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LECTURE III

<u>Scattering</u> Important tool to study the <u>structure</u> of matter
- study nature of interaction among particles
- produce new, unstable particles
- isolate specific, rare reactions

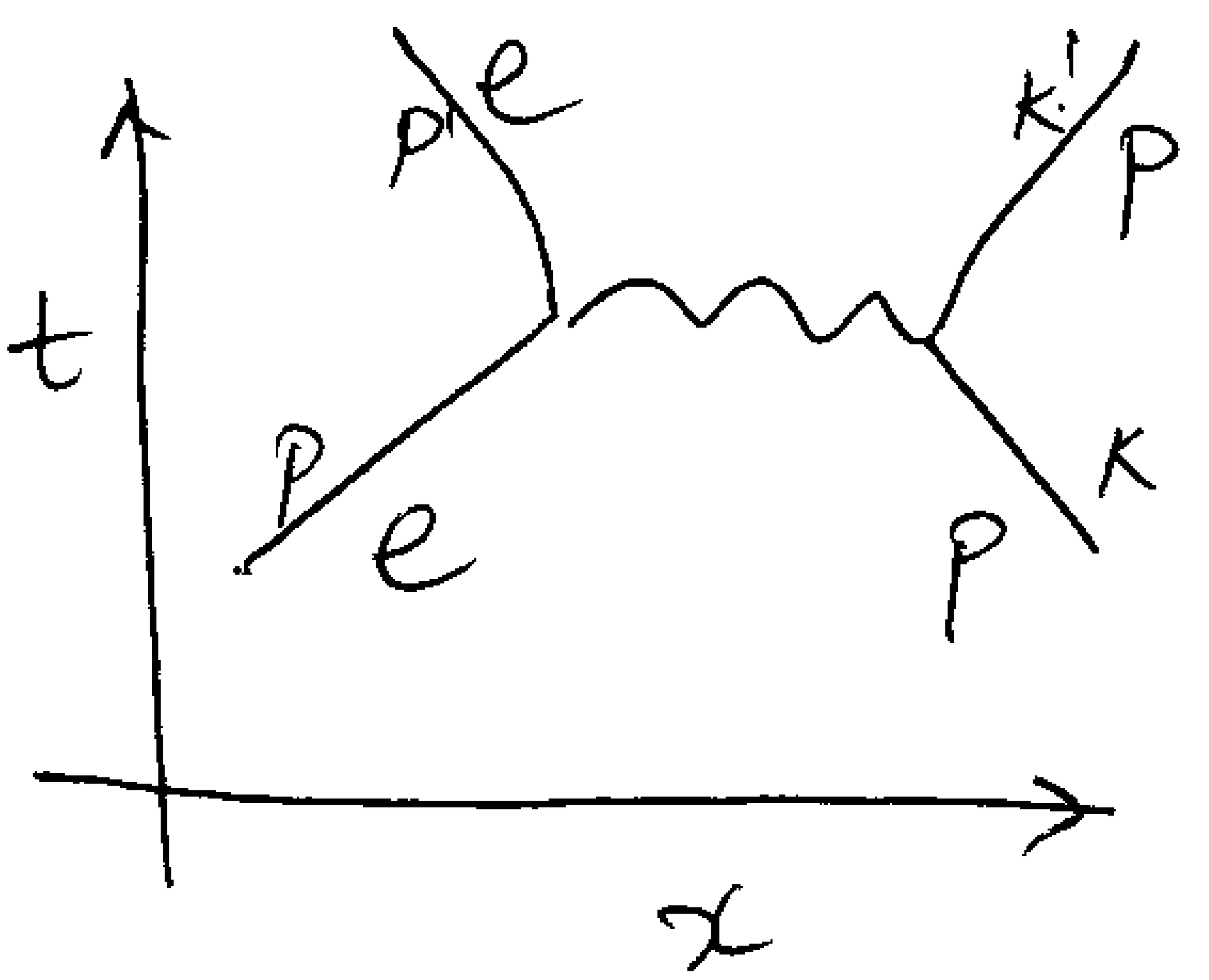
Elastic scattering $\equiv a+b \rightarrow a+b$
- Species remains the same
- Energy momentum ALWAYS conserved.

Inelastic scattering $a+b \rightarrow c+d+e \ldots$

Inclusive scattering $a+b \rightarrow a+X \ldots$

Exclusive scattering $a+b \rightarrow c+d+e$

(track all particles)



$$p - p' = k' - k$$

$$q^\mu = p^\mu - p'^\mu$$

Can prove: $q^2 \neq 0$!

"Virtual" particle exchange i.e. $E^2 - p^2c^2 \neq m^2c^4$

- No action at a distance.
- In quantum theory: exchange of virtual quanta
- Quantum carries energy & momentum.

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Energy-momentum NON-conservation allowed for a time period consistent with the uncertainty principle $\Rightarrow \Delta t \leq \hbar/\Delta E$

Higher Energy $\Rightarrow$ shorter time $\Rightarrow$ smaller range

For a massless particle: small E, p possible $\Rightarrow$ longrange force (e.g. electromagnetism)

For a massive quantum: Rest energy restricts range:

If $m=0$: $V \sim 1/r$
If $m \neq 0$: $V \sim (1/r)\, e^{-r/R}$; $R = \hbar/mc$

$R = 10^{-15}\,m \Rightarrow m = 100\ MeV/c^2$ (strong force)

$R = 10^{-18}\,m \Rightarrow m = 100\ GeV/c^2$ (Weak force)

To observe structure, look at de Broglie wavelength.

$$\lambda = h/q$$

$10^{-14}\,m \Rightarrow 20\ MeV$ (nuclear size)
$10^{-15}\,m \Rightarrow 200\ MeV$ (nuclear structure)

Several GeV $\Rightarrow$ proton structure (quarks)
100 GeV $\Rightarrow$ Weak force quantum

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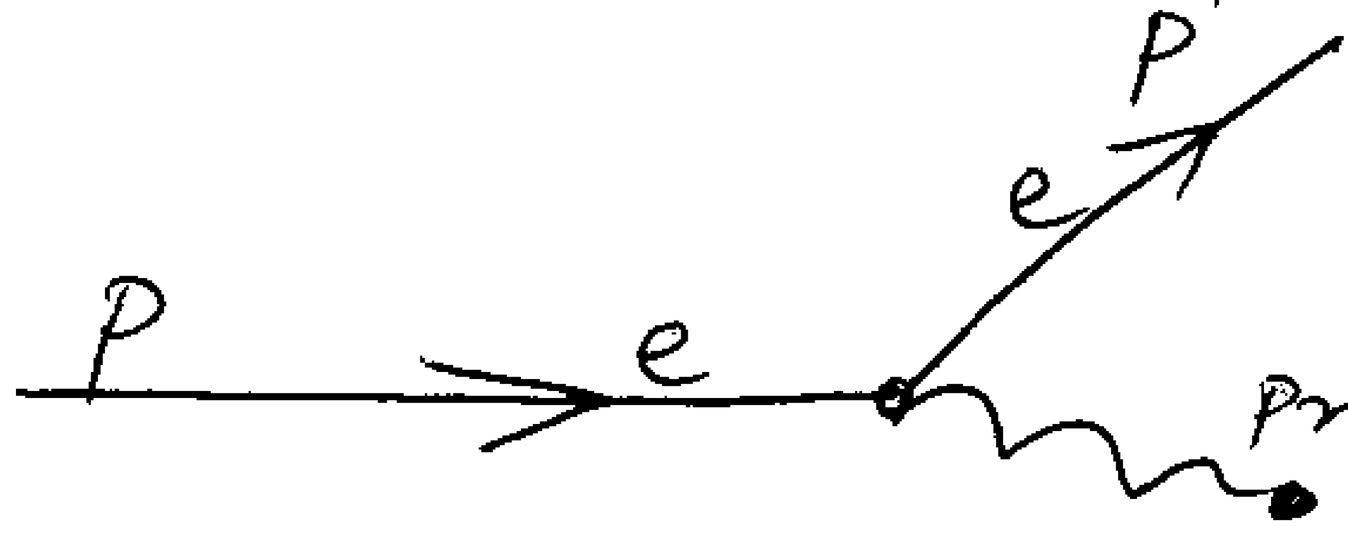
Calculation of q for a scattering reaction:



'Fixed' target reaction.

$K \simeq K'$ (electron-proton scattering)

4-momenta p, p' (electron), K, K' (proton)

$p - p' \equiv q \equiv$ 4-momentum transferred.

$$(p-p')^2 = p^2 + p'^2 - 2pp' = m_e^2c^2 + m_e^2c^2 - 2\left(\frac{EE'}{c^2} - \vec{p}\cdot\vec{p}\,'\right)$$

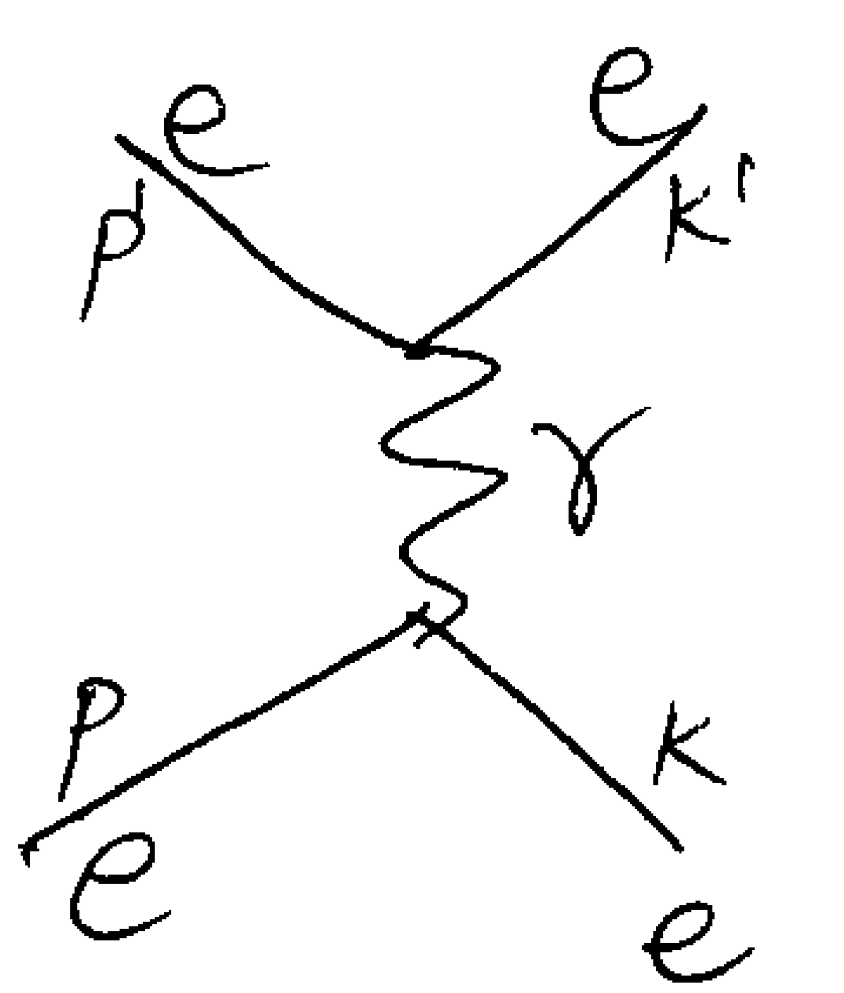
Extreme Relativistic Limit (ERL)

$$E, E' \gg m_e c^2, m_p c^2 \implies \begin{array}{l} E \simeq |p|c \\ E' \simeq |p'|c \end{array}$$

$$\implies q^2 \simeq -\frac{2EE'}{c^2}(1-\cos\theta) < 0$$

When $q^2 < 0 \implies$ 'space-like' reaction.

Calculation of q for an annihilation reaction:



$$q = p + K \implies q^2 = p^2 + K^2 + 2pK$$

If one particle at rest, then

$$q^2 \simeq 2m_e E$$

Maximum mass of new particle?

$$q^2 = m_{new}^2 c^2 \implies m_{new} \simeq \sqrt{2m_e E}$$

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For colliding beams: $\xrightarrow[e^+]{E}$ $\xleftarrow[e^-]{E}$

$$q^2 = p^2 + K^2 + 2pK \simeq 2E_{e^+}E_{e^-} - 2\,\vec{P}_{e^+}\cdot\vec{P}_{e^-}$$

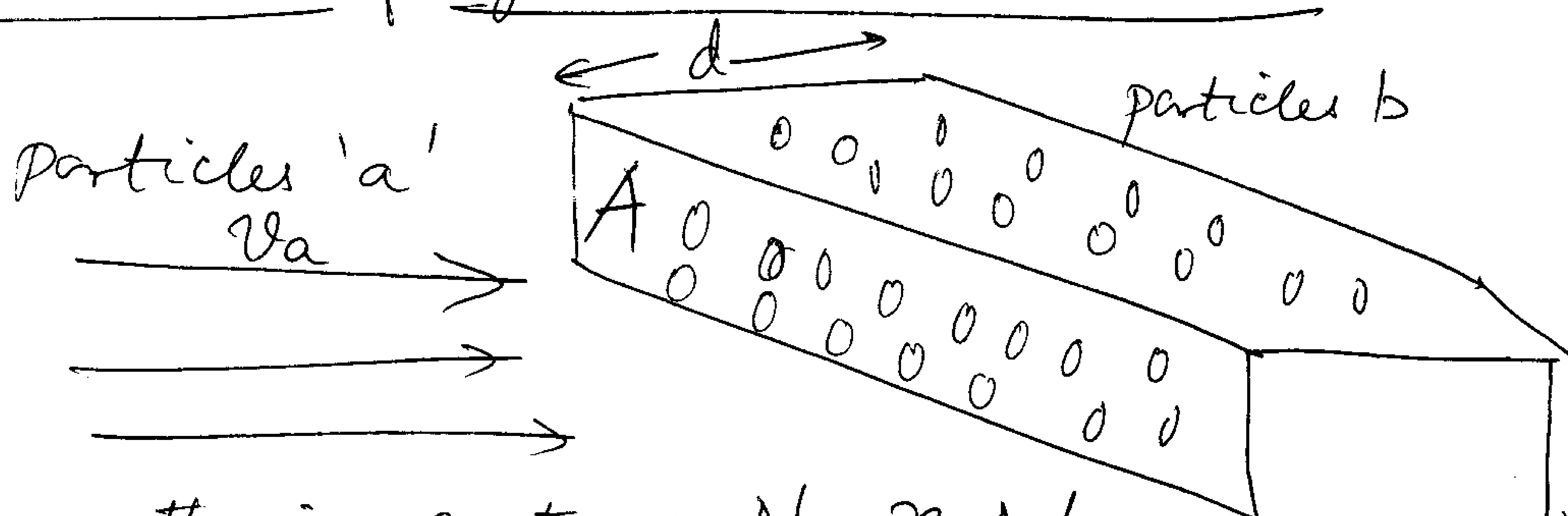
$$\because\ \vec{P}_{e^+}\cdot\vec{P}_{e^-} = -|P_{e^+}|^2 \Longrightarrow q^2 \simeq 4E_{e^+}E_{e^-}\ \ (\text{ERL})$$

Thus $m_{new}c^2 \simeq 2E_e$

Fixed target : $m_{new} \propto \sqrt{E}$
Colliders : $m_{new} \propto E$

- Colliding beams suitable for finding new particles.
- Fixed target better suited to study structure.

<u>Calculation</u> of <u>Geometric Cross-Section</u>



N_b scattering centers : $N_b = n_b A d$ (n_b = density)

$\sigma_b \equiv$ Cross-sectional area / scattering center

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Bombard target 'b' with particles 'a'
- Measure $\frac{dN}{dt}$ = # reactions / time

Flux of a $\Phi_a \equiv \frac{n_a A d}{A t} = n_a v_a$

$$\frac{dN}{dt} = \Phi_a N_b \sigma_b \qquad \Phi_a N_b \equiv \mathcal{L} \text{ or Luminosity.}$$

$$\frac{\#\text{reactions}}{\text{time}} = \frac{\text{flux of 'a'} \times \#\text{ of 'b'} \times \text{area of 'b'}}{}$$

$$\Rightarrow \text{Area of 'b'} \equiv \sigma_b = \frac{\#\text{ reations/time}}{\frac{\#\text{ projectiles}}{\text{time} \times \text{area}} \times \frac{\#\text{scattering centers}}{}}$$

$\frac{\#\text{ projectiles}}{\text{time}}$ = beam current.

$\frac{\#\text{scattering centers}}{\text{area}} \equiv$ target thickness

Nucleus : 10^{-14} m $\Rightarrow$ 1 barn $= 10^{-28} m^2$

High Energy collisions: $\sigma_{pp}(10 GeV) \simeq 10$ mb

$\sigma_{\nu p}(10 GeV) \simeq 100$ fm

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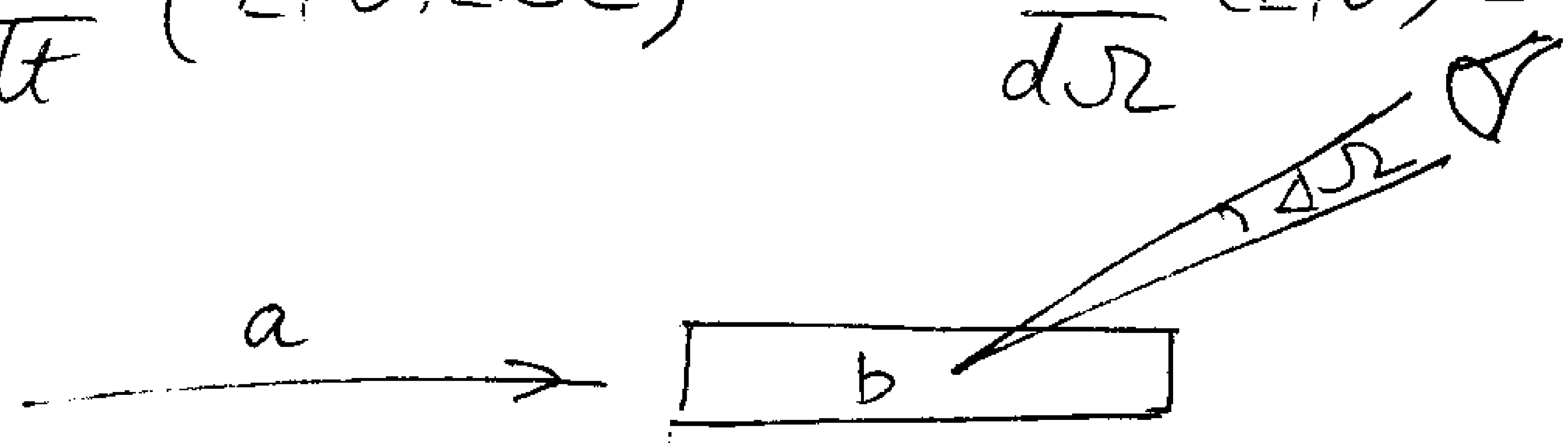
$$\sigma_b = \frac{\#\,\text{reactions}/\text{time}}{\dfrac{\#\,\text{projectiles}}{\text{time}} \times \dfrac{\#\,\text{scattering centers}}{\text{area}}} = \frac{\text{Rate}}{\text{Luminosity}}$$

In a real experiment, hard to measure 'total' rate to obtain 'total' cross-section

Differential Count rate $\Longrightarrow$ differential X section

$$\frac{dN}{dt}(E, \theta, \Delta\Omega) = \mathcal{L}\, \frac{d\sigma}{d\Omega}(E, \theta)\, \Delta\Omega$$



Sometimes, you cannot measure all energies.

$$\frac{d^2\sigma}{d\Omega dE'} \Longrightarrow \sigma_{tot} = \int_0^{E'_{max}} \int \frac{d^2\sigma}{d\Omega dE'}\, d\Omega\, dE'$$

We now extract an expression for scattering of a spinless projectile scattering of a spin-less nucleus using semi-classical methods.

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LECTURE IV

Calculation of Rutherford Cross Section

Fermi's Second Golden Rule:

Reaction rate depends on the form of the potential.
In Time-Dependent-Perturbation theory,
one defines a transition matrix element

$$M_{fi} \equiv \langle \psi_f | H_{int} | \psi_i \rangle$$

$$W \equiv \frac{\text{reaction rate}}{\#\text{beam} \times \#\text{target}} = \frac{2\pi}{\hbar} |M_{fi}|^2 \rho(E')$$

$$\rho(E') \equiv \frac{dn(E')}{dE'} \Longleftrightarrow$$ Reaction rate depends on # of available final states.

If one particle is detected: must sum over all 4-momenta of other particles allowed by energy momentum conservation.

Quantization: Each particle occupies a volume h^3 in 6D ~~x~~-p space.

In a volume V & with a momentum range dp',

$$dn(p') = \frac{V\, 4\pi p'^2 dp'}{(2\pi\hbar)^3}$$

$\frac{dn}{dE}$ calculation is straightforward & algorithmic

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LECTURE IV

$$W = \frac{dN/dt}{N_b \cdot N_a} = \frac{\phi_a N_b d\sigma_b}{N_b \cdot N_a} = \frac{n_a v_a \sigma_b}{N_a} = \frac{d\sigma_b v_a}{V}$$

$$\frac{d\sigma v_a}{V} = \frac{2\pi}{\hbar} |M_{fi}|^2 \frac{dN}{dE'}$$

$$\frac{d\sigma}{d\Omega} = \frac{V^2 E'^2}{(2\pi)^2 (\hbar c)^4} |M_{fi}|^2 \quad \left(\begin{array}{l} E \simeq pc, \\ v_a \simeq c \end{array} \right)$$

Rutherford Scattering

No spin, no recoil, interaction is a small perturbation.
(Born Approximation) $\Longrightarrow$
Incoming & Outgoing particles are plane waves.

$$\psi_i = \frac{1}{\sqrt{V}} e^{ipx/\hbar} \qquad \psi_f = \frac{1}{\sqrt{V}} e^{ip'x/\hbar}$$

V is finite but large compared to interaction range

$$H_{int} = -e\phi \qquad \nabla^2\phi = \rho/\epsilon_0$$
(Electromagnetism)

Define $q^\mu = p^\mu - p'^\mu$ (4-momentum transfer)

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$$M_{fi} = \int \psi_f^* H_{int} \psi_i \, d^3x = \frac{e}{V} \int \phi(x) \, e^{iq x/\hbar} d^3x$$

Define $\rho(x) \equiv Ze\, f(x)$

Use $\int (u \nabla^2 v - v \nabla^2 u) \, d^3x = 0$

$$\Rightarrow \quad M_{fi} = \frac{e\hbar^2}{V|q|^2} \int \nabla^2 \phi \, e^{iqx/\hbar} d^3x$$

$$= \frac{Ze^2\hbar^2}{\epsilon_0 V|q|^2} \underbrace{\int f(x) \, e^{iqx} d^3x}_{F(q) \,\equiv\, \text{Form Factor}}$$

Define $\dfrac{e^2}{4\pi\epsilon_0 \hbar c} = \dfrac{1}{137} \equiv$ Fine Structure Constant.

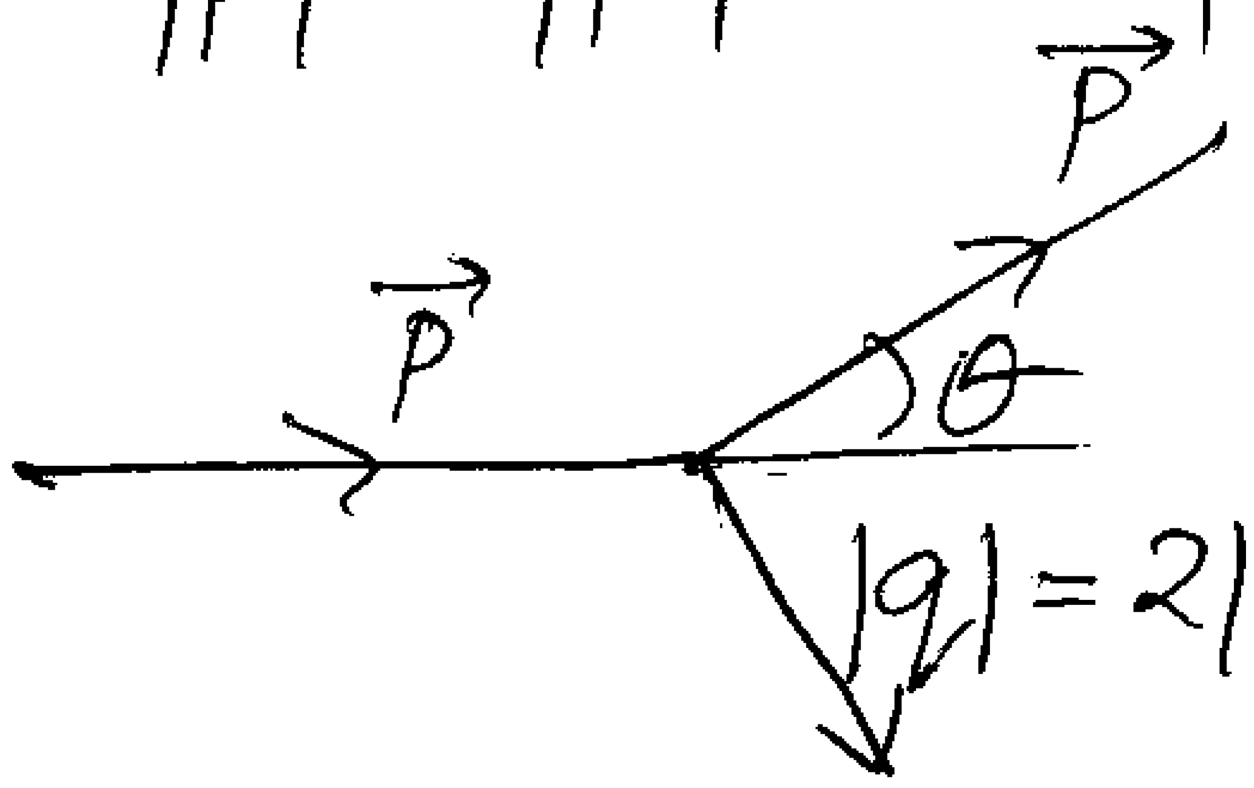
For a point heavy charge, $f(x) = \delta^3 x \Rightarrow F(q) = 1$

$$\Rightarrow \quad \frac{d\sigma}{d\Omega} = \frac{Z^2 \alpha^2 E'^2}{|q|^4}$$

To obtain Rutherford formula: use $E = E'$

$\Rightarrow$ 3-momentum transfer = 4 momentum transfer.

$|p| = |p'|$



$|q| = 2|p| \sin\theta/2$

$$\frac{d\sigma}{d\Omega} = \frac{Z^2 \alpha^2}{16 E^2 \beta^4 \sin^4 \theta/2}$$

$$\left(E = \gamma m c^2, \quad p = \gamma m \beta c, \quad \frac{E}{p} = \frac{1}{\beta} \right)$$

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LECTURE IV

What happens with all the $\hbar$'s & c's?
Use natural units: $\hbar = c = 1$

$$\Downarrow$$

LENGTH $\Longleftrightarrow$ TIME $\Longleftrightarrow$ 1/MASS

MASS $\Longleftrightarrow$ MOMENTUM $\Longleftrightarrow$ ENERGY

Need only 2 constants:
$\hbar c = 197$ MeV$\cdot$fm $\qquad \hbar^2 c^2 = 0.389$ GeV^{2}mb

For Rutherford Scattering, take 4 MeV α particles

$\Longrightarrow \beta \ll 1 \Longrightarrow E \simeq m_\alpha c^2$

$$\Longrightarrow \frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2}{16(m_\alpha c^2)^2\beta^4 \sin^4(\theta/2)}(\hbar^2c^2)\ \text{mb}$$

$$\Longrightarrow \approx 10^{-23}\,m^2$$

* Fit very well with the hypothesis of a nucleus with charge $+Ze$ smaller than 10^{-14} m
  - Few scatters at small angles ($\sigma \ll 10^{-18}$ m^2)
  - Significant scattering with $\theta > 90°$

$\Longrightarrow F(q) \simeq 1$

* Did not fit Thomsen (plum pudding) model.

LECTURE IV

- Rutherford formula is valid for $\beta \ll 1$ & spinless beam and a spinless target nucleus.

- For a spin ½ relativistic projectile, need to modify formula to incorporate conservation laws.

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{Z^2\alpha^2 E'^2}{q^4}$$

However $\frac{d\sigma}{d\Omega}(\theta=180°) \rightarrow 0$ for spin ½ beam particles.

$$\Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_R \cos^2\theta/2$$

Experimentally Mott cross section was found to be systematically smaller $\Rightarrow F(q) < 1$

$$\left(\frac{d\sigma}{d\Omega}\right)_{NUCLEUS} = \left(\frac{d\sigma}{d\Omega}\right)_M |F(q)|^2 \quad \left(F(q) \equiv \int e^{iqx} f(x)\, d^3x\right)$$

$f(x)$ is the nuclear charge distribution.

Next lecture $\Rightarrow$ analyze electron scattering data.

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LECTURE V

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Nucleus}} = \left(\frac{d\sigma}{d\Omega}\right)_M |F(q)|^2$$

$$F(q) = \int e^{iqx} f(x)\, d^3x$$

As q increases, one samples only a fraction of charge distribution since de Broglie wavelength becomes smaller than nuclear size.

For a uniform sphere of charge, $f(x) \to f(r) = \theta(r)$
θ = constant for $r < R$ & $\theta = 0$ for $r > R$

This leads to diffraction patterns (see slide 2)
First minimum at $|q| \cdot R/\hbar \cong 4.5$

Inspect slide: For ^{12}C, first minimum at $26° = \theta/2$
Since $E = 420$ MeV, $q \simeq 2E\sin\theta/2$
$\Longrightarrow R = 2.4\ fm$

- Different forms of ~~$\rho(\theta)$~~ $f(x)$ produce different $F(q)$
- Detailed measurements show that nuclei are diffuse spheres of charge.

- One can get a good estimate of ~~a~~ nuclear size with one single $d\sigma/d\Omega$ measurement at small, non-zero q value. (no detailed knowledge of $F(q)$ required) $\Longrightarrow$

LECTURE V

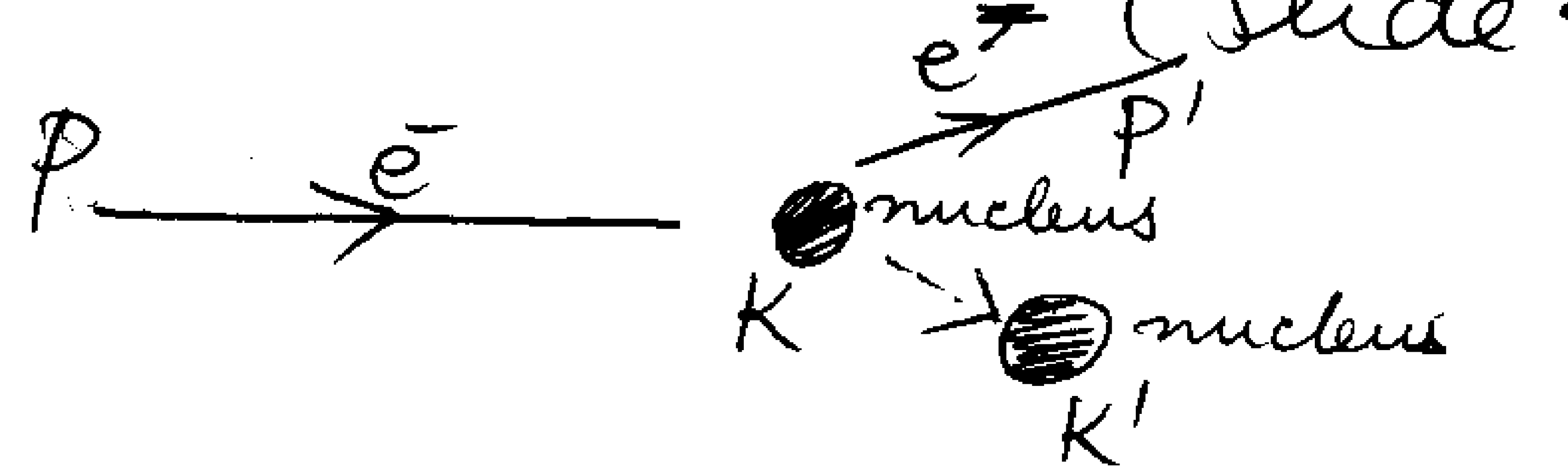
If $q \cdot R/\hbar \ll 1$, then

$$F(q^2) \simeq 4\pi \int_0^\infty f(r)\, r^2 dr - \frac{1}{6}\frac{|q|^2}{\hbar^2} 4\pi \int_0^\infty f(r)\, r^4 dr \ldots$$

Since $4\pi \int_0^\infty r^2 f(r)\, dr = 1$ & $\langle r^2 \rangle = 4\pi \int_0^\infty r^2 f(r)\, r^2 dr$

$$\Longrightarrow F(q^2) = 1 - \frac{1}{6}|q|^2 \frac{\langle r^2 \rangle}{\hbar^2} \Longrightarrow \langle r^2 \rangle = -6\hbar^2 \frac{dF(q^2)}{dq^2}$$

- Analysis of precision spectrum of ^{12}C electron scattering (Slide 4)



$E = 495$ MeV $\quad \theta = 65.4°$

What is peak at 482 MeV?

$$p + K = p' + K' \Longrightarrow p^2 + K^2 + 2pK = p'^2 + K'^2 + 2p'K'$$

For elastic scattering, $p^2 = p'^2$ & $K^2 = K'^2 \Longrightarrow pK = p'K'$

$$\Longrightarrow pK = pp' + p'K - p'^2$$

$p \equiv (E, \vec{p}) \quad p' \equiv (E', \vec{p}\,') \quad K = (M_N, 0)$

If $m_e \to 0$, then $E = |p|$ & $E' = |p'|$

$$\Longrightarrow pK = EM_N = EE' - |p||p'|\cos\theta + E'M_N - \cancel{m_e^2}$$

$$\Longrightarrow E' = E\left[\frac{1}{1 + \frac{E}{M_N}(1 - \cos\theta)}\right]$$

For $M_N = 12 M_p$, $E' = 482.6$ MeV $\Longrightarrow$ elastic peak

i.e. ^{12}C nucleus remains intact.

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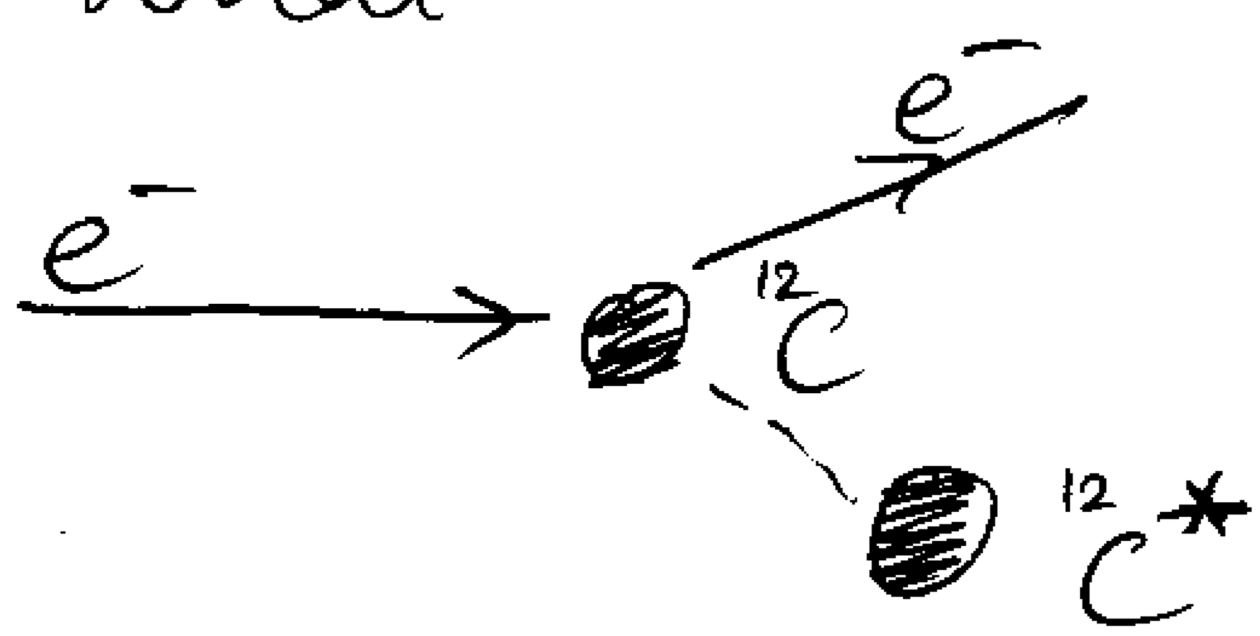
LECTURE V

What are the other peaks?



^{12}C nucleus can be excited.

$E'_{elastic} - E'_{inelastic} =$ energy of excited state

$\Rightarrow$ study nuclear structure.

- Large resonance at roughly 463 MeV/c is called the "giant dipole" resonance
- Rising continuum at lower E'?

Sometimes a proton or neutron is knocked out

$\Rightarrow$ quasi-elastic scattering.

- Must first give up 6 to 7 MeV to break up nucleus.

- $E' = 378$ MeV if $M_N = M_p$ (mass of proton)

- peak is very broad. Why?

$\Rightarrow$ Protons & neutrons are confined to a space of ~~reg~~ roughly a sphere of radius 2.5 fm.

$$\Delta p \simeq \hbar/\Delta x \Longrightarrow \Delta p = 100 \text{ MeV for } \Delta x \sim 10^{-15} m$$

- If energy of incident electron is greater than a few GeV, then one can break up the proton and ~~infer~~ its structure.
- This was done for the first time in the mid-1960s

$\Rightarrow$ DEEP INELASTIC SCATTERING

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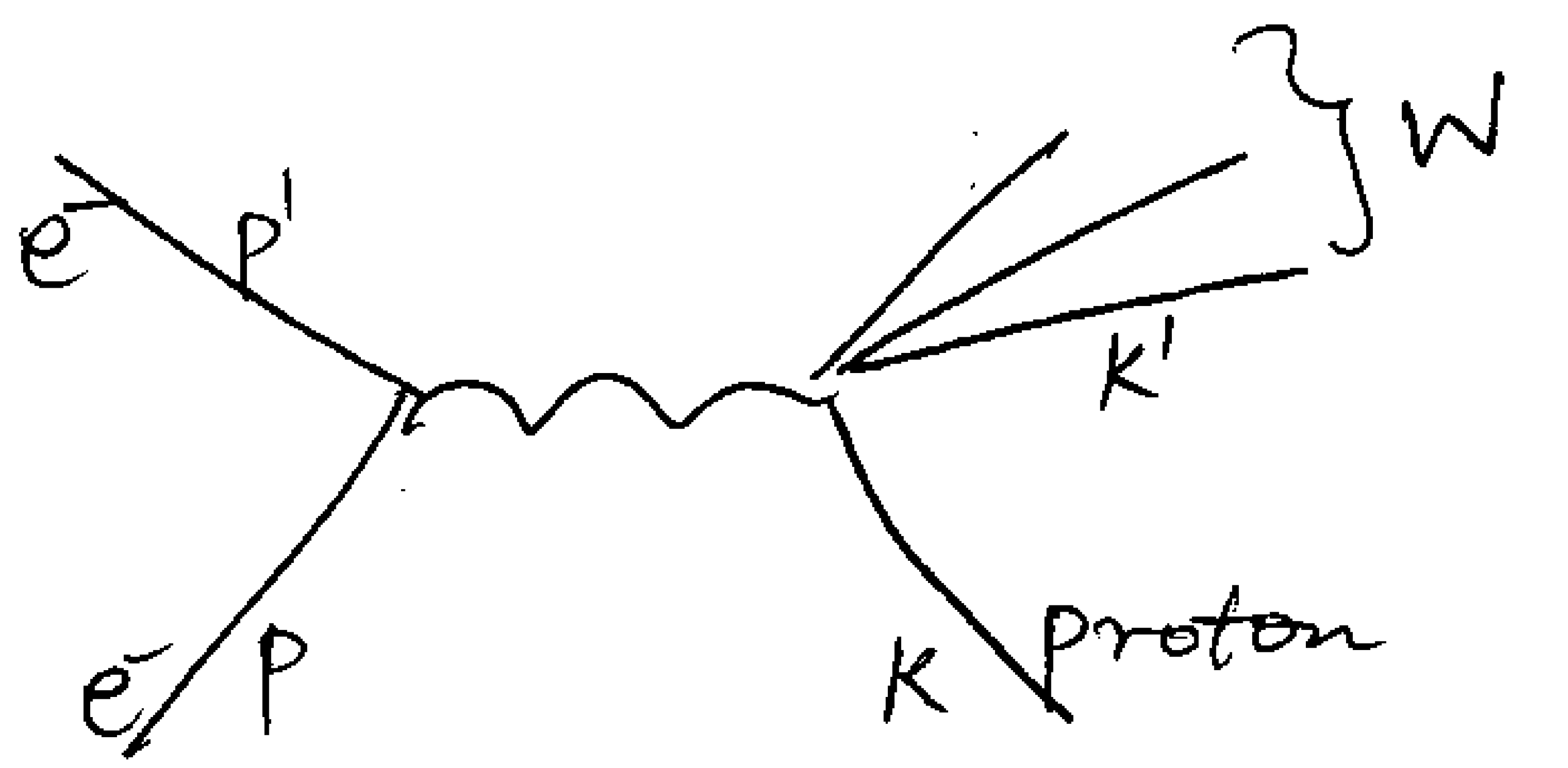
LECTURE V



$$p + k = p' + k'$$

$$k'^2 = W^2 = (p + k - p')^2 = (q + k)^2$$

$\frac{d\sigma}{d\Omega}$ is now a function of q^2 & W^2

- The Cross Section is seen to rise as $q^2 > 1\ GeV^2$
  $\Longrightarrow$ Contrary to expectation since $F(q)$ is getting smaller!
- Prametrize $\left(\frac{d\sigma}{d\Omega}\right)_{DIS} = \left(\frac{d\sigma}{d\Omega}\right)_{M} W_2(x, q^2)$

$$x = \frac{-q^2}{2kq} = \frac{-q^2}{2M(E-E')} \Longrightarrow W_2(x) \text{ independent of } q^2!$$

- x is the fraction of proton momentum ~~followed~~ carried by struck quark.
  $\Longrightarrow$ $F(q) = 1$ again!

$\Longrightarrow$ Conclusion: proton has several hard objects with no structure that are travelling at very high speeds.

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LECTURE V

UNSTABLE PARTICLES: How does one describe them? MASS & LIFETIME.

- Consider an assembly of identical particles each with decay probability λ.

$$\Rightarrow \quad dN = -\lambda N dt \Rightarrow N = N_0 e^{-\lambda t}.$$

$\tau = 1/\lambda \equiv$ lifetime i.e. time interval for N_0 to reduce to N_0/e

- Unstable particle does not have definite energy.

$\Rightarrow$ Consider a free particle at rest.

i.e. $\psi(t) = \psi_0 e^{-iE_0 t/\hbar}$, $(E_0 = mc^2)$

- $|\psi(t)|^2 = |\psi(0)|^2 \Rightarrow$ disagrees with 'unstable' model.

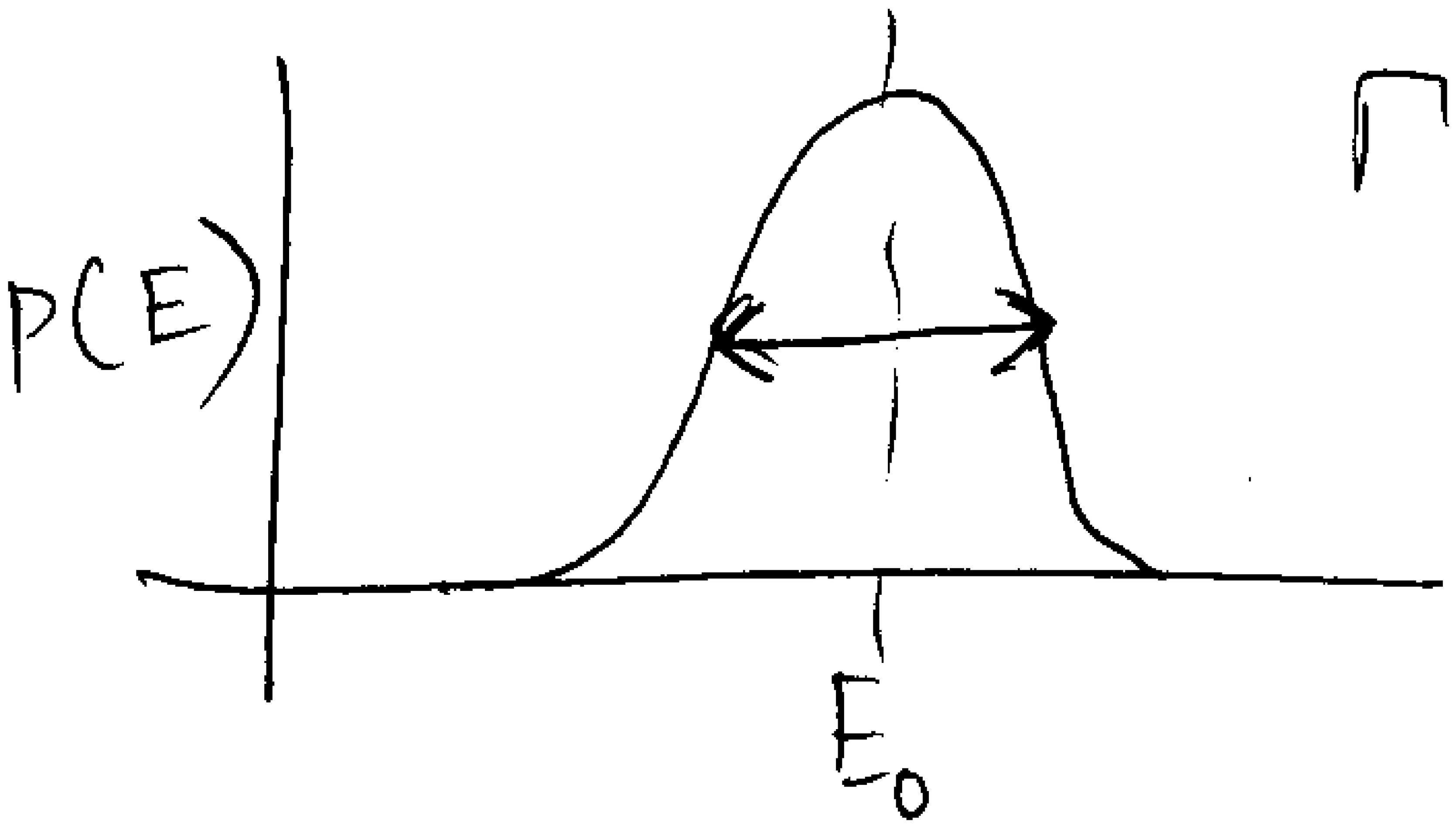
- Introduce $E = E_0 - \frac{i\Gamma}{2} \Rightarrow |\psi(t)|^2 = |\psi(0)|^2 e^{-\Gamma t/\hbar}$

$\Rightarrow \Gamma = \lambda\hbar = \hbar/\tau$ What does it mean?

Introduce $\omega = E/\hbar$ & the fourier transform

$$\Rightarrow \quad \psi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(\omega) e^{-i\omega t} d\omega$$

$$P(E) \propto g^* g \Rightarrow P(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-E_0)^2 - (\Gamma/2)^2}$$



$\Gamma \equiv$ width $\quad \tau = \hbar/\Gamma$

$\Delta E \Delta t \gtrsim \hbar \Rightarrow \Delta E \sim \Gamma \gtrsim \hbar/\tau$

Small lifetime $\Leftrightarrow$ Large Width.