and beings in the world of logic as coextensive with the division between particulars and universals stems from epistemology.

To see this, I must first ward off a misapprehension one might come to when connecting passage (G) to its context, which might look as if it favored Levine's reading. Russell is there assessing whether or not there are any logical grounds for accepting the axiom of infinity as true. Russell's answer is negative: whether or not there are infinitely many individuals turns out to be empirical, and cannot be solved on *a priori* grounds alone. One might come to the conclusion that Russell must have conceived the question as one about particulars, or existents, on the grounds that they are what is empirical and contingent, as opposed to timeless, purely rational and known independent from experience. But this seems to me to be too hasty of a conclusion, as I shall try to explain.

Russell explicitly claimed that without the axiom of infinity, most of mathematics would turn out to be trivial [25, p. 282]. In *PM*, as Levine notes, the infinity axiom is not really used as an “axiom”, but rather as a hypothesis, or antecedent on various results. The conditional used in *PM* is the material conditional, so if the antecedent is false, the whole conditional is trivially true. Without the infinity axiom, in the mathematical logic of *PM*, one cannot even obtain the fourth Peano postulate to the effect that no two natural numbers have the same successor.¹⁴ So without an infinity of individuals, not even the mathematics of the natural numbers is obtained outright, or in a non-hypothetical way. By not finding a proof, or at least a justification, for the axiom of infinity, Russell leaves his entire logicist project open to the charge that it has not been put on a secure enough of a foundation. Indeed, many of the most influential criticisms that have in fact been given of Russell’s philosophy of mathematics focus on this very sore spot (among others; see e.g., [50]: 130–31.) After years of what Russell himself describes as intellectual stalemate, struggling with “insolubilia” old and new, it is not plausible to suppose that he would have left any promising looking stones unturned. Those of us familiar with these mesmerizing but frustrating manuscripts he left behind know that he does seem to have experimented with nearly every theory imaginable.

Let us suppose that Levine is right and that logic is about universals, and math is logic: wouldn’t it be more natural to try to get infinity out of universals rather than particulars? In the “Paradox of the Liar” manuscript, where he does consider placing universals in a separate logical category from individuals, he considers *both* the question of whether there is *a priori* argumentation for an infinity of individuals and the question as to whether there is *a priori* reason to believe in an infinity of universals. He first comes to the conclusion that “no reason appears why the number of individuals in the universe should not be finite”, and then, in considering predicates, he writes:

It seems obvious that predicates have some kind of being, and that whatever kind they have is independent of existence. We ought to be able to make classes of predicates without assuming that anything exists. ... But, so far as I can see, there is no logical absurdity in supposing the total number of predicates and relations in intension to be finite. This elicits a new kind of empirical truth, one not concerned with existence in the narrower sense. To account for this, we shall have to say, I think, that we become acquainted with predicates, as well as with individuals, through

¹⁴See Russell’s own very accessible explanation of why this is the case in [32, chap. 13].

perception: what we perceive is that an individual has a predicate, and our acquaintance both with the individual and the predicate is derived by analysis from the one perception that the individual has the predicate. Thus all *acquaintance* is empirical; and no existence-theorem can be known without acquaintance, unless we can find some other way of discovering existence-theorems than by means of actual instances. [24]

Here Russell makes it clear that he thinks that our knowledge of the existence of universals, like that of the existence of particulars, is in general¹⁵ dependent on experience, and thus *a posteriori*. We become aware of the existence of universal through analyzing particular experiences. This suggests that Russell's metaphysics of universals was, at least at this time, sparse. Hence, there is no logical reason to believe in the existence of universals, and thus, no route to be found here either for the infinity needed to ground mathematics.

Levine agrees that Russell did not at that time believe that there was a logical guarantee that an infinity of universals exist.¹⁶ Yet, if our acquaintance with universals is in general not logical, but dependent upon perception, it would be odd to identify wholesale the world of universals with the "beings of the world of logic", as Levine seems to. Levine also does not have an explanation for why, in 1911, Russell does not even broach the issue (even if the answer would be negative) of whether or not there might be reason for believing in an infinity of universals. On my reading, by 1911, Russell was back to considering universals to be individuals, and hence, the question he raises there about an infinity of individuals includes the issue about whether or not there are an infinity of universals, and since there is no logical guarantee of the existence of either particulars or universals, and two finite numbers added together yield only another finite number, the issue as to the existence of individuals remains an empirical one.

The third, and I think, most important reason not to identify the distinction Russell is making in (G) with the distinction between the world of existent particulars and the world of subsistent universals is that Russell *tells us* in passage (G) itself what the beings of the world of logic are supposed to be: he tells us that Here "the word *individual* contrasts with class, function, proposition, etc." These are the denizens of the world of logic that Russell has in mind. While the status of functions is a matter of dispute between us (which we'll

¹⁵In fact, he seems to suggest something stronger: that it is always dependent on experience. I am, however, not convinced that *that* aspect of his view persevered into the *PM* period.

¹⁶This has been confirmed again in personal discussion.

return to sec. 6), it is uncontroversial that Russell does not understand at this point classes and propositions to be entities that merely subsist rather than exist; they are mere logical fictions or logical constructions. Hence it is implausible to understand the distinction Russell is making there as lining up with the distinction between the existent and the subsistent.

I believe the real distinction in Russell's mind between individuals and "the rest" is the distinction between logical atoms and entities which are to be analyzed in terms of logical atoms. Individuals are the atoms; they are what everything else is analyzed in terms of.¹⁷ This comes out very explicitly in passage (D); they are, as he puts it there, "genuine constituents", ones which are not "analyzed away". As he puts it in passage (C), they are what exist on their own account, i.e., they are what makes up the *ultimate* furniture of the world. Putting aside complexes, which again, Russell changes his mind about, the contrasting group from logical atoms would be "logical constructions" or "logical fictions". This is not a distinction in the ordinary sense. It is not as if there are two kinds of things in Russell's ontology at this time: genuine things, and non-genuine things. Non-individuals are not really there at all, but are just the appearance of things which are suggested by language, "linguistic conveniences," as Russell sometimes puts it. The only sense of being that they have is the sense in which we can construct a language that "looked as though" they were real.

This kind of "being" which non-individuals have must not be taken too literally. It is easy to find passages in Russell's writings in which he writes as if there are "beings" which are not individuals, but he makes it clear that this kind of "being" is very different. Consider, e.g., this early passage from the "Types" manuscript:

When we consider classes, it seems evident that, if they have being at all, they have being of a very different kind from that of individuals. When we have counted up all the things there are in the world, we don't get new things by taking them in groups. Thus in some sense classes must be non-entities, and statements about classes must be reducible to statements about their members. ... [43]

Taken literally, this passage seems to suggest a kind of reverse Meinongianism. Whereas Meinong countenanced entities without being, Russell here seems to consider a special kind of being applicable to non-entities. I think it better

¹⁷It is interesting to note that this interpretation coheres with the account Wittgenstein gives of both Tractarian "objects" and "Russell's individuals" in §46 of the *Philosophical Investigations*, as the primary elements of which the rest of reality is composed.

to read this kind of being as the thinnest possible, i.e., just the *appearance* of an entity produced by a certain kind of logical form. To say that these “objects have being” is I think, nothing more than to say that we can make sense of talk about them, and quantification over them, but in a way that does not take that talk or quantification at face value; these sorts of “beings” are ontologically eliminable in favor of “genuine” entities: individuals. If I am right, then this applies not just to classes, but the other sorts of “things” Russell contrasts with individuals: propositions and, yes, propositional functions.

The “actual world” of (G), is I think, the world that is genuine in the sense of not being a product of misleading logical grammar or logical form; it is what is really there, after all. “The beings of world of logic”, then, do not include universals, as Russell at this time did not believe that universals could be analyzed away or treated as logical constructions. The distinction is rather between what Russell later calls “the raw material of the world” and that which “has smooth logical properties” or “neat properties”, which analysis reveals to have been “constructed artificially in order to have [those properties]” (see the Introduction to the second edition of [17], p. xi; cf. [34, pp. 326–29]). During the subsequent atomist period, Russell seems to equate “having a taste for mathematics” with the tendency to “like symbolic constructions” [30, p. 281], suggesting that the subject matter of mathematics and logic is such symbolic constructions. These of course are not entities, but they are the closest as one comes something like denizens of the “world of logic”.

One additional complicating factor must be mentioned with regard to Russell’s conception of individuals. When he had his philosopher’s hat on, Russell of course had a theory about what the ultimate logical atoms or individuals were: sensibilia (sense-data) and their properties and relations. But nothing in his *logical or mathematical project* puts any constraints at all on how we are to conceive of individuals. The logically relevant thing about individuals is that they are *not* analyzed, not that they could not be. Hence, Russell himself often stresses that the important thing is relative types (see, e.g., [26, p. 76] and [39]), and that anything at all can be treated as an individual. Even classes and numbers, given the very limited and purely linguistic “being” they have, may be *treated as* individuals in a symbolic logic, with the main expense being that by doing so, they are deprived of their “neat logical properties” as logical constructions. Thus, we may formulate a symbolic logic in which expressions for numbers are treated like names of individuals, but in such a logic, the basic properties of numbers will appear as substantive truths rather than the purely logical principles they are revealed as when numbers are analyzed in

the Russellian fashion.

5 **The Theory of Knowledge Manuscript**

Levine and I agree in essentials about Russell's views on logical grammar in the 1913 *Theory of Knowledge (ToK)* manuscript. There, Russell adopts the view that while universals can occupy subject position in complexes, the positions they occupy are not accessible to particulars, and hence that there is no logical type subsuming both particulars and universals. We differ, however, on whether or not this was a new development in this manuscript, or whether it is largely in keeping with the view defended in the *PM*-period and *RUP*. Levine is aware that certain aspects of Russell's views have changed of course, but does not seem to notice that many of these changes are *just what one would expect* given the changes of mind about the logical status of universals as not falling into the same logical type as particulars.

Levine makes note, for example, that in *ToK* (p. 92), Russell claims that it is not actually possible to say of a particular subject that it is "like or unlike" a predicate, since nothing which can be meaningfully said about one can be said about the other. But Levine does not seem to acknowledge how utterly different this is from his attitude in *RUP*, which as we have seen, is largely dedicated to the task of explaining what makes particulars unlike universals. Similarly, Russell now acknowledges (pp. 92–93) that there is no sense of "there are" which ranges over both universals and particulars, which I argued earlier (sec. 3.2), does not seem to have been his attitude when he wrote *RUP*. Finally, unlike many of the works from the 1910–1912 period which embrace an existence/subsistence distinction, Russell now claims (p. 138) that there is no sense of "exists" which can be meaningfully predicated of a named (rather than described) individual or particular. The impetus for this change on my own view is clear. "Exists" and "subsists" are worthwhile as predicates of individuals only if they are not pleonastic: that is, if there are some cases of false predication. On my own reading of the 1910–1912 view, Russell held that it was meaningful, but false, to assert of a universal that it exists, and meaningful, but false, to assert of a particular that it merely subsists, since these predicates could meaningfully be predicated of both universals and particulars. In *ToK*, Russell now believes that no predicate can be meaningfully asserted of both, and so he rejects these predicates outright. All of these changes signify to me an abrupt change of mind, which cannot, I think, be understood on Levine's reading of the earlier period.

Russell of course not only advocated the view in *PoM* that the category of individual or logical subject was univocal and all-embracing, he *argued* for it. It is not plausible to suppose that he would have given up this position unless there was some other intellectual problem or issue pressuring him to do so. One thing I find lacking in Levine's paper is a clear explanation for why Russell, in the earlier period, would have placed universals in a distinct logical category from particulars, while at the same time holding both to be potential logical subjects. Universals and particulars are different all right, but why a logical difference in what can significantly be asserted about them, especially if it is acknowledged that universals can occur as subjects rather than always in a predicating way?

I think what motivated Russell to make this change in precisely 1913 was misgivings he had about the original and simpler exposition of the multiple relations theory of judgment found in works such as [27] and [29]. According to that earlier view, when a subject S believes that aRb , the logical form of the fact can be understood as $Bel(S, a, R, b)$. Here, belief, or " Bel ", is a four-place relation between a believer, a subject of belief, a relation, and an object of belief. The relation R occurs here as a relatum of this relation, and on my own reading, this position is the same logically as any other logical subject position, and in principle accessible to any other entity, or individual, particular or universal. The belief fact is of the same form as any other four-place atomic complex: a relation holding between four individuals. Famously, Russell came to believe that this version of the multiple relations theory of judgment was subject to certain "direction" problems (see, e.g., [5]) i.e., trouble explaining what the difference is between believing that aRb and believing that bRa , as well as trouble explaining why it should be impossible to believe that Socrates Platos Aristotle, where the relation believed is replaced by a particular. There is of course a large literature already on these direction problems, the role Wittgenstein played in pushing them on Russell, and I do not wish to delve into them too far.

However, in *ToK*, Russell now advances a much more sophisticated view, according to which, for S to believe that aRb is analyzed as S believing there is a complex C in which a precedes in C , b succeeds in C and R relates in C . It is important to the account that each of the atomic parts of the belief be non-permutative, or heterogeneous, so there can be only one complex composed of these parts. If "preceding" and "succeeding" could occur in complexes in the same positions as the particulars which bear them to complexes, then this would not be the case. E.g., for a given complex C , there would be two different

complexes made up of the parts C , succeeding and preceding, i.e., the complex that succeeds precedes in C , and the complex that precedes succeeds in C . Similarly, if a particular could replace R in the component part of the belief “ R relates in C ”, then it would seem possible to believe that Socrates Platos Aristotle. It is, I think, precisely in order to secure the desired results about non-permutability or heterogeneity in complexes, and provide an answer to the direction problems, that Russell changed his mind about whether or not predicates and relations could occur in complexes in the same positions accessible to particulars.

Of course, Russell ended up not entirely happy with this version of the multiple relations theory of judgment, and abandoned the *ToK* manuscript without publishing it. Russell’s 1918 “Philosophy of Logical Atomism” lectures (*PLA*) (chap. IV) take the position that beliefs and relations can only occur in predicative positions, never as logical subjects, but avoid taking a stance on the logical form of judgment or belief facts. By 1919, Russell settles on a very different analysis of judgment or belief (see esp. [33]).

Another piece of the puzzle involves sussing out the influence of Wittgenstein on the changes to Russell’s theory of judgment. The exact influence of Wittgenstein is not easy to pin down, and I think it is especially isn’t clear whether or not the changes Russell made to his philosophy were in any way an attempt to appease Wittgenstein rather than to appease his own doubts, which were perhaps reinforced by Wittgenstein’s. Levine does not a nice job establishing that due to the dates involved, including when Russell reported “paralysis” to Ottoline over Wittgenstein’s criticisms, that Wittgenstein’s was critical not just of the earlier theory of judgment that came prior to *ToK*, but also to the view of *ToK* itself. On my own reading, Wittgenstein’s influence seems to have been a factor—though almost certainly not the only factor—both in explaining why Russell changed his views for *ToK* itself, and also why he abandoned *ToK*. This should be uncontroversial, as the surviving letters strongly indicate on the one hand that Wittgenstein played a role in Russell’s decision to abandon *ToK*, and, on the other, in *ToK* itself (p. 46), Russell credits Wittgenstein with having shown that a judgment fact is not one in the series: subject-predicate form, dual relations, triple relations, etc., and so Wittgenstein has clearly already influenced the view of *ToK* itself. As I suggested above, on Russell’s early theory, the fact making it true that S believes that aRb can be understood as a four-place atomic fact like any other, $Bel(S, a, R, b)$, where R occupies a position that could also be occupied by a particular. We know from the *Tractatus* that Wittgenstein’s chief worry with Russell’s position was that

it did not rule out that we could judge nonsense (5.5422), and in letters from as early as January 1913, Wittgenstein accuses a theory of types in which universals could be named as not being able to explain why “mortality is Socrates” is nonsensical [49, p. 122].¹⁸ In response to these criticisms, I believe Russell first changed his view to one according to which while universals can be named, their names cannot occupy the same positions as the names of particulars, which is the view of *ToK*. This response was seen by Wittgenstein, I think rightly, as an ad hoc dodge of the problems: if universals can be named, and occupy subject positions, what makes their subject positions different from that of particulars? Russell response seems to be the circular one that they are occupied by different types of things, but of course what makes one thing be of a different logical type from another is precisely the issue regarding what can be significantly asserted of them.

In the end, Russell himself came to the conclusion that the position of *ToK* was poorly motivated and came to believe instead that universals could not be named at all, and expressions for them must always occur predicatively. As Levine notes, in a couple places, he credits Wittgenstein with this realization, such as in *PLA* (p. 205), as well as in a letter to Moore dated October 2, 1921, where he states it as the view that “there is never any relations between a universal and a particular and never any proposition in which a universal appears as subject or term of a relation”. But another piece of evidence, more revealing for my disagreement with Levine, involves a letter written from Ramsey to Wittgenstein from Feb. 20, 1924, where he described his discussions with Russell over Wittgenstein’s work:

Of all your work he seems now to accept only this: that it is nonsense to put an adjective where a substantive ought to be which helps in his theory of types. [14, p. 219]

¹⁸Levine gives a detailed analysis of this letter, some of which I agree with, and some of which I disagree with. Most importantly, Levine reads the view Wittgenstein begins with there, in which the complex “Socrates is mortal” is analyzed as “ $\epsilon_1(\text{Socrates}, \text{mortality})$ ” as already a view according to which Socrates and mortality are treated as falling in wholly different logical types. But this makes a mystery as to why Wittgenstein used variables of the same kind when representing the basic form “ $(\exists x, y)\epsilon_1(x, y)$ ”. If Socrates and mortality are not both being treated as individuals, it’s unclear why this is not written as “ $(\exists x, \alpha)\epsilon_1(x, \alpha)$ ” instead. Part of Wittgenstein’s complaint is that the approach does not respect that “what seem to be *different kinds of things*” should be treated differently. But I agree, hypothetically, with what Wittgenstein’s reaction would have been to the suggestion that one could simply stipulate that these are different kinds of expressions *because* they have different kinds of meanings, and this is important for understanding why Wittgenstein was not satisfied with the changes Russell did make to his philosophy.

This letter seems to indicate that prior to Wittgenstein’s influence, Russell not only believed that words for universals (“adjectives”) could occur in non-predicating ways, but also that they could occur in positions that would otherwise be occupied by names of particulars (“substantives”).

6 Propositional Functions

Having argued that Russell’s conception of “individual” is not all-embracing given that, on his reading, universals are not individuals, Levine is more willing than I am to read Russell as countenancing other entities which are not individuals. Although he puts less stress on it, Levine is willing to attribute to Russell a realism about propositional functions as mind- and language-independent entities as well. My own view, of course, is that Russell’s ontology does not endorse a reality of propositional functions as anything but open sentences; they, like class-talk, are merely a way of speaking. I do not want to delve into this as far as the importance of the issue deserves. For one reason, the development of Russell’s thought on this matter was full of twists and turns and uncertainties, and ought to be sketched in some detail. For another, I have written about the subject at more length in other recent papers (especially [9] and [11]), and I do not want to repeat myself here. I merely want to note that my [11] is devoted almost exclusively to explaining in what sense Russell believed there could be propositions “about” propositional functions, i.e., propositions of the form “ $\psi(\phi\hat{x})$ ” without interpreting those propositions as involving a non-linguistic entity named by the “ $\phi\hat{x}$ ”. This is important because the bulk of Levine’s argument in favor of reading Russell as having an ontology of non-linguistic propositional functions stems from passages in which Russell speaks of propositional functions as being “subjects” of certain propositions, but I have provided elsewhere an interpretation of just these propositions which allows for this locution to be apt without the corresponding ontology.

I do want to point out here, however, that there are other passages, from both before *PM* and afterwards, in which Russell explicitly speaks as if there are propositions that can be described as being “about” propositional functions, or have them as subjects, yet without taking them to be entities on their own. We have already seen one in the opening passage from the “Types” manuscript quoted above. There, Russell explicitly claims that propositions of the form $f!(\phi!\hat{x})$ can be regarded as “subject-predicate” in form, so that the “ $\phi!\hat{x}$ ” is thought of as subject and “ $f!...$ ” as the predicate. But this follows a

paragraph in which functions are explicitly called “incomplete symbols”, by which Russell means an expression that contributes to the meaning of a proposition in which it occurs without having its own semantic value, or any one entity that it contributes to the meaning. As Russell claims there, expressions of the form $\phi!x$ are needed only in argument position. While not all aspects of the views in the “Types” manuscript are preserved in the published *PM*, the idea that circumflex constructions are needed only in argument position to a higher-type function variable is one I think is maintained there. I have argued elsewhere [11] that once the semantics of such constructions are understood, we need not understand the argument function expression as naming something, but rather as indicating something about the propositions in terms of which the truth of the general propositions containing such variables are recursively defined.

We find similar sentiments expressed in others of Russell’s manuscripts from the period. In an earlier manuscript, Russell wrote:

What do ϕx and ϕy have in common? Suppose ϕx is “ x is a man”. Then what they have in common is expressed by “... is a man”. But this, plainly, is not anything; it can be part of a significant proposition, but by itself it is nothing.

...

That is to say, we can make a sort of dictionary in which phrases containing ϕx in the way that classes usually occur are interpreted so as to be significant, without having to assume that ϕx means anything by itself. Thus:

“ x is a member of ϕx ” means “ ϕx is true”

“ ϕx is contained in ψx ” means “for any value of x , ϕx implies ψx ”

...

Functions such as those we have just been considering are functions of functions. [42]

While this is a relatively early manuscript, and Russell certainly vacillated in between, it is a view like this that I believe he gravitated back towards in *PM* itself, where a propositional function is described as “not a definite object” (p. 48).

The attitude is expressed more strongly in works after *PM*, such as *PLA*, where we find Russell writing:

A propositional function is nothing, but, like most of the things

one wants to talk about in logic, it does not lose its importance through that fact. [30, p. 230; cf. p. 234]

Here we find Russell placing propositional functions squarely in the same camp with classes and other would-be denizens of “the world of logic” as logical fictions, much like he already did in passage (G) from 1911. In 1919’s *Introduction to Mathematical Philosophy*, Russell writes:

We do not need to ask, or attempt to answer, the question: “What is a propositional function?” A propositional function standing all alone may be taken to be a mere schema, a mere shell, an empty receptacle for meaning, not something already significant. [32, p. 157]

Yet, despite these clear disavowals of a realism of language-independent “propositional functions”, in these works Russell is happy to speak of quantified formulas, or the analyzed statements about classes, as if somehow they were “about” propositional functions:

When you take any propositional function and assert of it that it is possible, that it is sometimes true, that gives you the fundamental meaning of ‘existence’. You may express it by saying that there is at least one value of x for which that propositional function is true. Take ‘ x is a man’, there is at least one value of x for which it is true. That is what one means by saying that ‘There are men’, or that ‘Men exist’. Existence is essentially a property of a propositional function. [30, p. 232]

When we say “there are men,” that means that the propositional function “ x is a man” is sometimes true. When we say “some men are Greeks,” that means that the propositional function “ x is a man and a Greek” is sometimes true. [32, p. 159]

When we use a variable, and speak of a propositional function, ϕx say, the process of applying general statements about ϕx to particular cases will consist in substituting a name for the letter “ x ,” assuming that ϕ is a function which has individuals for its arguments. [32, p. 173]

Statements about all a -classes (i.e. all classes defined by a -functions) can be reduced to statements about all a -functions of the type τ . [32, p. 191]

I do not think it is charitable to read Russell however, as holding seriously that quantified formulas are “about” expressions or schemas, so that, e.g., “ $(\exists x).fx$ ” is to be understood as asserting that the open sentence “ fx ” is satisfiable. I take these passages, as well as the similar passages in *PM* itself which Levine cites, as loose talk aimed at explaining the basics of the syntax of quantification understood in terms of variable binding operators to an audience which, unlike the contemporary readers of Russell’s works, would likely not have been familiar with contemporary quantifier-logic. In passing, in *IMP*, Russell catches himself, and suggests that we “more correctly” speak of “functions of functions” rather than “statements about functions” (p. 186).

In an even later work [35, p. 192], Russell makes it clear that he does not conceive of higher-order quantification realistically, writing that “[i]n the language of the second order, variables denote symbols, not what is symbolised”. Of course, it remains open to Levine to read Russell has having changed his mind after *PM*, but I can think of no evidence to suggest that this is the case.

Perhaps most importantly, however, Russell himself in *My Philosophical Development*, when writing of his own views *at the time of PM*, interprets *himself* as shying away from an extra-linguistic ontology of propositional functions even then, writing that “Whitehead and I thought of a propositional function as an expression” [37, p. 124], making it clear that it is nothing more than that:

A propositional function is nothing but an expression. It does not, by itself, represent anything. But it can form part of a sentence which does say something, true or false: ‘ x was an Apostle’ says nothing, but ‘there are twelve values of x for which “ x was an Apostle” is true’ is a complete sentence. [37, p. 169]

He also explicitly likens the status of propositional functions to classes, again placing them in the category of expressions that can contribute to the meaningfulness of propositions in which they occur without having their own semantic values:

The propositional function itself is only an expression. ... A class, equally, is only an expression. It is only a convenient way of talking about the values of the variable for which the function is true. [37, p. 82]

And yet, once again, he allows himself to speak as if quantified statements, and higher-order statements, are “about” propositional functions:

One is a characteristic, not of things, but of certain propositional functions, namely, of those propositional functions which have the following property: ... [37, p. 69]

The proposition 'the golden mountain does not exist' becomes 'the propositional function "x is golden and a mountain" is false for all values of x.' [37, p. 84]

I take these to be telling indications that Russell's allowing himself to speak of propositional function as "subjects" or as things propositions can be "about" is not ontologically loaded, as Levine seems to think.

A puzzle which arises for those who do, like Levine, take propositional functions in Russell's philosophy to be language-independent abstract objects of some sort, is to give an account of the sorts of *facts* or complexes of which they form a part. Yet, Russell does not seem to speak of anything except elementary complexes in *PM*, writing of $(x).\phi x$ that it "does not point to a single corresponding complex: the corresponding complexes are as numerous as the possible values of x " [46, p. 46]. But if quantified propositions don't correspond to complexes containing propositional functions as entities, what propositions do? And if there are no metaphysical complexes containing these entities, in what sense could there be such things? On my own reading, it is important for understanding the birth of Russell's logical atomism that at the time of *PM*, he had become convinced that all truths are recursively grounded in elementary propositions, which he tells us, do not presuppose functions. This is just another way of saying that Russell's ontological picture includes only that which is involved at the elementary level, and hence, not anything like an ontological correlate of a propositional function.

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