11/30/04 # Lecture XXII ①

NUCLEI: made up of protons & neutrons

Z protons & electrons defines Atomic Number
$\Longrightarrow$ chemistry
N neutrons defines Neutron Number
$A = Z + N$ defines Atomic Mass Number

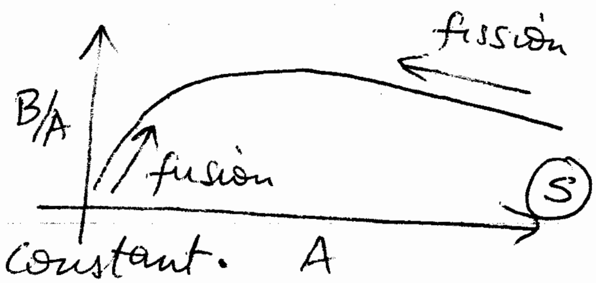
$$^{A}_{Z}X_{N} \qquad X \text{ defines } Z \Longrightarrow {}^{A}X$$

^2H, ^2H, ^3H are isotopes, also ^3He, ^4He
Same A $\Rightarrow$ Isobars   Same N $\Rightarrow$ Isotones

Binding Energy $B = M_N - \sum_Z M_p - \sum_N M_n - \sum_Z m_e$

- B roughly constant $B \sim 7\text{-}8$ MeV/nucleon for heavy nuclei

Higher B, lower energy state.



B/A constant $\Longrightarrow$ nuclear density constant.

LIQUID DROP model: $E_V = a_v A$
Corrections: - Surface nucleons : $E_s = -a_s A^{2/3}$
- Coulomb energy : $E_C = -a_c Z^2 A^{-1/3}$
- Pauli's principle : $E_s = -a_s \frac{(Z-N)^2}{A}$

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Liquid Drop Model Limitations: Mean free path comparable to size ⟹ quantum liquid.

Other Extreme: Nucleons move independent of each other except for obeying the exclusion principle.
⟹ degenerate Fermi gas

# of spatial states in volume V & between p & p+dp:

$$dn \text{ or } dp = \left(\frac{4\pi p^2 dp}{h^3}\right) V \quad (2 \text{ nucleons/state})$$

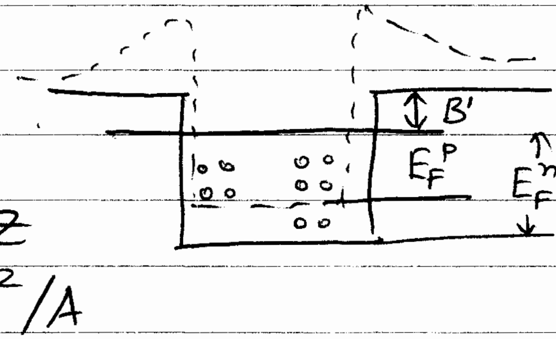
$$\Rightarrow A = Z + N = 4\int_0^{p_F} \frac{4\pi p^2 dp}{h^3} V = \frac{16\pi}{3h^3} p_F^3 V$$

$$V = 4/3 \pi r_0^3 \times A \quad \& \quad r_0 = 1.21 \, fm$$

$$\Rightarrow p_F \cong 250 \, MeV \quad \& \quad E_F \simeq \frac{p_F^2}{2M} \sim 33 \, MeV$$

Electron scattering (quasi-elastic) validates this. (S)

Picture of potential:



(S)

Total E_F minimizes for $N = Z$
Leading term $\propto (N-Z)^2/A$

Fermi Gas Model usually works for large N
⟹ electrons in solid ⟹ quantization neglected.

System of nucleons ⟹ small N ⟹ definite angular momentum

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Emperical Observation: N or Z = 2, 8, 20, 28, 50, 82 or 126 are exceptionally stable nuclei with large B'
- Large # of stable isotopes (S)
- First excited state has large E (S)
- Isotopic abundances (S)

<u>Shell Model</u>: Each nucleon assigned to a specific energy level ⟹ analog with atomic levels

Central Potential with Spherical symmetry

$$\psi \sim R_{nl}(r)\, Y_{lm}(\theta, \phi)$$

$n = 1, 2, 3, 4 \ldots$ # nodes + 1

$l = s, p, d, f, g$

nl levels are $2\cdot(2l+1)$ degenerate; $P = (-1)^l$

For low Z nuclei, V is that of 3-D harmonic oscillator.

$$E = (N + 3/2)\hbar\omega \; ; \quad N = 2(n-1) + l$$

For heavy nuclei: $V(r) = -V_0 \Big/ 1 + e^{-(r-R)/a}$

Breaks degeneracy and defines energy levels. (S)

This works well for ~~Z~~ A $\lesssim$ 20 but for large A; Spin-orbit coupling plays a big role.

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$$V(r) = V_c(r) + V_{ls}(r)\langle ls \rangle/\hbar^2$$

$$\Delta E_{ls} = \frac{2l+1}{2}\langle V_{ls}(r)\rangle$$

$\langle V_{ls}\rangle < 0$ & $\langle V_{ls}\rangle \simeq E_N - E_{N-1}$!

⇒ Changes heigrarchy of single particle energy level

⇒ Explains magic numbers (S)

J^P of nuclei can be studied in experiments to verify predictions of the shell model.

Closed shells: $J^P = 0^+$ $N, Z = 2, 8, 20, \ldots$

Examples: ${}^{16}_{8}O$, ${}^{40}_{20}Ca$, ${}^{48}_{20}Ca$, ${}^{208}_{82}Pb$

One particle & One Hole States: Last nucleon or "hole" defines J^P

Example: ${}^{15}_{7}N$, ${}^{15}_{8}O$ "mirror" nuclei

8 nucleons ⇒ closed shell

7 nucleons ⇒ "hole" in $1P_{1/2}$ ⇒ $J^P = 1/2^-$

First Excited State: nucleon to $1d_{5/2}$ or $2S_{1/2}$ ⇒ large binding energy

Example: ${}^{17}_{8}O$, ${}^{17}_{9}F$ "mirror" nuclei

1 nucleon in $1d_{5/2}$ ⇒ $5/2^+$

I Excited state: ⇒ $1/2^+$, small binding energy

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NUCLEAR STABILITY:

- α DECAY: In a heavy nucleus, some probabilty for 2 neutrons & 2 protons to bind into ^4He.

  ^4He is very stable: $B = 28.3$ MeV

  If $B(A,Z) < B(A-4, Z-2) + 28.3$ MeV

Rate of decay depends sensitively on B.

$\Rightarrow$ Stable nuclei until ^{209}Bi

Larger nuclei $\Rightarrow$ U & Th isotopes have long τ.

- FISSION: Heavy nucleus splits into 2 roughly equal pieces spontaneously.

  $\Rightarrow$ energetically favorable.

- Induced FISSION: Depends on binding energy:

^{235}U: 0 energy neutron induces fission

^{238}U: 1.4 MeV of energy needed.

- Fission produces more neutrons: $\nearrow$ ^{235}U
- Chain reaction: large fission probability + critical mass
- Controlled Chain reaction: ^{238}U + small ^{235}U + moderator

Bomb vs nuclear reactor.

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β-DECAY: Nuclei of equal mass number but different Z have a minimum with a quadratic dependence for $M(A,Z)$

If $M(A,Z) > M(A,Z+1)$ then β^- decay occurs
$n \rightarrow p + e^- + \bar{\nu}_e$ $\quad ({}^{101}_{43}Tc \rightarrow {}^{101}_{44}Ru + e^- + \bar{\nu}_e)$

If $M(A,Z) > M(A,Z-1)$ then β^+ decay occurs
$p \rightarrow n + e^+ + \nu_e$ $\quad ({}^{101}Rh \rightarrow {}^{101}_{44}Ru + e^+ + \nu_e)$

In this case another possibility:

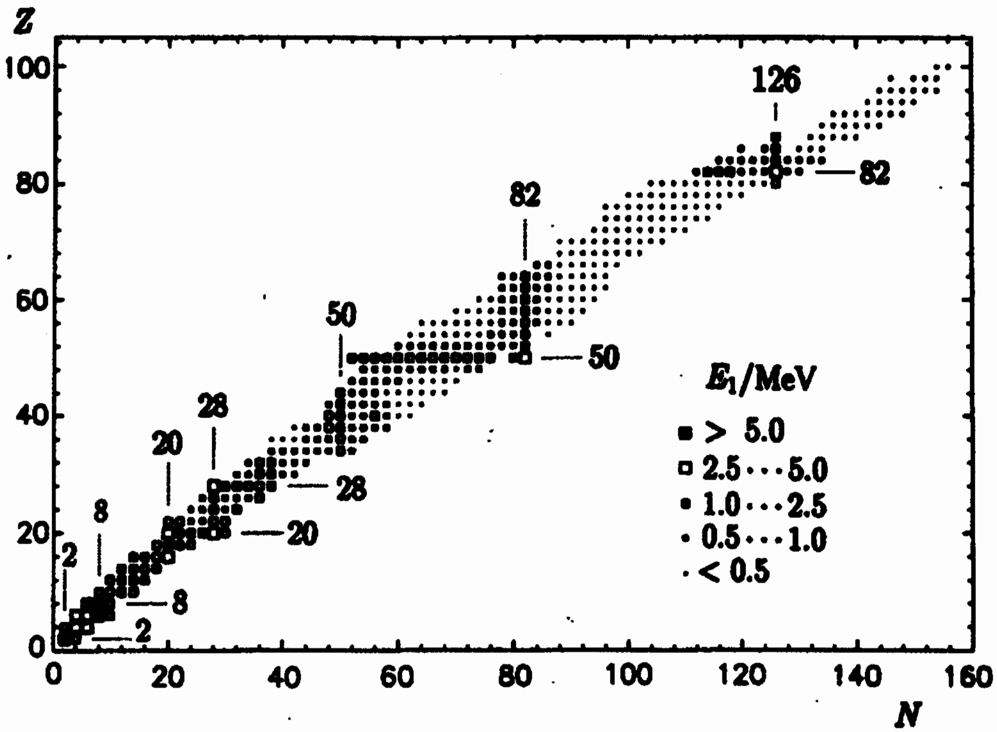
$M(A,Z) - M(A,Z-1) < 2m_e$ then,
Electron capture: $p + e^- \rightarrow n + \nu_e$

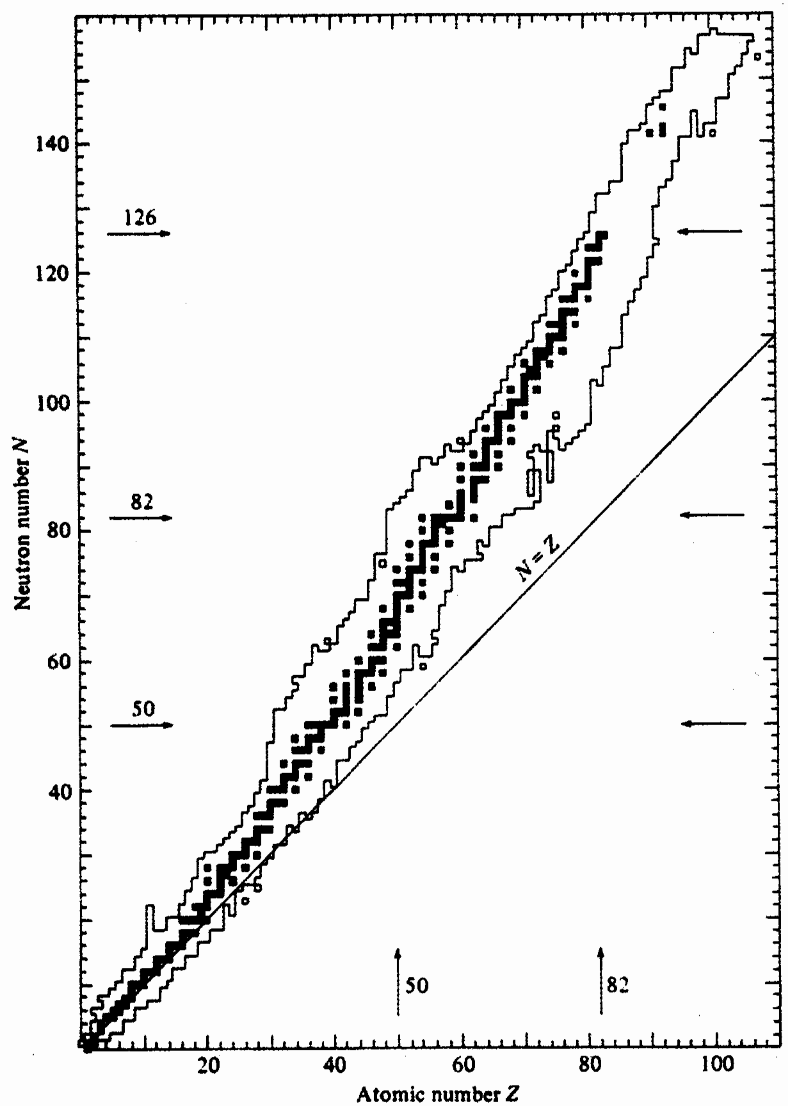
Heavy nucleus: - Nuclear radius large
- electronic orbits small.
- K shell electrons have maximum at nuclear center

- Lifetimes of unstable nuclei depend on ~~on~~ energy released (phase space) & J^P of mother & daughter nucleus.

eg: ${}^{40}K$ has long half life.
89% β^- to ${}^{40}Ca$
11% K capture to 2^+ excited state of ${}^{40}Ar$
10^{-3} β^+ decay to 0^+ G.S. of ${}^{40}Ar$

Magic Numbers





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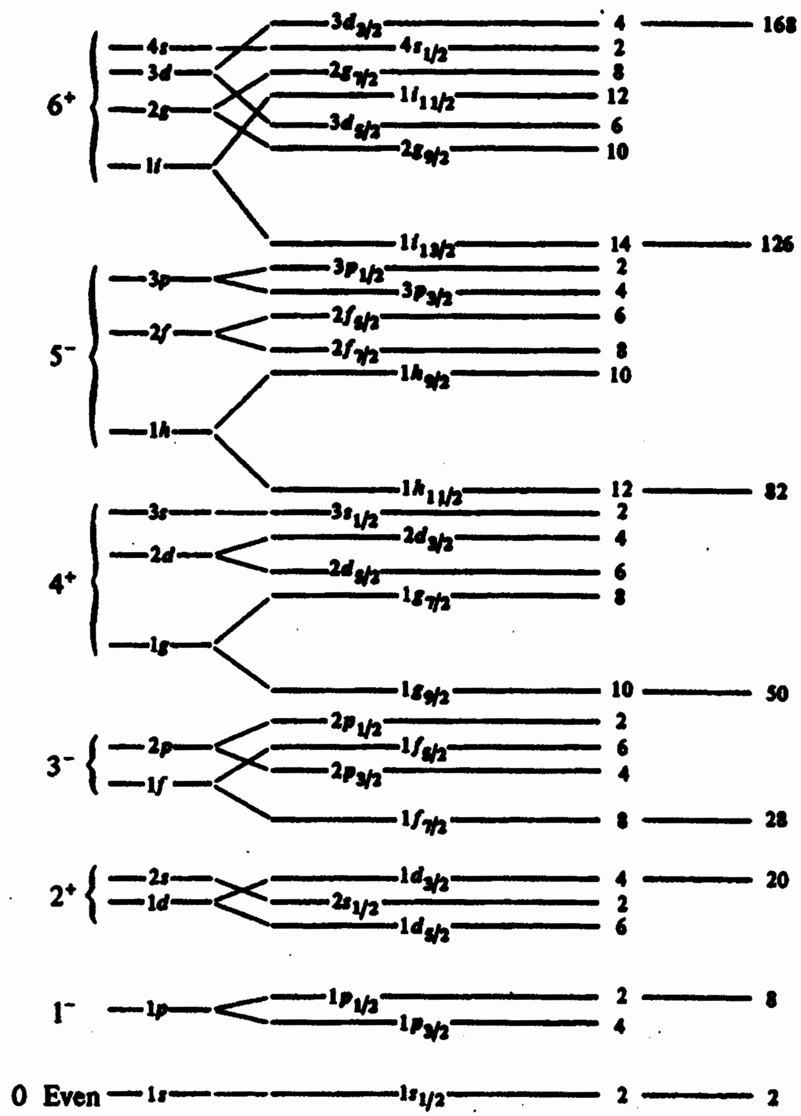
Shell Structure



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Nuclear β-Decay – β^- decay
– β^+ decay
– electron capture.

– Recall neutron decay: like μ decay except G_A was modified $1 \rightarrow 1.24$

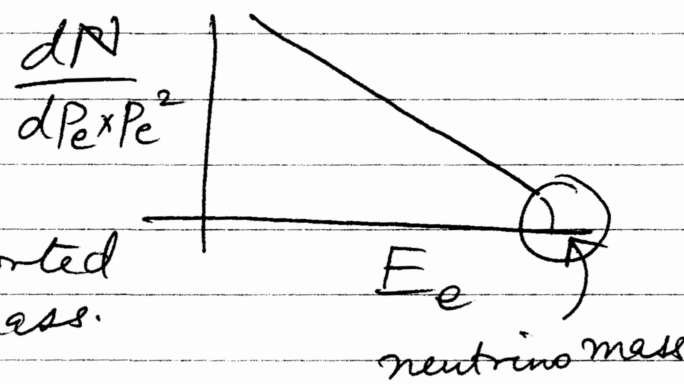
– Nuclear decay: 3 main differences
* M_{fi} contains overlap of initial & final wave f^{n}s
* phase space (β^+ or β^-) determined by binding energies
* Coulomb corrections as βs exit the nucleus.

$$d\Gamma = \frac{2\pi}{\hbar}|M_{fi}|^2 \frac{dn}{dE_0} \qquad E_0 = E_e + E_\nu + E_R \simeq E_e + E_\nu$$

In experiments, typically the βs are detected.

$$d\Gamma = \frac{2\pi}{\hbar}|M_{fi}|^2 (4\pi)^2 (E_0 - E_e)^2 p_e^2 \, dp_e$$

$$\sqrt{\frac{d\Gamma}{p_e^2 \, dp_e}} \propto |M_{fi}|(E_0 - E_e)$$



endpoint would be distorted by small neutrino mass.

Define $f \equiv \int dp_e \, F_\pm(Z, p_e) \, p_e^2 (E_0 - E_e)^2$
$F_\pm \equiv$ Fermi function.

$$\Gamma = \frac{1}{\tau} = \frac{1}{2\pi^3}|M_{fi}|^2 f \Rightarrow f\tau = \frac{2\pi^3}{|M_{fi}|^2}$$

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$f\tau$ is a measure of $|M_{fi}|$

	τ	$f\tau$
$n \to p$	10 minutes	1100
$^6He \to {}^6Li$	0.8 s	810
$^{14}O \to {}^{14}N$	1 minute	3076
$^{115}I \to {}^{115}Sn$	6×10^{14} years	7×10^9 !

Nonrelativistic QM $\quad j_\mu A^\mu = \rho\phi$

e.g. e-p scattering: $\quad \rho_e = -e\,\psi_f^*(r)\,\psi_i(r)$

$$\phi_p = \int \frac{\rho_p(r')}{|r'|}\,d^3r'$$

$$M_{fi} = \int \rho_e(r)\,\phi^p(r)\,d^3r$$

β-Decay: $\quad M_{fi} = \int \psi_e^*\,\psi_\nu\,\phi_W^N\,d^3r$

$$\phi_W^N \cong \int \frac{G_F}{\sqrt{2}}\,\psi_p^*\,\psi_n\,d^3r'$$

$$M_{fi} = \frac{G_F}{\sqrt{2}} \int \psi_e^*(r)\,\psi_\nu(r)\,\psi_p^*(r)\,\psi_n(r)\,d^3r \qquad \text{FERMI TRANSITION}$$

But weak interaction is V−A

$$M_{fi} = \frac{G_F}{\sqrt{2}} \int \psi_e^*\,\sigma\,\psi_\nu\,\psi_p^*\,\sigma\,\psi_n\,d^3r \qquad \text{GAMOW-TELLER TRANSITION.}$$

$$\psi_e^*(r)\,\psi_\nu(r) \sim \exp\left(\frac{i\vec{q}\cdot\vec{r}}{\hbar}\right) = 1 - \frac{\vec{q}\cdot\vec{r}}{\hbar} + \frac{1}{2!}\frac{(\vec{q}\cdot\vec{r})^2}{\hbar^2}\ldots$$

$\vec{l} = \vec{r}\times\vec{p}$ so expansion is with increasing l.

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$q \simeq$ few MeV/c $\quad r \simeq$ few fm $\Longrightarrow |q|R/\hbar \simeq 10^{-2}$

Each increase in l reduces M_{fi} by 10^{-4} to 10^{-3}

Fermi Decays – $e\nu$ in spin 0 state.
– nucleon spin unaffected.

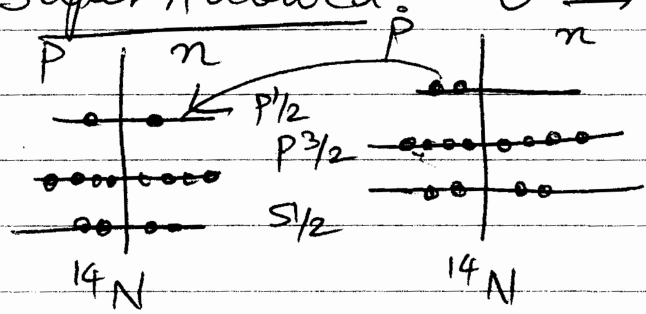
$\Delta P = 0, \Delta J = 0$ Selection rule.

$0^+ \longrightarrow 0^+$ Super allowed, $M_{fi} \simeq 1$.
$\Longrightarrow$ Complete wave-function overlap.

Gamow-Teller Decays $e\nu$ in spin 1 state.
$\Delta P = 0, \quad \Delta J = 0, \pm 1$.

$\Delta l = 0$: Allowed $\quad \Delta l = 1$: "once forbidden"
$\Delta l = 2$: "twice forbidden"....

Examples: Super Allowed: $^{14}O \longrightarrow {}^{14}N \quad 0^+ \longrightarrow 0^+$



If isospin symmetry $\Rightarrow$ no change of wavefn

– ^{14}C formed from ^{14}N; $\quad ^{14}C \longrightarrow {}^{14}N(GS)$
$0^+ \longrightarrow 1^+ \Longrightarrow t_{1/2} \sim 5000$ YRS

– $^{115}In \rightarrow {}^{115}Sn \qquad \Delta l = 4$!

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FUSION B/A rises with A for $A < 20$
$\Rightarrow$ fuse 2 light nuclei $\Rightarrow$ release energy.

- Must overcome Coulomb barrier ~ few MeV
- Scattering? No efficient
- Heating? Need 10^{10} K!

However tails of Maxwellian velocity distributions provide good probability for $\sim 10^7$ K $\Rightarrow$ stars.

SUN $M \sim 10^{30}$ kg $\Rightarrow$ 5×10^{56} H atoms

$$^1H + {}^1H \longrightarrow {}^2H + e^+ + \nu_e + 0.42\ MeV$$

Weak interaction! Otherwise no stars!

$$^1H + {}^2H \longrightarrow {}^3He + \text{more energy}$$

$$^3He + {}^3He \longrightarrow {}^4He + 2 \times {}^1H + 12.86\ MeV$$

$$\Longrightarrow \quad 4({}^1H) \longrightarrow {}^4He + 2e^+ + 2\nu_e + \text{energy}.$$

Solar luminosity $\simeq 4 \times 10^{26}$ W $\Rightarrow$ 5×10^{55} H atoms
$\Rightarrow$ 10% of mass turned so far!

CNO Cycle

$$3({}^4He) \longrightarrow {}^{12}C + \text{energy}$$

$$^{12}C \rightarrow N \rightarrow O \longrightarrow C + {}^4He$$

$$\Longrightarrow \quad {}^{12}C + 4({}^1H) \longrightarrow {}^{12}C + {}^4He + 2e^+ + 2\nu_e + \text{energy}.$$

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Also: $^3He + ^4He \rightarrow ^7Be + \gamma$

$^7Be + e^- \rightarrow ^7Li + \nu_e$

$^7Be + ^1H \rightarrow ^8B$

$^8B \rightarrow ^8Be + e^+ + \nu_e$

large energy range. ←

Net result: Clean solar neutrino spectrum with thresholds for different reactions.

5 MeV ν_e easy to measure,
0.5 MeV very hard.

<u>Pioneering Solar ν experiment</u>: ^{37}Cl Homestake

* Birth of "Underground" physics.
  - Large tank of oil (C_2Cl_4)
  - ν_e capture: $\nu_e + ^{37}Cl \rightarrow ^{37}Ar + e^-$
  - cosmic ray interactions minimized.
- ^{37}Ar has $t_{1/2} = 35$ days. $\rightarrow$ ^{37}Cl electron capture.

* Removal of ^{37}Ar: - Add ^{38}Ar as carrier,
  - remove Ar by circulating He and condensing.
  - Ar transferred to an apparatus for counting.

* Counting ^{37}Ar: - $^{37}Cl^*$ emits X-rays & electrons.

*

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- Backgrounds: - Cosmic rays ⇒ measured at shallow depth
  - neutrons from ^{238}U in rock ⇒ shielded by water

From 1970 to 1990: 339 events!!
0.5 atoms/day!!!

⇒ 3 times lower than predicted
⇒ Solar-neutrino problem.