PHY 556/714: Handout 4

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RE: *Topics, Reading & Assignment, lectures 15 thru 19, 11/02/07 thru 11/16/07*

In Lecture 15, we began a discussion of weak interactions. In lecture 16, we discussed the similarities and differences between the structure of weak and electromagnetic interactions. We then wrote down a Lagrangian for massless leptons and introduce the weak and electromagnetic interactions via local gauge invariance. After introducing the basic interaction Lagrangian involving W and Z bosons, we discuss how such bosons are produced at colliders and study their properties. Finally, we will conclude with a discussion of low energy weak interactions.

Main Reading:

Griffiths: Chapter 10, Sections 11.1 thru 11.4.

Supplementary Reading:

Kane, Chapters 9 and 10.

Homework Assignment 4 *(due 11/27)*

1) Make a table for left- and right-handed electrons and quarks and for neutrinos with the following information: charge, weak isospin and the left- and right-handed coupling to Z bosons.

2) Draw the relevant Feynman diagrams for the following processes:
$\nu_e e^- \to \nu_e e^-$, $\bar{\nu}_e e^- \to \bar{\nu}_e e^-$, $\nu_\mu e^- \to \nu_\mu e^-$, $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$, $\nu_\mu e^- \to \nu_e \mu^-$

3) Use helicity conservation in the high energy limit to infer the angular dependence, in the W rest frame, of the emitted electron in the reaction $d + \bar{u} \to W^- \to e^- + \bar{\nu}_e$. Use θ to be the angle between the incoming *d* quark and the emitted electron.

4) Use the Table from problem 1 to find the branching ratios for Z decays to u-anti-u pairs, electron-positron pairs and neutrino-antineutrino pairs. Use $\sin^2\vartheta_W = 0.23$. How would these ratios change if the value were 0.25?

5) Griffith's problem 10.6

6) Consider the decay process $Z^0 \to \tau^+\tau^-$ observed in electron-positron collisions at the Z resonance. In the Z rest frame, find the average distance the tau lepton travels before it decays.

Use the following information: tau rest mass = 1.7 GeV/c^2, the muon decay width formula is $\Gamma_\mu = G_F^2 m_\mu^5 / 192\pi^3$ and the branching ratio for $\tau^- \to e^-\bar{\nu}_e\nu_\tau$ is 18%.

7) The total cross-section for muon-neutrino electron scattering to produce a muon is approximately $G_F^2 E^2$, where E is the incoming neutrino energy in the *center-of-mass* frame. Calculate the cross-section (in Barns) for a *laboratory* neutrino beam energy of 10 GeV.

Following for 714 students only:

8) This is a followup to problem 6. In the same (Z rest-) frame, consider the subsequent 2-body weak charged-current decay $\tau^- \to \pi^-\nu_\tau$. The energy distribution of the pions in this frame reveals the polarization of the τ leptons in Z decays: $P_\tau = (g_R^2 - g_L^2)/(g_R^2 + g_L^2)$, where g_R, g_L are the right- and left-handed couplings of the τ lepton to the Z boson. Throughout, use the approximation $m_\pi^2 << m_\tau^2 << m_Z^2$

   a. Evaluate the tau polarization in Z decays for $\sin^2\vartheta_W = 0.23$ and 0.25

   b. Use angular momentum arguments (the neutrino is always left-handed) in the tau lepton rest frame to infer the angular distribution of the decay to be of the form $g_R^2 \sin^2(\vartheta/2) + g_L^2 \cos^2(\vartheta/2)$. Here, θ is the angle between the pion momentum and the spin quantization axis in the tau lepton rest frame.

   c. Obtain the relationship between θ and the energy E_π of the pion in the *laboratory* frame (Z rest frame), by applying the appropriate Lorentz transformation. (Boost along the quantization axis.)

   d. Using the results of parts b. and c., show that the distribution of observed pion energies in the laboratory frame (the Z rest frame) in Z decays to tau leptons is given by $\frac{1}{N}\frac{dN}{dx} = 1 + P_\tau(2x-1)$, where $x = E_\pi/E_{beam}$ and E_{beam} is the energy of the incoming electron and positron beams.