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Review: *Russell's Philosophy of Logical Analysis*. 1897–1905, by Jolen Galaugher

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Palgrave's *History of Analytic Philosophy* series has lately been releasing important books in Bertrand Russell studies at a pace that makes it seem as though it were making up for lost time. The series editor, Michael Beaney, has recently in another context identified the early 1990s as the period when serious historical scholarship on analytic philosophy finally began in earnest (Beaney, 2013, p. 54). Two groundbreaking works from that period, by Hylton (1990) and Griffin (1991), reminded us of Russell's academic beginnings in the British Idealist tradition. Since then, important studies on the development of Russell's technical philosophical and mathematical logic during the first decade of the 20th century have also emerged, such as Landini (1998) and Makin (2000). One of the newest contributions from Palgrave's series, Galaugher's fascinating *Russell's Philosophy of Logical Analysis: 1897–1905*, serves as a bridge connecting these different areas of Russellian scholarship. She traces themes in Russell's work that have their origins during his idealist period but which remained prominent through his early logicist period. These include identity and difference (and thus plurality), the analysis of complexity, the nature of relations, judgments and meaning. Her study makes it clear that while Russell's views were in a state of constant flux, many interests and concerns remained constant. His transition from idealism to realism was not a simple matter of turning his back on what had come before, but can perhaps be better understood as involving a series of steps of fine tuning and improving upon a single overarching approach to logical analysis.

Galaugher sets the stage in the first chapter with a discussion of Russell's abandonment of British Idealism. Russell himself later claimed that Moore led the way in this regard (Russell, 1958, p. 54), and Galaugher presents a nice summary of Moore's (1899) influential criticisms of Bradley's theory of judgment. As important as Moore's influence was on early Russell, Galaugher makes it clear that we must not consider that influence in isolation. Noting differences between Russell's eventual realist position and Moore's, Galaugher also considers Russell's transition in the light of tensions within his early philosophical views on the foundations of mathematics. In works such as his 1897 *An Essay on the Foundation of Geometry*, Russell had hoped to support the axioms of geometry using Kantian-style transcendental deductions. However, he wished to give an account of the synthetic a priori that made it a logical notion rather than a psychological one. This led Russell away from thinking in terms of the preconditions of our knowledge of geometric truths, and toward thinking instead of the logical features a system of relations must have in order for a geometry characterized by certain axioms to be relevant to it. The views on relations Russell held during this period, and in particular the version of the doctrine of internal relations according to which all relations depend on qualities of the relata or the relation itself, generated certain "paradoxes of relativity" when applied in mathematical contexts. Different geometrical points, for example, seem indistinguishable in their intrinsic qualities but nonetheless are distinguishable through their relations to one another. Galaugher also discusses the influence of Whitehead's *Universal Algebra* in bringing Russell to adopt a broader conception of the scope of mathematics. Finally, there are the relatively well-known difficulties Russell discussed in *The Principles of Mathematics* (chap. XXVI) with accommodating asymmetrical relations within a framework accepting the doctrine of internal relations. These various developments came to a head for Russell when he was working on his 1899 lectures on Leibniz (the basis for Russell, 1900). Russell attributed (perhaps wrongly) a similar view of relations to Leibniz and diagnoses problems as he sees them with Leibniz's views as stemming from this position. It was through a confluence of these forces, according to Galaugher, that Russell came to abandon his neo-Hegelian idealism in favor of the precise form of analytic realism that marked his early logicist period.

The importance of Russell's work on Leibniz for his evolving views on relations is given an even more detailed examination in Galaugher's second chapter. There she examines the complex relationships between the view attributed by Russell to Leibniz that all predication involves analyzing a substance—or, perhaps, the concept thereof—into constituent predicates, the identity of indiscernibles, and the contention that all relations are "intelligible things" brought about by the mind in some way. Those familiar with Russell's work on Leibniz are no doubt well aware of Russell's dissatisfaction with Leibniz's strict adherence to subject/predicate analyses of propositions, and his contention that it prevented Leibniz from giving adequate philosophical accounts of such notions and space and time. Galaugher argues that these complaints with Leibniz brought Russell to reject broadly similar commitments within his own form of idealism. In particular, Russell came to abandon the principle of identity of indiscernibles as interpreted to mean that all difference between things is grounded in difference of concepts applicable to them. Instead, Russell came to regard mere numerical diversity as logically prior to difference in predicates, and presupposed by all relational judgments. This took Russell a step further away from the idealist monists of his day, and indeed, a step further than Moore had taken by this point. Early on in his realist phase, Moore continued to regard a thing, or even a concept, as a whole of its properties (concepts), and maintained that the relation between a whole and its parts was internal. Russell, however, took the radical step of holding all propositions to be relational, and all relations to be external, differentiating a thing from the sum of its qualities. Along with this, Russell began to see the basic logical relationships also as relational and synthetic, and thus distinct from part/whole relationships. This development culminated and was reinforced when he adopted Peano's symbolic logic as a replacement for the more limited Boolean logic of containment relationships between classes.

Galaugher also stresses the importance of Russell's adopting an intensional view of relations, and one on which an asymmetrical relation is differentiated from its converse. It wasn't entirely clear to me how Galaugher understood the relationship between these two features of Russell's evolving views. Galaugher suggests, in both Chapters II and III, that early Russell believed that differentiating an asymmetrical relation from its converse, or equivalently, capturing the sense of an asymmetrical relation, requires an intensional view of relations. It was not clear to me why this should be. At least in contemporary parlance, the relations less than and greater than are not co-extensive, and thus they may be differentiated even on a fully extensional view of relations. A philosophical argument might be given to the effect that the extensions of these relations, considered, say, as sets of ordered pairs, can only be held different if we can account for the difference between such pairs as $\langle 3, 5\rangle$ and $\langle 5, 3\rangle$, and that doing so somehow requires intensional relations. However, there's nothing I know in Galaugher's exposition, or even in Russell's writings of the period, to suggest an argument along these lines.

In Chapter III, Galaugher discusses Russell's emerging logicist views in the 1901–03 period, the importance of Russell's adoption of Peano's symbolic logic, and his discovery of the logical paradoxes such as the antinomy now known as "Russell's paradox". She enters into the debates regarding whether or not Russell's logicism in the *Principles of Mathematics* should be regarded as having an "if-then"-ist or "conditional" form, differentiating her interpretation from those of Putnam (1975), Coffa (1981), Griffin (1982), Proops (2006), and Gandon (2011). Coffa interprets Russell's logicist treatment of geometry as taking the form of logically true conditionals from the axioms of a given geometry to the theorems. This sort of interpretation threatens to trivialize the logicist project, as the relationship between the axioms and theorems of any theory can always be understood as purely logical. Galaugher makes note of a response by Griffin stressing that the conditionals in Russell's logicism are better understood not as representing relationships between axioms and theorems, but rather as universally quantified statements making use of unrestricted variables where the antecedents and consequents state categorical conditions for the values of the variables to satisfy. Galaugher argues for a more nuanced position according to which Russell held a position aligning with Coffa's interpretation early on (in, e.g., the 1900 drafts of *Principles*), but one more in tune with Griffin's interpretation from mid-1901 onwards after fully integrating Peano's logic. Along with this change came a new attitude about the analysis of mathematical notions and definitions of mathematical terms. Russell no longer held that an analysis of a mathematical notion must preserve the intensional aspects of our pre-analytic understanding of the notion. Instead, he held that one may make use of any nominal definition preserving the formal features of the original notion. Russell became increasingly prone to giving nominal definitions making use of classes defined using purely logical propositional functions, making a fully logicist analysis of mathematical notions possible.

The chapter concludes with a discussion of Russell's changing views on logic. Russell did not take over Peano's logic uncritically. Russell was dissatisfied with Peano's extensional treatment of relations and sought to supplement it with his own intensional theory. He also criticized Peano's logic for failing clearly to distinguish between a proposition and a propositional function. The latter notion emerged from Russell's consideration of the Peanist notions of formal implication (implication for all values of a variable) and class abstraction. Galaugher goes on to discuss Russell's discovery of the various versions of "Russell's paradox", noting that the earliest manuscript in which it can be found gives a version stated in terms of predicates or class–concepts not predicable of themselves. Galaugher stresses, quite rightly, that one of the most important lessons Russell drew from the paradox early on was that not every propositional function corresponds to a class-concept.

Unfortunately, I think there are at least two ways in which Galaugher's discussion of these issues is not as clear as it might have been. Galaugher does not distinguish, as I think Russell does, between the claim that the "*functional part* of a propositional function is not an independent entity" (*Principles*, p. 88) from the simpler claim that the function itself is not an independent entity. As a result, it is a bit difficult to understand what Galaugher believes Russell's position in *Principles* was regarding the independent reality of propositional functions. Secondly, in contrasting Russell's reaction to the paradoxes to Frege's, Galaugher claims that Frege was not bothered by an "intensional version" of the paradox (p. 110). By this, she seems to mean that Frege's theory of levels of functions and concepts blocked the version involving a function not satisfied by itself, or a concept not falling under itself. But it is misleading to describe this as an "intensional version", however, as Frege held an extensional view of both functions and concepts. For example, Frege claims that concepts coincide when the same objects fall under them.[1] Recall that concepts are the references of predicates for Frege. "Intensional" versions for Frege would be ones involving his notion of sense. Apart from some inconclusive discussion of a paradox of thoughts in his correspondence with Russell (Frege, 1980, pp. 147–66), Frege did not consider intensional versions. Moreover, it is even misleading to suggest that Frege's theory of levels avoids a function or concept version of the paradox. Given that Frege identifies the extension of a concept with the "value-range" of the concept considered as a function, and holds that the extension of a concept "has its being" in the concept (Frege, 1906, p. 183), arguably Frege considers the extension of a concept simply to *be* the concept considered as a logical subject.[2] If this is right, then it is not possible to differentiate the classes (or extensions) version of the paradox in Frege's logic from one involving a concept taking itself-qua-logical-subject as argument.

The fourth chapter compares Russell's logicist views with those of Frege, particularly with regard to their nominal definitions of numbers as classes (or extensions) of classes of like-cardinality, and the relative priority of propositional and non-propositional (or "mathematical") functions. Galaugher believes that the widespread claim that Russell independently rediscovered Frege's definition of number is too simplistic, and cites what she sees as non-negligible differences between their accounts of number. Ultimately, the differences stem from wider disagreements over the nature of propositions, intensional relations, functions and classes. Galaugher notes that Russell complained that Frege had failed to give a full account of how classes are to be understood as entities, having only equated them with extensions or value-ranges of functions obeying his problematic Basic Law V. Galaugher goes on to discuss Russell's own rapidly changing views on classes and their relationships to functions in the 1902–1905 period. In so doing, she draws heavily upon Russell's very interesting correspondence with Couturat, which has not yet been published in English. There we find both praise of and frustration over Frege's more mathematical notion of a function. Galaugher nicely traces Russell's 1903 adoption of a view which eschewed classes altogether in favor of functions, various views of 1904 in which Russell sought to discover conditions under which some but not all functions determine classes, and Russell's eventual adoption—after the theory of descriptions of 1905—of a "substitutional theory" according to which classes, relations in extension, and even propositional functions are all treated as mere *façons de parler*.

In the final full chapter, Galaugher takes up the relationship between, on one hand, Russell's attempts to understand propositions involving functions and variables, and on the other, his views on meaning and denotation, culminating in his landmark "On Denoting" of 1905. Galaugher notes a similarity between the difficulty Russell pointed to in "On Denoting" concerning disambiguating between propositions about denoting concepts and propositions about their denotations, and an earlier puzzle about differentiating between what Russell had called a "propositional concept" in *Principles*, e.g., "the death of Caesar", and the proposition it represents—in our example, "Caesar died". In his 1904 work on Meinong, Russell was led to the view that the difference could not be maintained. Galaugher also explores in some detail Russell's dissatisfaction with how his earlier theory of denoting handled non-propositional "denoting functions", such as "the father of x". The issue was important for Russell's treatment of mathematics, as most mathematical functions, e.g., "the sine of x", "the sum of x and y" are of this type. Russell vacillated during the 1903–1905 period between the view he held both earlier and later that propositional functions are more fundamental than others, and the more Fregean view that all functions can be treated uniformly. The issues involved are quite complicated. It is impossible here to provide more than a crude summary even of Galaugher's exploration, much less Russell's. However, there are at least two sources of worry. One involves whether or not Russell can provide a coherent account of *aboutness* when denoting functions are involved. "The father of Russell was a political activist" appears to be *about* Russell. Notice here Russell only occurs as argument to a denoting function, and thus as a part of the denoting complex "the father of Russell". Yet, according to the theory of denoting concepts, the proposition is not at all about this complex or its parts, but only about what it denotes. Another worry involves Russell's attempts to do away with functions, both propositional and denoting, as distinct entities separable from their values. While at some points Russell was willing to consider the view that functions were separable from their values, at others he took denying this to be a promising route for solving functional versions of Russell's paradox. The hope was to replace the notion of the values of a function with the notion of different results of substitution within a complex. However, if p is a proposition containing a denoting complex such as "the father of Russell", does $p\frac{x}{y}$ represent the result of replacing y with x y even where y occurs only in the meanings in p, or just within the denotations of those meanings? It proved difficult to answer this and related questions coherently, while at the same time providing a uniform means for differentiating between complex meanings and their possibly complex denotations. Such complications were sidestepped by Russell's mature theory of descriptions which replaced denoting functions with descriptions to be interpreted within the scope of a greater propositional context.

The book ends with a short concluding postscript in which Galaugher summarizes the development of Russell's views on logical analysis in the years covered. There are some puzzling claims made here that seemed out of sorts with claims made in the body of the book. It is unclear whether these were mere infelicities of expression, or indicative of a deeper misunderstanding. For example, Galaugher writes (p. 176):

> While Russell initially subscribed to a naïve comprehension principle on which every predicate or class-concept determines some class, the contradiction of predicates not predicable of themselves and class–concepts not members of their own extensions led him to reject this principle . . .

However, what the body of the book had argued (p. 107), and what in fact Russell concluded from these versions of the paradox, was that not every class has a defining predicate or class-concept. This is not the same as the claim that not every predicate or class-concept defines a class. To my knowledge, Russell continued to maintain that every class-concept or (simple) predicate defines a class, even when denying that every class is so defined. He also, separately from this (I think), abandoned the (naïve abstractionist) view that every *propositional function* defines a class, but as Russell did not equate predicates or class–concepts with propositional functions, this would not require him to give up the view quoted above. Similarly, Galaugher writes (p. 175):

> While Russell initially held that relations in intension are to identified with class–concepts (PoM, p. 514), he came to hold that class–concepts are marked by intensional propositional functions.

If Galaugher had argued that Russell at one time held that relations in intension were class–concepts in the body of the book, I missed it. As far as I know, Russell never held such a view. Class–concepts are monadic qualities of individuals; to equate relations with class–concepts would seem to amount to the adoption of an extreme form of the doctrine of internal relations. She cites p. 514 of *Principles*, but I cannot find anything relevant on that page. Moreover, I do not know what it is for class–concepts to be "marked by" propositional functions, nor how that view would be contrary to the view previously held. I suspect, however, that these remarks and similar remarks in the summary section of the book are simply sloppily worded, and that Galaugher meant something different by them than what I have understood. Indeed, it is possible that some of my concerns over passages in earlier chapters are the result of misunderstandings brought on by otherwise minor infelicities of expression.

Overall, Galaugher's prose is dense, and I think it is fair to say that it is quite demanding on the reader. The topics are many and varied. In the space of a few pages, Galaugher moves between such difficult topics as Leibniz's theory of monads, Bradley's views on relations, the differences between projective and metric geometries and logicist theories of cardinal number. Her writing assumes that the reader has at least a basic understanding of all these topics. Unfortunately, not everyone is the sort of polyglot Russell himself was. While this may limit the book's audience, it is not meant as a criticism. Only by delving into so many issues is Galaugher able to draw connections usually unnoticed between diverse areas of Russell's thought, which is one of the chief merits of Galaugher's work. Two such contributions stand out as particularly valuable. Firstly, there is Galaugher's excellent discussion in the first two chapters about the importance of Russell's confrontation with Leibniz for understanding both his abandonment of idealism and his newly emerging views on relations and propositional analysis. Secondly, Galaugher's discussion, especially in the fifth chapter, makes it abundantly clear that the secondary literature so far has underestimated the connection between Russell's work on the theory of descriptions and his greater logicist project. These two contributions will, I predict, make a lasting impact on Russell studies and Galaugher's book would be worth a read for them alone. However, for the reader up to the challenge, the book offers many additional insights into the development of Russell's philosophy as well.

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Notes

[1] See Frege (1979a, p. 122). There are other places as well in which Galaugher seems to attribute to Frege an intensional view of functions. For example, she claims that Frege is committed to taking the identity relation to be a relation "in intension" in order to solve the belief puzzles (p. 120). It is unclear to me what she means by this, or how she reaches this conclusion. The received (and probably correct) interpretation of Frege is that the cognitive difference between "$a = b$" and "$a = a$" stems from the differing senses of "a" and "b" which contribute to the sense of (i.e., thought expressed by) the whole equations. This view does not require him to take the *references* of "a" and "b" to be different when flanking "=", nor to insist that "=" *refer* to some kind of intensional relation, or relation between intensions.

[2] For discussion, see Cocchiarella (1987, chap. 2) and Klement (2012, sec. 4).

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