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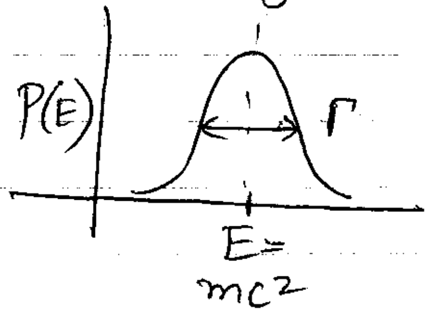
LECTURE VI

Unstable particles: characterized by mass & lifetime.

$\Gamma = \hbar / \tau$ $\qquad$ $W \propto \Gamma$



Heirarchy of Interactions:

Decay by the strong force: 10^{-23} s $\Longleftrightarrow \Gamma \sim 100$ MeV

Weak force: 10^{-8} s

electromagnetic: 10^{-16} s

Density of states or Phase space causes variations:

Free neutron : 10 s

Nuclear isotopes : 1000s of years

Free muon : 10^{-6} s

All are weak decays!

By studying the lifetime of various allowed decay modes:
learn about forces & interactions.

* If a decay mode is allowed, it will happen.

$\mu^- \longrightarrow e^- \bar{\nu}_e \nu_\mu$ (allowed)

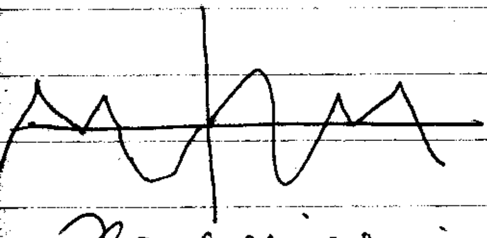
$\mu^- \longrightarrow e^- \gamma$ (not observed)

LECTURE VI

Symmetries

Classical physics: Consider an odd $f^{\underline{n}}$.

We know a few things:



$$\int_{-3}^{3} f(x)\,dx = 0 \qquad \int_{-7}^{0} f(x)\,dx - \int_{0}^{7} f(x)\,dx$$

No cosines in Fourier series, odd powers in Taylor series $\Longrightarrow$ symmetry allowed for a lot of guesses

In physics, intuition can suggest symmetries
$\Longrightarrow$ exploit symmetries to understand underlying physics

Example: Space is homogeneous $\Longrightarrow$
$\mathcal{L}$ invariant under translation
$\mathcal{L}(r_i, \dot{r}_i, t) \longrightarrow r_i \longrightarrow r_i + \vec{a}$

$$\delta \mathcal{L} = \sum_i \frac{\partial \mathcal{L}}{\partial r_i} \cdot \delta r_i = \vec{a} \cdot \sum \frac{\partial L}{\partial r_i} = 0$$

$$\Rightarrow \sum \frac{\partial L}{\partial x_i} = 0 \Longrightarrow \frac{d}{dt}\left(\sum \frac{\partial L}{\partial \dot{r}_i}\right) = 0 \Longrightarrow \frac{dp_i}{dt} = 0$$

$$\because \frac{d}{dt}\left(\sum \frac{\partial L}{\partial \dot{r}_i}\right) - \frac{\partial L}{\partial x_i} = 0 \quad (\text{E-L Eqn})$$

$$\Longrightarrow p_i = \text{constant}$$

LECTURE VI

translational invariance $\Longleftrightarrow$ momentum conservation
time invariance $\Longleftrightarrow$ energy conservation
rotational invariance $\Longleftrightarrow$ angular momentum

Noether's Thm (Sweeping implications for dynamics)
If E-L Eqn is invariant under any coordinate transformation, $\exists$ an integral of motion
$\Longrightarrow$ conserved quantity or observable

Not just space-time symmetries, also internal symmetries i.e. invariance of Hamiltonian or Lagrangian

Example: $|\alpha\rangle \longrightarrow \psi_\alpha(\vec{r},t)$: If $\vec{r} \to \vec{r} + \vec{\xi}$
then $\psi'_\alpha(\vec{r},t) \longrightarrow U\psi_\alpha(\vec{r},t)$
with $U = e^{-(\vec{\xi}\cdot\vec{\nabla})}$ $\Rightarrow$ $U = e^{-(\vec{\xi}\cdot\vec{P}/\hbar)}$

Invariance under Schroedinger Eqn: $i\hbar\frac{\partial\psi}{\partial t} = H\psi$
$\Longrightarrow$ $[U, H] = 0$
For translations: $[H, P] = 0$

In general: $[H, F] = 0$

which implies invariance under $e^{i\epsilon F}$

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LECTURE VI

Example: Electric Charge If $[Q, H] = 0 \Rightarrow \frac{d\langle Q\rangle}{dt} = 0$

$Q\psi = q\psi$ (ψ is a simultaneous eigenf^n of H & Q)

This implies that $\psi' = e^{i\epsilon Q}\psi$ leaves H invariant
$\Rightarrow$ a "global" gauge tranformation
i.e. ϵ is a constant.

Profound Implications: Inspect Hamiltonian,
predict $\sum q_i = \sum q_f$

Since charge is quantized, define charge# $q = Ne$

If $a + b \longrightarrow c + d + e$
$N_a + N_b = N_c + N_d + N_e$
(Additive conservation law)
e.g. $e \longrightarrow \nu\gamma$ is forbidden.

Baryon # $A = 1$: $p \quad n \quad \Lambda \quad \Sigma^+ \quad \Xi^- \quad \Omega$

$A = -1$ for antiparticles

$p \longrightarrow e^+\pi^0$ (Violates Baryon#)

Hence proton is stable (lucky us!)

Origin of baryon# conservation unknown.

LECTURE $\overline{\underline{VI}}$

$$\Lambda^0 \to n\,\pi^0$$
$A=1 \qquad A=1$

$$\Sigma^+ \to p\,\pi^0$$
$A=1 \qquad A=1$

$$\Sigma^- \to n\,\pi^-$$
$A=1 \qquad A=1$

$$\pi^- p \to \Lambda^0 + K^0$$
$A=0 \quad A=1 \qquad A=1 \quad A=0$

Baryons are made up of 3 quarks

Some amount of baryon# violation needed to explain preponderence of matter in the universe.

<u>Lepton number</u> $\gamma^* \to e^+e^-$ or $p\bar{p}$ but not e^+p

$$n \to p\,e^-\,\overrightarrow{\nu_e}$$
$A=1 \qquad A=1$
$L=0 \qquad L=0 \quad L=1 \quad L=-1$

$\bar{\nu}_e$ observed for a first time in a reactor experiment.

$$\bar{\nu}_e\,p \to e^+ n$$
$L \quad -1 \qquad -1$

Not seen: $\bar{\nu}_e\,n \to e^-\,p$ $\qquad$ $\nu_e\,p \to e^+ n$
$L \quad -1 \qquad +1 \qquad\qquad +1 \qquad -1$

Seen: $\nu_e\,n \to e^-\,p$
$+1 \qquad +1$

LECTURE VI

Neutrinos & anti-neutrinos have opposite lepton #
(Defer discussion of massive neutrinos)

We are obliged to introduce several lepton #s. Why?
Consider $\mu^- \to e^- \nu \bar{\nu}$
If there was only one lepton number, then
$\mu^- \to e^- \gamma$ would be allowed.
NOT OBSERVED.

$\Rightarrow$ Assign $L_\mu(\mu^-) = 1$ $L_\mu(\mu^+) = -1$ $L_\mu(e^-) = 0$ etc

	μ^-	$\longrightarrow$	e^-	ν_μ	$\bar{\nu}_e$
L_e	0		1	0	-1
L_μ	1		0	1	0

Both L_e & L_μ are independently conserved

$\Rightarrow$

	π^-	$\longrightarrow$	μ^-	$\bar{\nu}_\mu$		π^+	$\longrightarrow$	μ^+	ν_μ
L_μ	0		1	-1		0		-1	1

$\Rightarrow$ ν_e & ν_μ are distinct particles. How do you prove it? Make ν beam from π-decay.

If π^- used, then $\bar{\nu}_\mu$ beam, if π^+ used, then ν_μ beam.

$\Rightarrow$ $\bar{\nu}_\mu p \to \mu^+ n$ $\qquad$ $\nu_\mu n \to \mu^- p$
OBSERVED!
$\bar{\nu}_\mu p \to e^+ n$ $\qquad$ $\nu_\mu n \to e^- p$
NOT OBSERVED!
1962 Nobel Prize

LECTURE VI

Symmetry Operation: leaves system invariant.
Set of all operations with Closure Identity Inverse & Associativity (Note: Commutativity not required)

Finite group $\Longrightarrow$ discrete symmetry
Infinite group $\Longrightarrow$ continuous symmetry.

In physics, groups can be represented by matrices.
e.g. Lorentz transformations, rotations

$SO(n)$: Group of n-D rotations
$SO(3) \Longleftrightarrow SU(2)$

$SU(2)$ is an important group in particle physics

Invariance under $SU(2)$: Angular momentum conservation
$\Rightarrow$ Next week, introduce formalism.

Summary on Lepton Number

$L_e = +1$: ν_e, e^- $\qquad$ $L_e = -1$: $\bar{\nu}_e, e^+$
$L_\mu = +1$: ν_μ, μ^- $\qquad$ $L_\mu = -1$: $\bar{\nu}_\mu, \mu^+$
$L_\tau = +1$: ν_τ, τ^- $\qquad$ $L_\tau = -1$: $\bar{\nu}_\tau, \tau^+$

Antiparticles in general have mass & spin that are the same but additive quantum numbers that are the opposite sign.

LECTURE VII

Review of Angular Momentum Formalism:

<u>Spin-1/2</u> <u>System</u> $|1/2, 1/2\rangle \equiv \begin{pmatrix}1\\0\end{pmatrix}$; $|1/2, -1/2\rangle \equiv \begin{pmatrix}0\\1\end{pmatrix}$

General state $\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix}$

$|\alpha|^2 \equiv$ probability that S_z yields value $+\hbar/2$

$$|\alpha|^2 + |\beta|^2 = 1$$

To measure probability of measuring $\pm\hbar/2$ along S_x or S_y
$\Rightarrow$ Construct operators in matrix form.

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \quad \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\i & 0\end{pmatrix} \quad \hat{S}_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\frac{\hbar}{2}$$

$S_i \equiv \frac{\hbar}{2}\sigma_i \qquad \sigma_i \equiv$ Pauli spin matrices

$\chi_\pm \equiv$ eigenvectors of $\hat{S}_x \equiv \begin{pmatrix}1/\sqrt{2}\\\pm 1/\sqrt{2}\end{pmatrix}$

$$\begin{pmatrix}\alpha\\\beta\end{pmatrix} = a\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix} + b\begin{pmatrix}1/\sqrt{2}\\-1/\sqrt{2}\end{pmatrix}$$

$|a|^2$ is the probabaility of finding $+\hbar/2$ along S_x

$$|a|^2 + |b|^2 = 1$$

<u>Algorithm</u>:

- Find matrix for observable
- Find eigenvalues & eigenvectors
- Probability given by coefficients.

LECTURE VII

Spin-1/2 formalism occurs often in particle physics.
Example :- Free fermions, iso spin,
rotations in spinor space (2-D space)

$$U(\theta) = e^{-i(\theta\cdot\sigma)/2} \qquad \begin{pmatrix}\alpha'\\ \beta'\end{pmatrix} = U(\theta)\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

Similar to $\begin{pmatrix}x'\\ y'\\ z'\end{pmatrix} = R(\theta)\begin{pmatrix}x\\ y\\ z\end{pmatrix}$ rotations of 3-vectors

$U(\theta)$: 2-D representation of $SU(2)$
$R(\theta)$: 3-D representation of $SU(2) \Longleftrightarrow SO(3)$

Addition of Angular Momenta

$\vec{L}$ or $\vec{S}$: $L^2 \Longrightarrow l(l+1)\hbar^2$

L_z or S_z : $L_z \Longrightarrow m_l \hbar$ ($-l$ to l)

Particles are in a state of definite $\vec{L}$ or $\vec{S}$
$\vec{L}$ can be anything but $\vec{S}$ is a ground state property

Examples: π^0 has $S=0$, electron has $S = 1/2$

Example: Electron in hydrogen atom:

$|l\ m_l\rangle |S\ m_s\rangle \qquad l=-1$
$|2\ -1\rangle |\ 1/2\ 1/2\rangle \qquad S = 1/2 \qquad$ F state.

Often we need to know $\vec{J} = \vec{L} + \vec{S}$
What j states are allowed and with what probability?

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generally $\vec{J} = \vec{J_1} + \vec{J_2}$ $\quad |j_1 - j_2| < j < |j_1 + j_2|$

$m = m_1 + m_2$.

Probabilities can be looked up in table of Clebsch-Gordon coefficients.

$$|j_1 m_1\rangle |j_2 m_2\rangle = \sum_{j=|j_1-j_2|}^{|j_1+j_2|} C^{j\, j_1\, j_2}_{m\, m_1\, m_2} \, |j m\rangle$$

Suppose electron is in $|2, -1\rangle |\tfrac{1}{2}\, \tfrac{1}{2}\rangle$
$\qquad l \;\; m \quad s \;\; m_s$.

Possible values of j are $3/2$ & $5/2$

$m = m_1 + m_2 = -1 + 1/2 = -1/2$

Look at the <u>$2 \times 1/2$ table</u>

		j: $5/2$	$3/2$
		m: $-1/2$	$-1/2$
m_1	m_2		
0	$-1/2$	$3/5$	$2/5$
-1	$+1/2$	$2/5$	$-3/5$

$$|2, -1\rangle |\tfrac{1}{2}\, \tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{5}}\, |5/2\; -1/2\rangle - \sqrt{\tfrac{3}{5}}\, |3/2\; -1/2\rangle$$

<u>Example</u>: A $q\bar{q}$ meson has $l = 0$. What s values are allowed? $\quad 1/2 + 1/2 = 1$ or $1/2 - 1/2 = 0$

$\Longrightarrow$ spin 1 or spin 0.

Spin 1 mesons: $\rho^+, \rho^-, \rho^0, K_s^*, \phi, \omega$ $\qquad$ <u>vectors</u>

Spin 0 mesons: $\pi^+, \pi^-, \pi^0, K_s, \eta, \eta'$ $\qquad$ <u>scalars</u>

LECTURE VII

<u>Baryons</u>: 3 quark states: If $l=0$ then $\frac{1}{2}+\frac{1}{2}$ & $\frac{1}{2}-\frac{1}{2}$ combines with $\frac{1}{2}$ to give $\underline{+\frac{1}{2}}$ or $\underline{+\frac{3}{2}}$

If mesons have non-zero l, then $l+1$, l & $l-1$ are possible
If baryons have non-zero l, then $l-\frac{3}{2}$, $l-\frac{1}{2}$, $l+\frac{1}{2}$ & $l+\frac{3}{2}$

<u>Concrete Example of Formalism</u>: ISOSPIN

After neutron discovery (1932), Heisenberg noted:
$m_n \simeq m_p$ ∴ if electromagnetism is ignored then n & p are identical ∴ most of nucleon mass comes from strong force binding.

Propose: $p \equiv \begin{pmatrix}1\\0\end{pmatrix}$ $n \equiv \begin{pmatrix}0\\1\end{pmatrix}$ are two states of the same particle.

General state: $N = \begin{pmatrix}\alpha\\\beta\end{pmatrix} = \alpha p + \beta n$

Introduce new quantum number I or Isospin (in analogy with spin angular momentum)
<u>What is the physics?</u> That strong interaction Hamiltonian is invariant under isospin rotations. (Internal symmetry)

Protons & neutrons are in 2-D representation of isospin rotations: $SU(2)_I$

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3-D representation: <u>triplet</u>

π^+ $|1,1\rangle$
π^- $|1,0\rangle$
π^0 $|1,-1\rangle$

<u>Quadruplet</u>

Δ^{++} : $|3/2, 3/2\rangle$
Δ^{+} : $|3/2\ 1/2\rangle$
Δ^{0} : $|3/2\ -1/2\rangle$
Δ^{-} : $|3/2\ -3/2\rangle$

Goes beyond classification:

<u>Example</u> Combine two nucleons: p & n.

Can form state of total $I=1$ (triplet) or 0 (singlet)

TRIPLET
$$\begin{cases} |1,1\rangle = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle = pp \\ |1,0\rangle = \frac{1}{\sqrt{2}}\left[|\tfrac{1}{2}\ \tfrac{1}{2}\rangle|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle + |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle\right] \\ \qquad\qquad = \frac{1}{\sqrt{2}}(np+pn) \\ |1,-1\rangle = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle = nn \end{cases}$$

SINGLET $$|0,0\rangle = \frac{1}{\sqrt{2}}\left[|\tfrac{1}{2}\ \tfrac{1}{2}\rangle|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle - |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle\right]$$
$$= \frac{1}{\sqrt{2}}(pn-np)$$

In nature, only one bound 2-nucleon state:
Deuteron $\Longrightarrow$ singlet ($I=0$)

<u>Speculation</u>: $H_{int} = K\,\vec{I}_1\cdot\vec{I}_2$ since

$$\langle I_1\cdot I_2\rangle = +1/4 \text{ (triplet) or } -3/4 \text{ (singlet)}$$

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LECTURE VII

Application to scattering: Example 1.

$$|1,1\rangle \Leftrightarrow p+p \longrightarrow d+\pi^+ \Leftrightarrow |1,1\rangle \quad \mathrm{I}$$
$$\frac{1}{\sqrt{2}}\left[|1,0\rangle+|0,0\rangle\right] \Leftrightarrow p+n \longrightarrow d+\pi^0 \Leftrightarrow |1,0\rangle \quad \mathrm{II}$$
$$|1,-1\rangle \Leftrightarrow n+n \longrightarrow d+\pi^- \Leftrightarrow |1,-1\rangle \quad \mathrm{III}$$

Matrix element proportional to $\langle I_i | H_{int} | I_f \rangle = M_{fi}$

If I is conserved, then $M_{fi} \propto \delta_{if}$

$$\mathrm{I} \Longrightarrow M^2 \propto |\langle 1|H|1\rangle|^2$$
$$\mathrm{II} \Longrightarrow M^2 \propto \frac{1}{2}|\langle 1|H|1\rangle|^2 + \frac{1}{2}|\langle 0|H|1\rangle|^2$$
$$\mathrm{III} \Longrightarrow M^2 \propto |\langle 1|H|1\rangle|^2$$

$$\Longrightarrow \boxed{\sigma_{\mathrm{I}} : \sigma_{\mathrm{II}} : \sigma_{\mathrm{III}} \Longrightarrow 1 : \tfrac{1}{2} : 1}$$

Example II: πp scattering.

$$\pi^+ + p \longrightarrow \pi^+ + p \qquad \mathrm{I}$$
$$|1,1\rangle|\tfrac{1}{2}\tfrac{1}{2}\rangle = |\tfrac{3}{2}\tfrac{3}{2}\rangle \longrightarrow |1,1\rangle|\tfrac{1}{2}\tfrac{1}{2}\rangle = |\tfrac{3}{2}\tfrac{3}{2}\rangle$$

$$\pi^- + p \longrightarrow \pi^- + p \qquad \mathrm{II}$$
$$|1,-1\rangle|\tfrac{1}{2}\tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{3}}|\tfrac{3}{2}-\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}|\tfrac{1}{2}-\tfrac{1}{2}\rangle \longrightarrow \sqrt{\tfrac{1}{3}}|\tfrac{3}{2}-\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}|\tfrac{1}{2}-\tfrac{1}{2}\rangle$$

$$\pi^- + p \longrightarrow \pi^0 n \qquad \mathrm{III}$$
$$\sqrt{\tfrac{1}{3}}|\tfrac{3}{2}-\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}|\tfrac{1}{2}-\tfrac{1}{2}\rangle \longrightarrow |1,0\rangle|\tfrac{1}{2}-\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}|\tfrac{3}{2}-\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}|\tfrac{1}{2}-\tfrac{1}{2}\rangle$$

LECTURE VII

$$\mathrm{I} \Rightarrow M^2 \propto |\langle \pi^+ p | H | \pi^+ p\rangle|^2 = |\langle 3/2 | H | 3/2\rangle|^2$$

$$\mathrm{II} \Rightarrow M^2 \propto |\langle \pi^- p | H | \pi^- p\rangle|^2 = \left(\frac{1}{3}\right)^2 |\langle 3/2 | H | 3/2\rangle|^2 + \left(\frac{2}{3}\right)^2 |\langle 1/2 | H | 1/2\rangle|^2$$

$$\mathrm{III} \Rightarrow M^2 \propto |\langle \pi^- p | H | \pi^0 n\rangle|^2 = \left(\frac{\sqrt{2}}{3}\right)^2 |\langle 3/2 | H | 3/2\rangle|^2 - \left(\frac{\sqrt{2}}{3}\right)^2 |\langle 1/2 | H | 1/2\rangle|^2$$

In πp scattering, a large resonance (peak) is observed at invariant mass of about 1.1 GeV. The ratio $\sigma_{\mathrm{I}} : \sigma_{\mathrm{II}} : \sigma_{\mathrm{III}}$ is $9 : 1 : 2$.
(Δ resonance)
We deduce that the resonance has definite I

If $I = 1/2$ then : $\langle 3/2 | H | 3/2\rangle = 0$ &
$\sigma_{I} : \sigma_{II} : \sigma_{III}$ is $1 : \frac{4}{9} : \frac{2}{9}$ or $9 : 4 : 2$.

If $I = 3/2$ then $\langle \cancel{3}/2 | H | 1/2\rangle = 0$ &
$\sigma_{I} : \sigma_{II} : \sigma_{III}$ is $1 : \frac{1}{9} : \frac{2}{9}$ or $9 : 1 : 2$

We conclude that Δ resonance has $I = 3/2$

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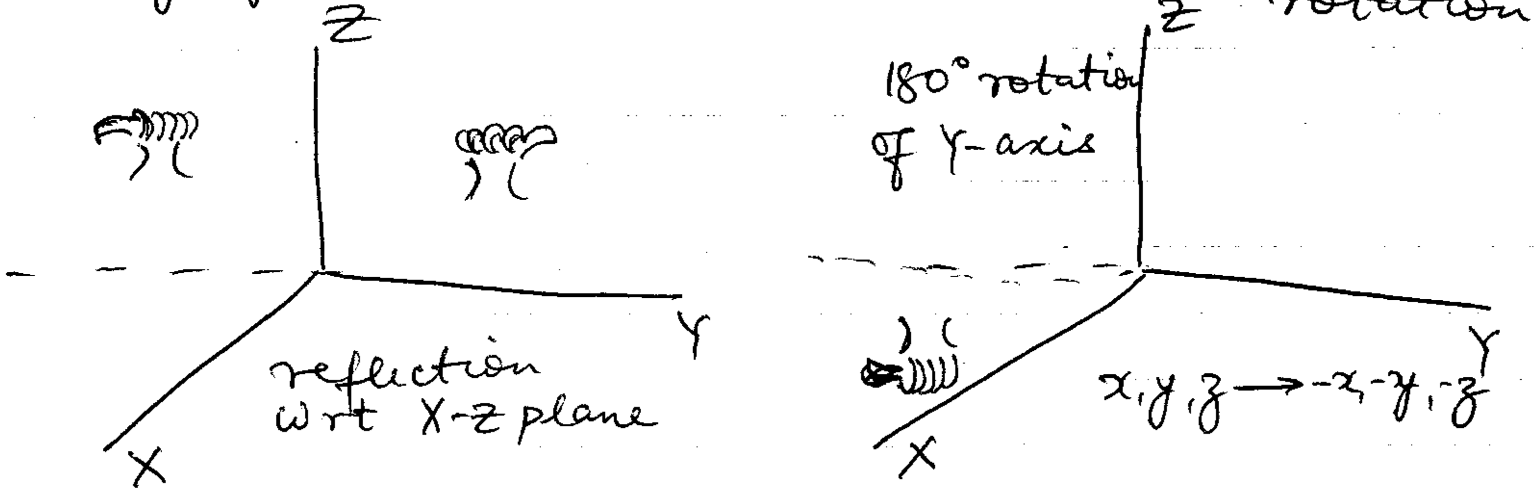
LECTURE VIII

There are quantum numbers associated with continuous symmetries & discrete symmetries

DISCRETE SYMMETRIES C, P, T

Parity operation $P \equiv$ Inversion $\equiv$ reflection + 180° rotation



$P\,\psi(\vec{r}) = \psi(-\vec{r}) \qquad \vec{X} \to -\vec{X} \qquad \vec{P} \to -\vec{P}$

But $\vec{L} \to \vec{L}$ because $\vec{L} = \vec{r} \times \vec{P}$

$\vec{r}$ & $\vec{P}$ are (polar) vectors

$\vec{L}$ is an example of an axial or pseudo vector.

Since $P^2\psi(x) = \psi(x)$, the group has 2 elements: P & I.

If $[H, P] = 0 \Rightarrow H\psi = E\psi$ & $P\psi = \pi\psi$, $\pi = \pm 1$

If the hamiltonian is invariant under parity transformations, the π is conserved & observable

Example: $Y_\ell^m(\theta, \phi)$ are parity eigenf^{ns} with $\pi = (-1)^\ell$

In a theory where parity is conserved, vectors & axial vectors do not get added.

e.g. $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

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LECTURE VIII

Implications of Parity Conservation:

Consider: $a + b \longrightarrow c + d$

$\pi_i = \pi_a \pi_b (-1)^l \qquad \pi_f = \pi_c \pi_d (-1)^{l'}$

l & l' reflect relative motion

- One can assign intrinsic parities to particles
- Start with $\pi_p = \pi_n = +1$

Consider $d\pi^- \longrightarrow nn$ ✓

$\longrightarrow nn\gamma$ ✓

$\longrightarrow nn\pi^0$ ×

$|nn\rangle$ state must be antisymmetric $= \psi_{space}\psi_{spin}$

spin symmetric $\Downarrow$ space antisymmetric $\Downarrow$ $l' = 1, 3 \dots$

spin antisymmetric $\Downarrow$ space symmetric $\Downarrow$ $l = 0, 2 \dots$

$d : L = 0, \; S = 1 \implies \pi_d = \pi_p \pi_n = +1$

(Assume $\pi_p = \pi_n$)

$d\pi^-$ is known to react in the S-state.

$= |\text{initial}\rangle = \pi_p \pi_n \pi_{\pi^-}$

Total angular momentum $= 1 \implies l' = 1$

$\implies |nn\rangle = \pi_n^2 (-1)^{l'} = -1$

$\implies \pi_{\pi^-} = -1$

One can show π_{π^0} & $\pi_{\pi^+} = -1$

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LECTURE VIII

Particles are assigned S^{π} (spin-parity)

π mesons:	0^-	pseudo-scalars
	0^+	scalars
ρ mesons	1^-	vectors
	1^+	pseudovector

As new particles were found, they were assigned π

1956: τ-θ puzzle

$$\theta^+ \longrightarrow \pi^+ + \pi^0 \qquad \tau^+ \longrightarrow \pi^+ + \pi^0 + \pi^0$$

$$P = +1 \qquad\qquad P = -1$$

Same mass but different parities.

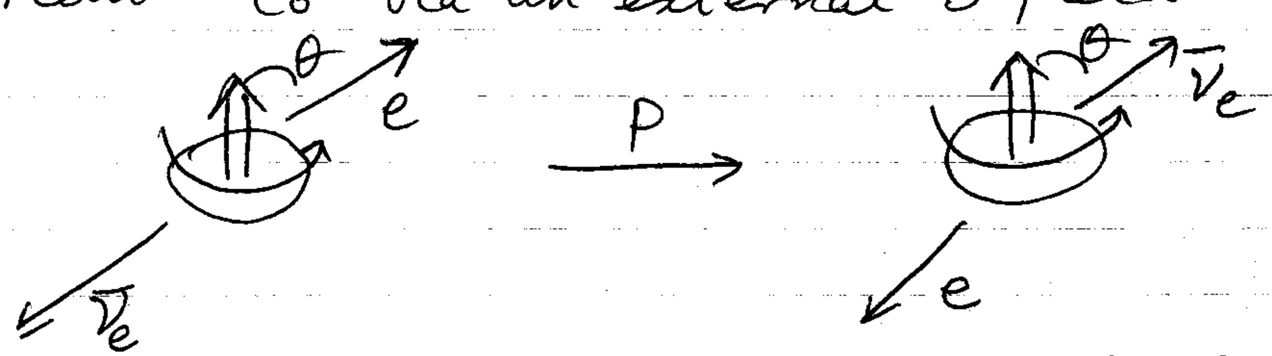
Proposal (Lee & Yang) Particles decayed by weak interaction (narrow width, long lifetime)

What if parity is conserved in strong interactions but not in weak interactions.

C.S. Wu et al. $\quad {}^{60}Co \longrightarrow {}^{60}Ni + e^- + \bar{\nu}_e$

$$\Delta J = 1$$

Orient ${}^{60}Co$ via an external B field.



If electron flux not symmetric in θ, then Parity violation

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LECTURE VIII

We now know the ν_e are left-handed and anti-neutrinos are right-handed.

X not possible Observed: electrons mostly left-handed.

CHARGE CONJUGATION Particle to anti-particle

$C|p\rangle = |\bar{p}\rangle$ All quantum numbers flip sign except m & s

Only particles that are their own anti particles are eigen states of C.

$C|\pi^0\rangle = \pm|\pi^0\rangle$ $\quad C|\gamma\rangle = -|\gamma\rangle$
(From Maxwell's equations)

$\pi^0 \rightarrow 2\gamma \Rightarrow C|\pi^0\rangle = +|\pi^0\rangle$

Hence $\pi^0 \rightarrow \gamma\gamma\gamma$ is forbidden.

Other examples: $p + \bar{p} \rightarrow \pi^+ + \pi^- + \pi^0$
The π^+ & π^- distributions must be identical if C is conserved in strong interactions.

LECTURE VIII

Charge Conjugation invariance is NOT obeyed by <u>Weak</u> interactions

Consider $\pi^+ \longrightarrow \mu^+ + \nu_\mu$ & $\pi^- \longrightarrow \pi^- + \bar{\nu}_\mu$

$$\overset{\nu_\mu}{\longleftarrow\!\!\Rightarrow}\ \pi^+\ \overset{\mu^+}{\Leftarrow\!\!\longrightarrow} \quad \xrightarrow{\ C\ } \quad \overset{\bar{\nu}_\mu}{\longleftarrow\!\!\Rightarrow}\ \pi^-\ \overset{\mu^-}{\Leftarrow\!\!\longrightarrow}$$

<u>Observed</u>   P   <u>not observed</u>

$$\overset{\mu^-}{\longleftarrow\!\!\Leftarrow}\ \pi^-\ \overset{\bar{\nu}_\mu}{\Rightarrow\!\!\longrightarrow}$$

<u>observed</u>

$\Longrightarrow$ CP is conserved.

$\Longleftrightarrow$ matter - anti-matter.

However, small amount of CP violation has been observed in special systems (separate topic)

<u>TIME REVERSAL</u> $\quad T\psi(t) = \psi^*(-t)$

$\Rightarrow$ All reactions are reversible if T is conserved.

$\not{CP} \Longrightarrow \not{T} \Longrightarrow$ permanant electric dipole moment non-zero for particles.

<u>CPT invariance</u> means all particles & anti-particles have the same mass & lifetime.

All Known Quantum Field Theories require CPT invariance

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LECTURE VIII

Resonance Scattering:
$\pi + p$ forms an intermediate state of definite mass, width, I, L, S, π. i.e. definite quantum #s.

Some of the new particles were "strange".

$$\pi^- + p \longrightarrow K^+ + \Sigma^-$$
$$\pi^+\pi^0 \leftarrow K^+ \qquad \Sigma^- \rightarrow n + \pi^-$$

These particles were very long-lived i.e. very narrow width.

$$\pi^- p \longrightarrow K^+\Sigma^-, \ K^0\Sigma^0, \ K^0\Lambda$$
$$\not\rightarrow \pi^+\Sigma^- \quad K^0 n$$

'Strange' particles produced in pairs.
$\pi, n, p \Rightarrow S=0 \qquad K_s : S=+1 \qquad \Sigma_s, \Lambda_s : S=-1$.

Propose: Strange particles produced in pairs via the strong force: S is conserved in strong reactions

Strange particles must decay by an interaction that does not conserve S: weak interaction.

	π^-	p	$\rightarrow$	Λ +	K^0	Σ^-	K^+
S	0	0		-1	$+1$	-1	1
I	1	1/2		0	1/2	1	1/2
I_3	-1	1/2		0	$-1/2$	-1	1/2

$$\begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \Longrightarrow C \Longrightarrow \begin{pmatrix} ? \\ K^- \end{pmatrix} \ \bar{K}^0 /$$

Unlike π^0! antiparticles!

10/4/05

LECTURE VIII

Categorization of hadrons:

BARYONS $\frac{1}{2}^+$

$S=0$		n		p		$I=\frac{1}{2}$	
$S=-1$	Σ^-		Σ^0		Σ^+	$I=1$	Λ^0 $I=0$
$S=-2$		Ξ^-		Ξ^0		$I=\frac{1}{2}$	
I	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1		

Along diagonal: same charge!

Increasing S : heavier mass
Same S : similar mass

MESONS 0^-

$S=1$		K^0		K^+	
$S=0$	π^-		π^0		π^+
$S=-1$		K^-		$\bar{K}^0$	
I	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1

Same pattern!

$$Q = I_3 + \frac{B}{2} + \frac{S}{2}$$

LECTURE VIII

$\frac{3}{2}^+$

$S=0$	Δ^- Δ^0 Δ^+ Δ^{++}	$I=3/2$
$S=-1$	Σ^{*-} Σ^{*0} Σ^{*+}	$I=1$
$S=-2$	Ξ^{*-} Ξ^{*0}	$I=1/2$
$S=-3$	Ω^-	$I=0$

Ω^- predicted: (mass & decay mode) by Gell-Mann

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LECTURE IX

Quark Model : Gell-Mann proposal

All hadrons (baryons & mesons) are composed of 3 elemantary particles called quarks : u, d, s

Why 3 quarks ? We wont go through the detailed group theory arguments. Instead we will take a simpler example of 2 quarks u & d.
with $I = 1/2 \Longrightarrow$

$$u = |1/2 \; 1/2\rangle \qquad d = |1/2 \; -1/2\rangle$$

$$\bar{d} = |1/2 \; 1/2\rangle \qquad \bar{u} = |1/2 \; -1/2\rangle$$

Propose that mesons are made of $q\bar{q}$ pairs with integral charge

$$\left.\begin{array}{lll} |1,1\rangle = u\bar{d} & \pi^+ & \rho^+ \\ |1,0\rangle = (u\bar{u} - d\bar{d})/\sqrt{2} & \pi^0 & \rho^0 \\ |1,-1\rangle = d\bar{u} & \pi^- & \rho^- \end{array}\right\} \text{TRIPLET}$$

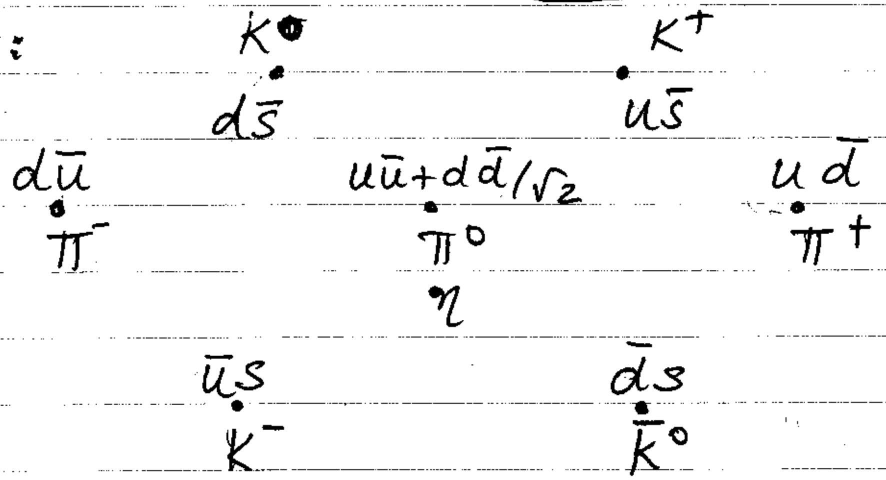
$$|0,0\rangle = (u\bar{u} + d\bar{d})/\sqrt{2} \qquad ?? \qquad \omega \qquad \text{SINGLET.}$$

In group theory language: $2 \otimes \bar{2} = 3 \oplus 1$

However: $3 \otimes \bar{3} = 8 \oplus 1$ octet & singlet

LECTURE IX

Octet:

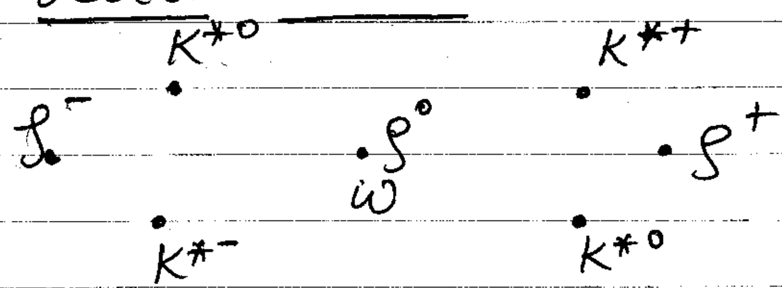


Pseudo-scalars

$$\eta \equiv \frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}} \qquad \eta' \equiv \frac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}$$

part of octet   singlet

For the vector mesons:



$$\omega = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}} \qquad \phi = s\bar{s} \text{ is not the singlet}$$

rather these two are mixed.

The reason the masses in a multiplet ~~have~~ different masses is because the s quark is much heavier than the u & d quarks.

~~Bar~~ For Baryons: $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$

Decuplet & octet are symmetric under exchange.

LECTURE IX

Total wave function $\psi = \psi(\text{space})\,\psi(\text{flavor})\,\psi(\text{spin})\,\psi(\text{color})$

For lightest baryons, $\psi(\text{space})$ is symmetric.

The observed baryons correspond to $\psi(\text{flavor})\,\psi(\text{spin})$ antisymmetric.

$\Rightarrow$ $\psi(\text{color})$ antisymmetric! QCD gives reason to believe the only colorless baryons can be observed $\Rightarrow$ asymptotic freedom (lack of free quarks)

$\frac{1}{2}^+$

	n $\dot{u}dd$	p $\dot{u}ud$	
Σ^- $dd\dot{s}$	Σ^0 $\dot{u}ds$	Σ^+ $u\dot{u}s$	Λ^0 $\dot{u}ds$
$\dot{\Xi}^-$ dss	Ξ^0 uss		

nonet
octet + singlet

$\frac{3}{2}^+$

ddd $\dot{\Delta}^-$	udd $\dot{\Delta}^0$	uud $\dot{\Delta}^+$	uuu $\dot{\Delta}^{++}$
dds $\dot{\Sigma}^{*-}$	uds $\dot{\Sigma}^{*0}$	uus $\dot{\Sigma}^{*+}$	
dss $\dot{\Xi}^{*-}$	uss $\dot{\Xi}^{*0}$		
sss $\dot{\Omega}^-$			

decuplet.

Quark content; not wavefunction.

e.g. $\Delta^0 \equiv \dfrac{ddu + dud + udd}{\sqrt{3}}$

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LECTURE IX

Part II of the course: Calculating with QED.

- Feynman Rules
- Feynman diagrams to calculate M
- Fermi's Golden Rule
- Cross Sections & Lifetimes

Origin of Feynman Rules?

- Write down eqns of motion for leptons & quarks
- Recognize internal symmetries of the Lagrangian
- Introduce force quanta via local gauge symmetry.

We start with:

- Relativistic Field Theory
- Dirac Eqn for free spin ½ fermions
- Gauge invariance

Lagrangian in classical mechanics:

$L(q_i, \dot{q}_i)$ can define the action $S \equiv \int_{t_1}^{t_2} dt\, L(q_i, \dot{q}_i)$

Hamilton's principle: $\delta S = 0 \Rightarrow$ Euler-Lagrange Eqns

$$\frac{\partial L}{\partial q_i} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) \quad : \text{ If } \quad L = \frac{1}{2} m \ddot{x}^2 - V(x)$$

then $-\frac{dV}{dx} = m\ddot{x} = F$

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LECTURE $\overline{\underline{IX}}$

<u>Field Theory</u>:

$\mathcal{L}$ is a functional of fields ϕ_i & its space derivatives.

$\Longrightarrow$ closed system with infinite degrees of freedom.

This is very general. In particle physics:

- The fields represent fundamental particles
- Composite objects are bound state soln^s of the theory.
- Kinetic energy terms depend on spins
- Potential energy terms contain interactions

Technically, $\mathcal{L}$ is a lagrangian density with

$$L = \int dt \int d^3x \, \mathcal{L}\left(\phi_i(\vec{x}), \frac{d\phi}{dx_i}\right)$$

<u>Relativistic Field Theory</u>: $\mathcal{L}$ is a Lorentz scaler.

with $c^2t^2 - x^2 = \text{const}$ with $c=1$.

$$a^\mu = (a^0, a^i) \qquad \mu = 0 \cdots 3, \qquad i = 1 \cdots 3$$

$$x^\mu = (t, x, y, z) = (t, x^i) = (t, \vec{x})$$

$$a_\mu = (a_0, a_i) = g_{\mu\nu} a^\mu = (a^0, -a^i)$$

$$x^\mu = (t, \vec{x}), \quad x_\mu = (t, -\vec{x}) \quad p^\mu = (E, \vec{P}) \quad P_\mu = (E, -\vec{P})$$

$$\partial^\mu = \frac{\partial}{\partial x_\mu} \equiv \left(\frac{\partial}{\partial t}, -\vec{\nabla}\right) \qquad \partial_\mu = \frac{\partial}{\partial x^\mu} \equiv \left(\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

$$\partial_\mu a^\mu = \frac{\partial a^0}{\partial t} + \vec{\nabla} \cdot \vec{a} \qquad \partial_\mu \partial^\mu = \frac{\partial^2}{\partial t^2} - \nabla^2$$

Define $S \equiv \int d^4x \, \mathcal{L}(\phi(x_\mu), \partial_\mu \phi(x_\mu))$

$$\delta S = 0 \Longrightarrow$$

LECTURE IX

$$\Rightarrow \frac{\partial \mathcal{L}}{\partial \phi} = \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right] \; ; \; \partial_\mu \phi = \frac{\partial \phi}{\partial x^\mu} \; ; \; \phi(x_\mu)$$

Example Real Scalar field (far from Source)

E.
$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu \phi)(\partial^\mu \phi) - m^2 \phi^2\right]$$

E-L Eqn $\Rightarrow \partial_\mu \partial^\mu \phi + m^2 \phi = 0$

This describes motion of a free, spin-0 particle.

Note: $E^2 = p^2 + m^2$ is automatically satisfied

$E = i\partial_0 \quad \vec{p} = -i\vec{\nabla} \Rightarrow E^2 - p^2 = -\partial_0^2 + \nabla^2 = -\partial_\mu \partial^\mu$

Thus $\partial_\mu \partial^\mu \phi$ is the kinetic energy term.

No potential energy term $\Rightarrow$ free particle

Quantum Field Theory: ϕ has definite energy ω & momentum $\vec{K}$ with $\omega^2 = K^2 + m^2$

$$\phi = \frac{1}{\sqrt{2\omega}}\left[e^{i(\vec{k}\cdot\vec{r} - \omega t)} + e^{-i(\vec{k}\cdot\vec{r} - \omega t)}\right]$$

Quantum Field Operator:

$$\phi = \frac{1}{\sqrt{2\omega}}\left[a e^{i(\vec{k}\cdot\vec{r} - \omega t)} + a^{+} e^{-i(\vec{k}\cdot\vec{r} - \omega t)}\right]$$

a^+ creates quanta associated with ϕ

a destroys quanta associated with ϕ

Eg: γ is the quantum of the EM field

e is the quantum of the Dirac field.