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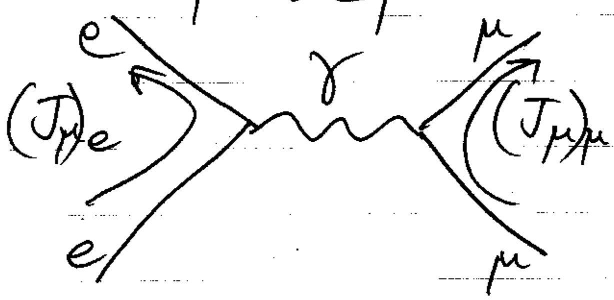
LECTURE XVI

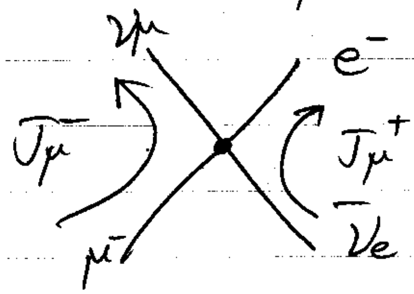
Unification of Weak & Electromagnetic Interactions:

3 issues on apparently large differences

I Issue: EM current is pure V while weak is V−A

$e\mu \to e\mu$ $\qquad\qquad$ $\mu^- \to e^- \bar{\nu}_e \nu_\mu$





$$A_\gamma \sim \frac{e^2}{q^2} J_{\mu e} J^\mu_\mu \qquad\qquad A_W \sim G\, J_\mu^+ J_\mu^-$$

$$J_\mu^+ = \frac{1}{2}\left(\bar{\nu}_e \gamma^\mu e - \bar{\nu}_e \gamma^\mu \gamma^5 e\right) = \bar{\nu}_{e_L} \gamma^\mu e_L$$

Thus:
$$j_\mu^{EM} = q_{EM}\, \bar{e}_L \gamma_\mu e_L + q_{EM}\, \bar{e}_R \gamma_\mu e_R$$

$$j_\mu^{W} = q_W\, \bar{\nu}_{e_L} \gamma_\mu e_L + 0\, \bar{\nu}_{e_R} \gamma_\mu e_R$$

II Issue

$$q_W \ll q_{EM}$$

For unification, we would ~~[illegible]~~ like

$$q_W = q_{EM}$$

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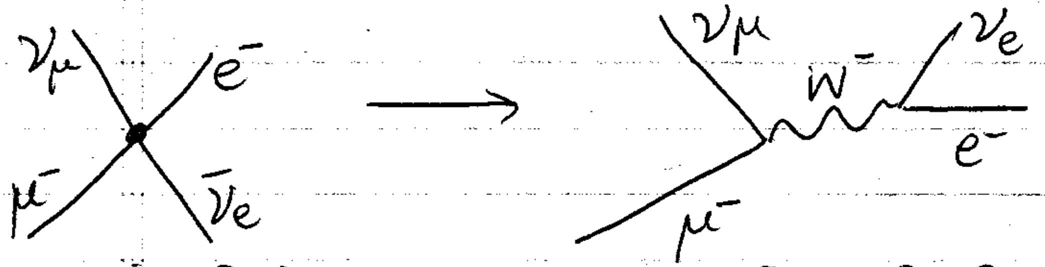
Strength disparity could be reconciled if the force carrier for the weak force is very heavy.



Massive spin-1 propagator: $\dfrac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \sim \dfrac{g_{\mu\nu}}{M_W^2}$

$$G = \frac{g^2}{M_W^2} \Rightarrow g \sim \sqrt{4\pi\alpha} \Rightarrow M_W \sim 50\text{–}100\,\text{GeV}$$

III Issue

- Why is $m_\gamma = 0$ while m_W is massive? (DEFER)

SU(2) & U(1) gauge transformations

EM gauge transformation is 1-D i.e. ~~U(1)~~
$\Rightarrow$ group of U(1) transf^n.

- Symmetry in Lagrangian between particles & antiparticles

If one now assumes electrons & neutrinos ~~to~~ of the same mass, then there is a new symmetry in the Lagrangian.

- Recall p & n states with isospin invariance
- Believed that with no EM, p & n states have same mass.

$$\mathcal{L} = \bar{p}\,(i\gamma^\mu \partial_\mu - m)\,p + \bar{n}\,(i\gamma^\mu \partial_\mu - m)\,n$$

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Define $\psi \equiv \begin{pmatrix} P \\ n \end{pmatrix}$ with ~~each~~ both p & n being bi-spinors -

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi$$

This Lagrangian has a new symmetry in 2-D.
Consider e^{iH} where $H = \theta I + \tau_i a_i$ (2×2 matrices)

Global SU(2) phase transf^n: $\psi \to e^{i\vec{\tau}\cdot\vec{a}}\psi$
$\vec{\tau}$ is a set of 3 orthogonal 2×2 matrix set.
One choice: Pauli spin matrix

Consider an infinitesimal tranf^n: $\delta\psi = \frac{i}{2}\vec{\alpha}\cdot\vec{\tau}\,\psi$

$$\delta\left(\partial_{\mu}\psi\right) = \frac{i}{2}\left(\vec{\alpha}\cdot\vec{\tau}\right)\partial_{\mu}\psi$$

Demand $\delta\mathcal{L} = 0 \Rightarrow \partial_{\mu}\vec{J}^{\mu} = 0$
(Noether's Theorem)

$\vec{J}^{\mu} = \bar{\psi}\gamma^{\mu}\vec{\tau}\psi$ are a set of 3 conserved currents

Now we demand LOCAL phase invariance
$\Rightarrow$ leads to 3 new interactions.

U(1)	SU(2)
$D_{\mu} \equiv \partial_{\mu} + iqA_{\mu}$	$D_{\mu} \equiv I\partial_{\mu} + ig\,\vec{\tau}\cdot\vec{A}_{\mu}$
$D_{\mu} \to e^{i\alpha(x_{\mu})}D_{\mu}$	$D_{\mu} \to e^{i\vec{\tau}\cdot\vec{\alpha}}D_{\mu}$
provided	provided
$A_{\mu} \to A_{\mu} - \partial_{\mu}(\alpha/q)$	$\vec{A}_{\mu} \to \vec{A}_{\mu} - \partial_{\mu}(\vec{\alpha}/g) - (\vec{\alpha}\times\vec{A}_{\mu})$

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U(1)

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$
$$= \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + q\bar{\psi}\gamma^\mu\psi A_\mu$$

$A_\mu \equiv$ new gauge field

$$\mathcal{L}_{free} = F_{\mu\nu}F^{\mu\nu}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

SU(2)

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$
$$= \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - g(\bar{\psi}\gamma^\mu\vec{\tau}\psi)\circ\vec{A}_\mu$$

$\vec{A}_\mu \equiv$ 3 new gauge field

$$\mathcal{L}_{free} = \vec{F}_{\mu\nu}\circ\vec{F}^{\mu\nu}$$

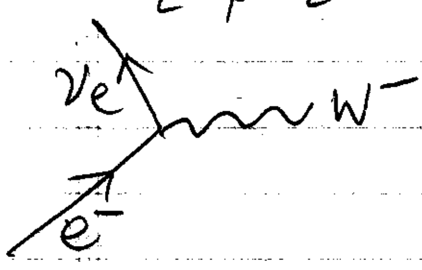
$$\vec{F}_{\mu\nu} = \partial_\nu\vec{A}_\mu - \partial_\mu\vec{A}_\nu + ig(\vec{A}_\mu \times \vec{A}_\nu)$$

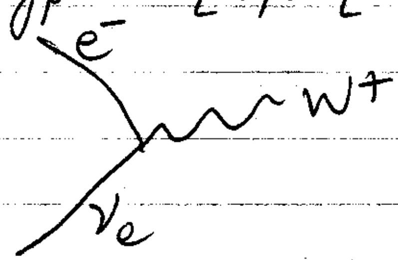
Note: $m^2\vec{A}_\nu\circ\vec{A}^\nu$ breaks invariances!
New quanta have to be massless!

Result: 3 new gauge bosons
Problem: Bosons are massless (DEFER)

Explicit construction for I generation leptons

$$j_\mu^- = \bar{\nu}_L\gamma_\mu e_L \qquad\qquad j_\mu^+ = \bar{e}_L\gamma_\mu\nu_L$$





Define $\psi \equiv \begin{pmatrix}\nu_e \\ e\end{pmatrix}_L$: Left-handed Isospin doublet

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Define $\sigma^{\pm} \equiv \frac{1}{2}(\tau^1 + i\tau^2)$: $\sigma^{+} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $\sigma^{-} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$$j^{\mu\pm} = \bar{\psi}\gamma^{\mu}\tau^{\pm}\psi$$

Start with $\mathcal{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu})\psi$ & impose local gauge invariance to produce 3 new fields W_{μ}^{i} ~~invariance~~

$$\mathcal{L}_{int} = \bar{\psi}\gamma^{\mu}\left[g\frac{\tau}{2}^{i}W_{\mu}^{i}\right]\psi$$

$$\bar{\psi} \equiv (\bar{\nu}_{e_L}, \bar{e}_L) \qquad \psi \equiv \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \qquad I = \pm 1/2$$

$$\tau^{i}W_{\mu}^{i} = \begin{pmatrix} W_{\mu}^{3} & W_{\mu}^{1} - iW_{\mu}^{2} \\ W_{\mu}^{1} - iW_{\mu}^{2} & -W_{\mu}^{3} \end{pmatrix}$$

Define $W_{\mu}^{\pm} = (W_{\mu}^{1} \pm W_{\mu}^{2})/\sqrt{2}$

$$\mathcal{L}_{int} = -\frac{g}{2}\left[\bar{\nu}_L\gamma^{\mu}\nu_L W_{\mu}^{3} - \sqrt{2}\,\bar{\nu}_L\gamma^{\mu}e_L W_{\mu}^{+} - \sqrt{2}\,\bar{e}_L\gamma^{\mu}\nu_L W_{\mu}^{-} - \bar{e}_L\gamma^{\mu}e_L W_{\mu}^{3}\right]$$

Need EM interactions & right handed electrons
$\Longrightarrow$ Introduce new $U(1)$ phase. (NOT $U(1)_{EM}$.) that couples to both right & left-handed particles.

$$\mathcal{L}_{int} = -\bar{\psi}\gamma_{\mu}\left(\frac{g'}{2}Y_L B_{\mu}\right)\psi + \bar{e}_R\gamma^{\mu}\left(g' Y_R B_{\mu}\right)e_R$$

$$= -\frac{g'}{2}\left[(Y_L\bar{\nu}_L\gamma^{\mu}\nu_L)B_{\mu} + (Y_L\bar{e}_L\gamma^{\mu}e_L)B_{\mu} + (Y_R\bar{e}_R\gamma^{\mu}e_R)B_{\mu}\right]$$

We demand: $\mathcal{L}_{SU(2)} + \mathcal{L}_{U(1)}$ contain ~~$\mathcal{L}$~~ QED
A_{μ}^{EM} must not couple to νs
W_{μ} does not couple to e_R

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LECTURE XVI

$\Rightarrow Q = I + \frac{Y}{2} \Rightarrow Y_L = -1$ & $Y_R = -2$

Since $\underbrace{\nu_L, e_L}_{\text{Isospin doublet}}$ have $I = \pm 1/2$ & e_R has $I = 0$.
Isospin Singlet.

B_μ & W_μ^3 are neutral fields $\Rightarrow$ they can mix.

$$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W$$
$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W$$

Collect $\bar{e}e$ terms:

$$g/2\, \bar{e}_L \gamma^\mu e_L W_\mu^3 - g'/2\, Y_L \bar{e}_L \gamma^\mu e_L B_\mu - g'/2\, Y_R \bar{e}_R \gamma^\mu e_R B_\mu$$

From here, we must extract:

$$g_e A_\mu [\bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R]$$

$$\Rightarrow g = \frac{g_e}{\cos\theta_W} \qquad g' = \frac{g_e}{\sin\theta_W}$$

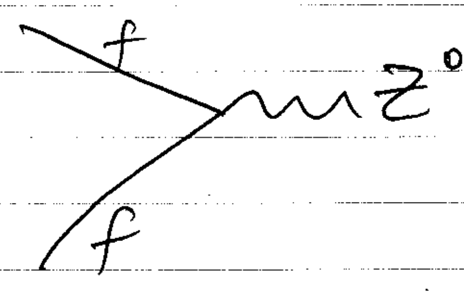
Collecting all W_μ^3 & B_μ terms with rotated fields:

$$g_e(\bar{e}\gamma^\mu e)A_\mu - \frac{g_e}{\sqrt{2}\sin\theta_W}\bar{\nu}_L \gamma^\mu e_L W_\mu^+ - \frac{g_e}{\sqrt{2}\sin\theta_W}\bar{e}_L \gamma^\mu \nu_L W_\mu^-$$

$$\pm \frac{g_e}{2\cos\theta_W \sin\theta_W}\left[(\bar{\nu}_L \gamma^\mu \nu_L) Z_\mu + \left[-\tfrac{1}{2} + \sin^2\theta_W\right]\bar{e}_L \gamma^\mu e_L Z_\mu + \left[-\sin^2\theta_W\right]\bar{e}_R \gamma^\mu e_R Z_\mu\right.$$

First term is the EM interaction, ~~followed~~ followed by two terms for the charged weak interaction, followed by three terms for the neutral weak interaction.

New prediction $\Rightarrow$



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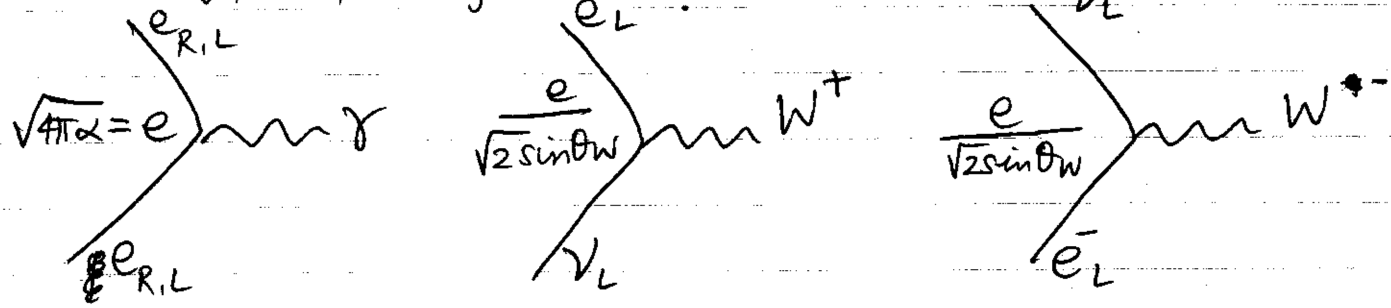
LECTURE XVII

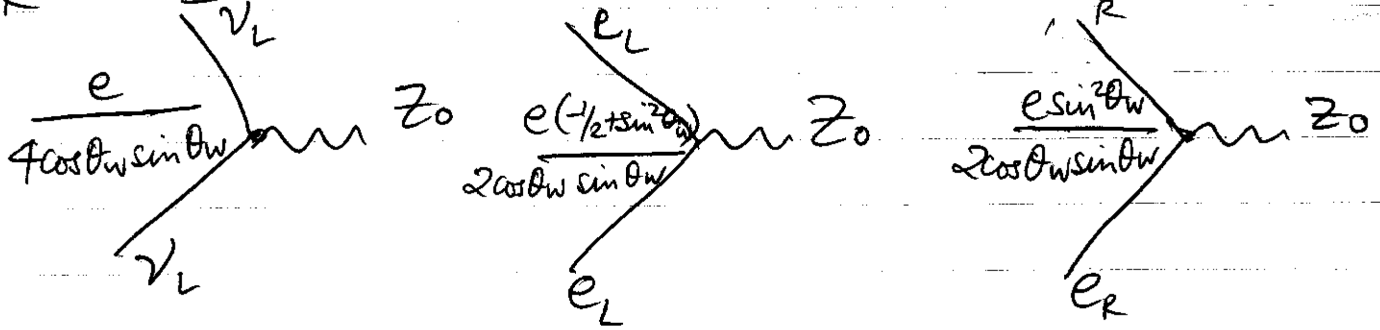
Electroweak Lagrangian with $SU(2)_L \times U(1)_Y$ local gauge invariance produces 4 gauge bosons:

γ, W^+, W^- & Z^0.



Photon is massless and does not couple to νs $\Longrightarrow$
Choose appropriate linear combination of B_μ & W_μ^3
$\Longrightarrow$ weak mixing angle $\sin^2\theta_W$

	Q_e	I	I_3	Y	$Q_Z = I_3 - Q\sin^2\theta_W$
ν_L	0	1/2	1/2	-1	1/2
e_L	-1	1/2	-1/2	-1	$-1/2 + \sin^2\theta_W$
e_R	-1	0	0	-2	$\sin^2\theta_W$



W's, Z & fermions obtain mass by the Higgs mechanism $\Longrightarrow$ $m_W/m_Z = \cos\theta_W$

Make connection to experiment: $\frac{G_F}{\sqrt{2}} = \frac{e^2}{8\sin^2\theta_W M_W^2}$

G_F from muon decay. Now need θ_W

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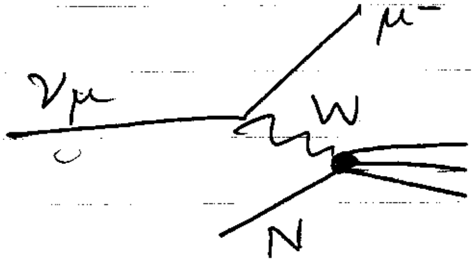
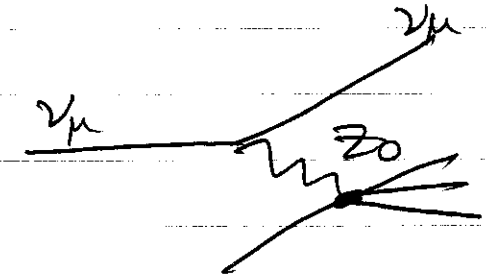
How to measure θ_W? Look for a ~~neutrino~~ NEUTRAL current interaction:





Charged current — neutral current.

Ratio of cross sections is a function of $\sin^2\theta_W$

$$\frac{\sigma(\nu_\mu N \to \nu_\mu X)}{\sigma(\nu_\mu N \to \mu^- X)} = f(\sin^2\theta_W)$$

Once θ_W is known; one can predict m_W & m_Z

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8\sin^2\theta_W M_W^2} \qquad m_Z^2 = \frac{m_W^2}{\cos^2\theta_W}$$

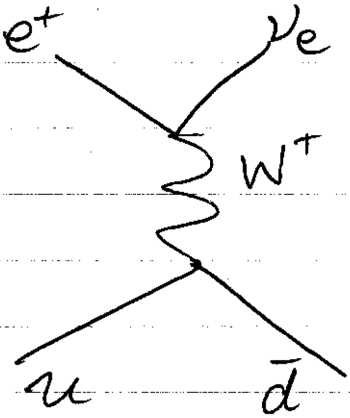
$\Rightarrow$ $m_W \sim 80$ GeV $\qquad m_Z \sim 91$ GeV

How to observe W & Z bosons directly?

Fixed Target: $\xrightarrow{E}$ $\bullet\, m$ $\Rightarrow M^2_{new} = 2ME$

Collider $\xrightarrow{E} \xleftarrow{E}$ $\Rightarrow M^2_{new} = 4E^2$

W production in $p\bar{p}$ collisions $\Rightarrow u + \bar{d} \to W^+ \to e^+ \nu_e$



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$$\mathcal{L}_{int} = \frac{e}{\sin\theta_W\sqrt{2}}\left(\bar{e}_L \gamma_\mu \nu_L W^{+\mu} + \bar{d}_L \gamma_\mu u_L W^{+\mu}\right)$$

$$-iM = \frac{e^2}{2\sin^2\theta_W}\left[\bar{e}\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)\nu\right]\frac{g_{\mu\nu} - q_\mu q_\nu/M_W^2}{q^2 - M_W^2}\left[\bar{d}\gamma_\nu\left(\frac{1-\gamma^5}{2}\right)u\right]$$

But real Ws are produced at $q^2 = M_W^2$!

Recall: E ~~is~~ is uncertain in W rest frame.

$$\tau = \hbar/\Gamma \qquad P(E) = \frac{\Gamma}{2\pi}\frac{1}{(E-m_W)^2 + (\Gamma/2)^2}$$

Consider $A + B \to R \to C + D + E \cdots$

$\Gamma = \sum_i \Gamma_i$ where Γ_i defines the probability for the i^{th} particle decay channel.

$\Gamma_{AB} \equiv$ Probability for $R \to A+B$ & $\Gamma_f \equiv R \to C+D+E\cdots$

Most general result:

$$\sigma_{Peak} = \frac{16\pi\,(2S+1)\,C_R}{(2S_A+1)(2S_B+1)\,C_A C_B}\,\frac{\Gamma_{AB}\,\Gamma_f}{m_R^2\,\Gamma^2}$$

$S \equiv$ spin of R $\quad C_R, C_A, C_B$ are color factors

S_A, S_B are spins

Define $\frac{\Gamma_{AB}}{\Gamma} =$ Branching fraction or B.R. $(R \to A+B)$ is the relative probability.

$$\Rightarrow \sigma_{Peak}(\bar{u} + d \to W^+ \to e^+\nu_e) = \frac{16\pi \;\; 3\times 1}{2\times 2\times 3\times 3}\;\frac{B.R.(W \to \bar{u}+d)\,B.R.(e^+\nu_e)}{M_W^2}$$

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LECTURE XVII

Branching ratios for W decays are easy to calculate ~~Sca~~ (in the massless limit) since all charge current couplings are the same:

$$W^+ \to e^+\nu_e,\ \mu^+\nu_\mu,\ \tau^+\nu_\tau,\ u\bar{d},\ c\bar{s},\ \cancel{t\bar{b}}$$

1, 1, 1, 3, 3, too heavy.

$\Rightarrow$ B.R.$(e^+\nu_e) = 1/9$ B.R.$(u\bar{d}) = 1/3$

$$\Rightarrow \sigma_{peak}(u\bar{d} \to W^+ \to \cancel{\ell}^+\nu_e) = \frac{4\pi}{81 M_W^2}(0.389)\,\mathrm{GeV}^2\,\mathrm{mbarn} = \underline{\underline{2.3\ \mathrm{n\,barn}}}$$

In practice $\sigma(p\bar{p} \to W^+ \to e^+\nu_e)$
$= 1$ n barn

If $p\bar{p}$ collider has luminosity $\mathcal{L} \sim 10^{27}/\mathrm{cm}^2/\mathrm{s}$
then $N = \mathcal{L}\sigma T = 10^{27} \times 10^{-9} \times 10^{-24} \times 10^{7} \Rightarrow 10$ events
The discovery of W & Z was made on a few events!

Experimental Challenge: $p\bar{p} \to X$ $\sigma \sim 40$ m barn
i.e. W & Z decays are one in a million triggers!

Topological Signature: W produced nearly at rest & ν_e must carry half the energy ~ 40 GeV.
$\Rightarrow$ Look of "unbalanced" events in the transverse plane $\Rightarrow$ "hermetic" detector.

*Challenge: Enough antiproton intensity to produce $\mathcal{L} = 10^{27}/\mathrm{cm}^2/\mathrm{s}$

LECTURE XVII

Z^0 production: $e^+e^- \to Z^0 \to l^+l^-, q\bar{q}, \nu\bar{\nu}$

$l = e, \mu, \tau \quad \nu = \nu_e, \nu_\mu, \nu_\tau \quad q = u, d, s, c, b$

$$\mathcal{L}_{int} = \frac{e}{\cos\theta_W \sin\theta_W}\left[\bar{f}_L \gamma^\mu f_L g_L + \bar{f}_R \gamma^\mu f_R g_R\right] Z_\mu$$

$f \equiv l, \nu, q$

$g_L = T_3 - Q\sin^2\theta_W \quad g_R = -Q\sin^2\theta_W.$

$$B.R.(f^+f^-) = \frac{g_L^{f\,2} + g_R^{f\,2}}{\sum g_L^2 + g_R^2}$$

Count $6(e^+e^-) + 6(u\bar{u}) + 9(d\bar{d})$

$\Rightarrow B.R.(e^+e^-) \sim 3.3\% \qquad B.R.(q\bar{q}) \sim 70\%$

$$\sigma_{tot}(e^+e^- \to Z \to \text{hadrons}) = \frac{16\pi}{M_Z^2}\,\frac{3}{2\times 2} \times B.R.(e^+e^-)\, B.R.(q\bar{q})$$

$= 40$ n barn

This is 200 times larger than $e^+e^- \to \gamma^* \to$ hadrons.

Thus, an e^+e^- collider tuned to the Z resonance is an exceptionally clean way to study Z^0 decays $\Longrightarrow$ Z-factory.

2 Z factories: CERN (Geneva) & SLAC (Stanford)

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LECTURE XVIII

LEP (Large Electron Positron Collider) at CERN:

$$e^+e^- \longrightarrow Z^0 \longrightarrow q\bar{q} \qquad \sigma_{Peak} \sim 40 \text{ n barn.}$$

LEP $\mathcal{L} \sim 10^{31}/cm^2/s \Longrightarrow$ 1 Z every few seconds!
No backgrounds!

$\sigma(E)$ proportional to $P(E)$

$$\sigma(E) \propto \frac{\Gamma_{ee}\Gamma_{ff}}{(E-M_Z)^2 + \Gamma^2/4} \Longrightarrow \text{ extract } \Gamma \;\&\; M_Z.$$

Γ is the total width: $\Gamma_{q\bar{q}} + \Gamma_{\ell^+\ell^-} + \Gamma_{invisible}$

Prediction: $\Gamma_{invisible} = n\Gamma_{\nu\nu}$ if there are n neutrino species.
Experiment: $n = 3.00 \pm 0.02$!
$\Longrightarrow$ - Indirect evidence for $Z \to \nu_\tau \bar{\nu}_\tau$
- Important cosmological implications.

<u>Angular</u> <u>distribution</u> <u>in Z decay</u>

Consider: $e^+e^- \longrightarrow Z$ in massless limit.

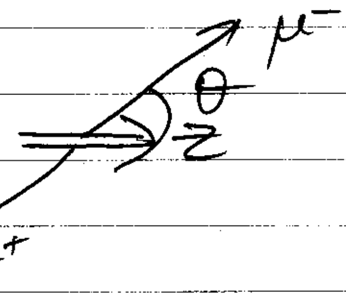
<u>Helicity</u> <u>conservation</u>

If $g_R \neq g_L$, then Zs are polarized.
i.e. More Z's are polarized one way than the other.

$$P_Z = \frac{N_R - N_L}{N_R + N_L} = \frac{g_R^2 - g_L^2}{g_R^2 + g_L^2}$$

LECTURE XVIII

Now consider $Z \to \mu^+\mu^-$
probabilities from $\langle jm'|e^{-i\theta J_y/\hbar}|jm\rangle$



$A_{1,1}$, $A_{1,-1}$, A_{-1-1}, $A_{-1,1}$ depend on θ, g_L & g_R.

$A_{11} \propto g_e^R g_\mu^R (1+\cos\theta)/2$ $\qquad$ $A_{1-1} \propto g_e^R g_\mu^L (1-\cos\theta)/2$

$A_{-11} \propto g_e^L g_\mu^R (1-\cos\theta)/2$ $\qquad$ $A_{-1-1} \propto g_e^L g_\mu^L (1+\cos\theta)/2$

$$\sigma \propto \sum A^2 \sim (g_e^{R^2} + g_e^{L^2})(g_\mu^{R^2} + g_\mu^{L^2})(1+\cos^2\theta) + (g_e^{R^2} - g_e^{L^2})(g_\mu^{R^2} - g_\mu^{L^2})\, 2\cos\theta$$

$$A_{FB} = \big(\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)\big) \big/ \big(\sigma(\cos\theta > 0) + \sigma(\cos\theta < 0)\big) = 3/4\, P_Z P_f$$

$$P_Z = \frac{g_e^{R^2} - g_e^{L^2}}{g_e^{R^2} + g_e^{L^2}} \qquad P_f = \frac{g_f^{R^2} - g_f^{L^2}}{g_f^{R^2} + g_f^{L^2}}$$

A_{FB} is the product of 2 polarizations
P_Z & P_f are each 0.15, so $A_{FB} \sim 1\%$.

One can separately measure P_Z & P_f if one can either polarize the initial state or analyze the final state.

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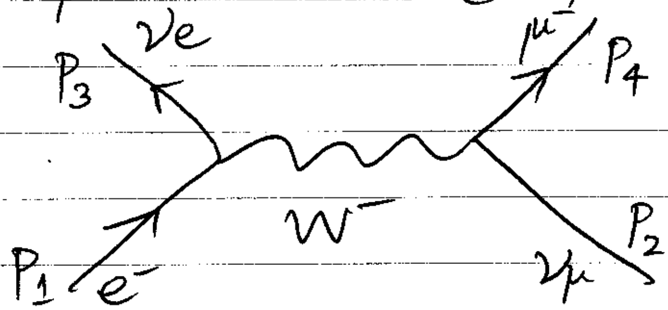
LECTURE XVIII

Consider weak interaction processes at $q^2 \ll M_W^2$

$\nu_\mu + e^- \longrightarrow \nu_e + \mu^-$



$q = P_1 - P_3$

$$\frac{-i\left(g_{\mu\nu} - q_\mu q_\nu / M_W^2\right)}{q^2 - M_W^2} \simeq \frac{+i\, g_{\mu\nu}}{M_W^2}$$

$$\mathcal{L}_{int} = \frac{g_e}{\sqrt{2}\sin\theta_W}\left(\bar{\mu}_L \gamma_\mu \nu_{\mu L} + \bar{\nu}_{e_L} \gamma_\mu e_L\right) W^{\mu -}$$

$$\Rightarrow M = \frac{g_e^2}{8 M_W^2 \sin^2\theta_W}\left[\bar{u}(3)\gamma^\mu(1-\gamma^5)u(1)\right]\left[\bar{u}(4)\gamma_\mu(1-\gamma^5)u(2)\right]$$

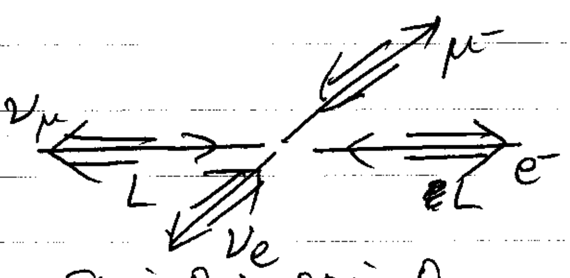
$$\Rightarrow \langle |M|^2 \rangle \Rightarrow \text{trace theorems} \Rightarrow \frac{4 g_e^4}{M_W^4 \sin^4\theta_W}(P_1 \cdot P_2)(P_3 \cdot P_4)$$

NOTE: Only 1 neutrino state.

In COM frame: in the high energy limit.

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E^2}{2 M_W^4 \sin^4\theta_W}$$

- $\frac{d\sigma}{d\Omega} \propto E_{COM}^2$
- $\frac{d\sigma}{d\Omega}$ independent of θ



spin 0 → spin 0
isotropic.

$E_{COM}^2 = 2 m_e E_{\nu(lab)}$

$\Rightarrow \sigma \propto E_{\nu(lab)}$

characteristic of ν cross-sections

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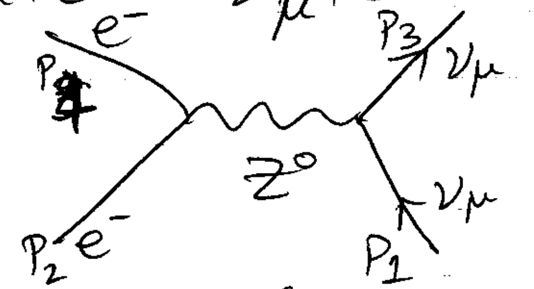
LECTURE XVIII

$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$ (electron in the final state)

$\nu_\mu e$ elastic scattering

$q^2 << M_Z^2$



$$\mathcal{L} = \frac{ge}{2\sin\theta_W\cos\theta_W}\left(\bar{e}_L\gamma_\mu e_L g_L + \bar{e}_R\gamma_\mu e_R g_R + \bar{\nu}_L\gamma_\mu\nu_L\right)Z^\mu$$

Define $g_L + g_R = C_V \quad g_L - g_R = C_A$ (Vector & axial-vector coupling)

$$\bar{e}_L\gamma_\mu e_L g_L + \bar{e}_R\gamma_\mu e_R g_R = (\bar{e}\gamma_\mu e)\frac{C_V}{2} - (\bar{e}\gamma_\mu\gamma^5 e)\frac{C_A}{2}$$

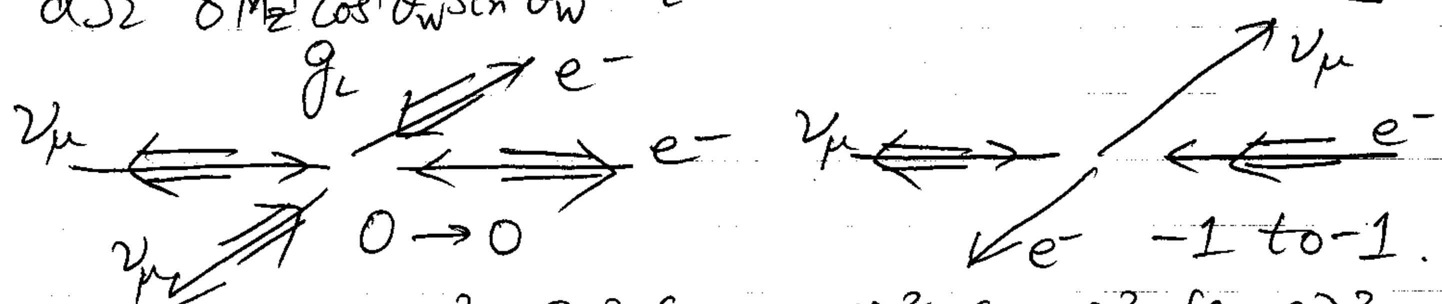
$$M = \frac{ge^2}{8M_Z^2\sin^2\theta_W\cos^2\theta_W}\left[\bar{u}(3)\gamma^\mu(1-\gamma^5)u(1)\right]\left[\bar{u}(4)\gamma_\mu(C_V - C_A\gamma^5)u(2)\right]$$

$\Longrightarrow$

$$\langle|M|^2\rangle = \frac{ge^4}{32M_Z^2\sin^4\theta_W\cos^4\theta_W}\left\{(C_V+C_A)^2(P_1\cdot P_2)(P_3\cdot P_4) + (C_V-C_A)^2(P_1\cdot P_4)(P_2\cdot P_3) - m^2(C_V^2-C_A^2)(P_1\cdot P_3)\right\}$$

High energy limit $m \rightarrow 0$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8M_Z^4\cos^4\theta_W\sin^4\theta_W}E^2\left[(C_V+C_A)^2 + (C_V-C_A)^2\cos^4\theta/2\right]$$



$$\sigma \propto g_L^2 + g_R^2(1+\cos\theta)^2 = (C_V+C_A)^2 + (C_V-C_A)^2\cos^4\theta/2.$$

<u>Precision Ratio experiment:</u>

$$R = \frac{\sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-)}{\sigma(\nu_\mu e^- \rightarrow \nu_e \mu^-)}$$

$R = f(\sin^2\theta_W) = 0.09$ for $\sin^2\theta_W = 0.23$

Expt: $R = 0.08 \pm 0.01$

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LECTURE IXX

Lifetime of an unstable particle:

Fermi's Golden Rule for decays: $1 \longrightarrow 2+3+\cdots n$

$$d\Gamma_i = |M|^2 \frac{S}{2m_1}\left[\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\cdots\cdots\frac{d^3\vec{p}_n}{(2\pi)^3 2E_n}\right]$$

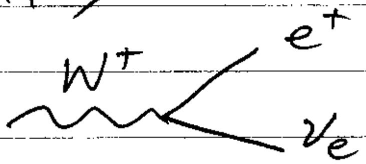
$$\Gamma_{tot} = \sum_i \Gamma_i \quad \text{(sum over all decay channels)}$$

$$\text{Lifetime } \tau = \frac{1}{\Gamma} \quad (\text{i.e. } \hbar/\Gamma)$$

2 Body Decays: $1 \longrightarrow 2+3$

$$\Gamma_i = \frac{S\,|p_3|}{8\pi m_1^2}\langle |M|^2\rangle$$

Consider: $W^+ \longrightarrow e^+\nu_e$



$$\mathcal{L} = \frac{g_e}{\sqrt{2}\sin\theta_W}(\bar{\nu}_L\gamma_\mu e_L)W^{\mu+} \qquad M = \frac{g_e}{\sqrt{2}\sin\theta_W}\epsilon^\mu\bar{\nu}\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)e$$

Trace Theorems $\Longrightarrow$ $\langle|M|^2\rangle = \frac{g_e^2}{\sin^2\theta_W}\frac{m_W^2}{3}$

$$S = 1 \qquad |p_3| = m_W/2 \Longrightarrow \Gamma_{e\nu} = \frac{g_e^2}{48\pi\sin^2\theta_W}M_W$$

$$\Gamma_{e\nu} = \frac{\alpha M_W}{12\sin^2\theta_W} = 0.23\ \text{GeV}$$

$$\Gamma_{tot} = \sum\Gamma_i = 3\times\Gamma_{l\nu} + 3\times 2\times\Gamma_{u\bar{d}} = 9\times 0.23\ \text{GeV}$$

Note: all particles have same W couplings.

LECTURE $\overline{IXX}$

$Z \to e^+e^-$ $\quad \mathcal{L} = \dfrac{g_e}{2\sin\theta_W\cos\theta_W}\left(\bar{e}_L\gamma_\mu e_L g_L + \bar{e}_R\gamma_\mu e_R g_R\right)$

$$\Gamma_{ee} = \Gamma_{e\nu} \times \frac{M_Z}{2M_W\cos^2\theta_W}\left(g_L^2 + g_R^2\right)$$

$$g = I_3 - Q\sin^2\theta_W$$

$$g_L = \pm\tfrac{1}{2} - Q\sin^2\theta_W$$

$$g_R = Q\sin^2\theta_W$$

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L \quad \begin{pmatrix}u\\ d\end{pmatrix}_L \begin{pmatrix}c\\ s\end{pmatrix}_L \begin{pmatrix}t\\ b\end{pmatrix}_L$$

$$e^-_R \quad \mu^-_R \quad \tau^-_R \quad u_R\ d_R\ c_R\ s_R\ t_R\ b_R$$

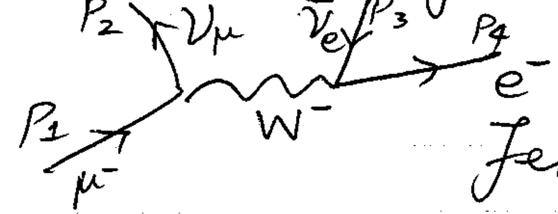
$$\Gamma_{tot} = \sum_i \Gamma_i = 3\Gamma_{ee} + 3\Gamma_{\nu\nu} + 3\times 2\times\Gamma_{uu} + 3\times 3\times \Gamma_{dd} \simeq 2.4\,\text{GeV}$$

$$B.R. = \frac{\Gamma_i}{\Gamma_{tot}} \qquad (\text{e.g. } BR(W^- \to e^-\bar{\nu}_e) = 1/9)$$

For $Z = f\bar{f} \Rightarrow B.R.(f\bar{f}) = \dfrac{\Gamma_{f\bar{f}}}{\Gamma_{tot}} = \dfrac{g_L^{f2} + g_R^{f2}}{\sum g_L^2 + \sum g_R^2}$

<u>Low Energy</u> <u>Weak</u> <u>Decays</u>: $q^2 << M_W^2$

<u>Muon Decay</u>: $\quad M = \dfrac{g_e^2}{8M_W^2\sin^2\theta_W}\left[\bar{u}(3)\gamma^\mu(1-\gamma^5)u(1)\right]\left[\bar{u}(4)\gamma_\mu(1-\gamma^5)v(2)\right]$



Fermi Theory: $\dfrac{G_F}{\sqrt{2}} = \dfrac{g_e^2}{8M_W^2\sin^2\theta_W}$

$$G_F = \frac{1}{\sqrt{2}}\frac{\pi\alpha}{M_W^2\sin^2\theta_W}$$

Historically: τ_μ gives G_F & νN scattering gives $\sin^2\theta_W$

from which one predicts M_W & M_Z

LECTURE IXX

$$\langle |M|^2 \rangle = \frac{2 g_e^4}{M_W^4 \sin^4\theta_W} (P_1 \cdot P_2)(P_3 \cdot P_4)$$

$P_1 \equiv (m_\mu, 0) \quad |P_4| = E_4 \quad P_2^2 = P_3^2 = P_4^2 = 0$

$$\langle |M|^2 \rangle = \frac{g_e^4}{M_W^4 \sin^4\theta_W} m_\mu^2 E_2 (m_\mu^2 - 2E_2)$$

$$d\Gamma = \frac{\langle |M|^2 \rangle}{2 m_\mu} \frac{d^3\vec{P_2}}{(2\pi)^3 2E_2} \frac{d^3\vec{P_3}}{(2\pi)^3 2E_3} \frac{d^3\vec{P_4}}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(P_1 - P_2 - P_3 - P_4)$$

δf^n gets used up in $\vec{P_3}$ integral

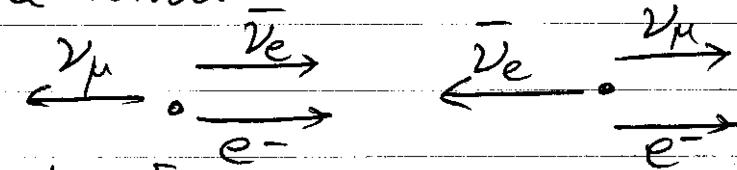
Then, $\vec{P_2}$ & $|P_4|$ integrals are performed to get $\frac{d\Gamma}{dE_e}$

One needs to specify the limits.

$E_{2\,max} = m_\mu/2$

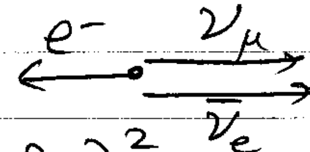
$E_2 + E_4 > m_\mu/2$

$\Rightarrow \quad E_{2\,min} = m_\mu/2 - E_4$



$E_{4\,min} = m_e \simeq 0$

To obtain $E_{4\,max}$ Consider:



$$(P_\mu - P_e)^2 = (P_{\nu_\mu} + P_{\bar{\nu}_e})^2$$

$m_\mu^2 + m_e^2 - 2 m_\mu E_e = 0 + 0 + 0 \Rightarrow E_{4\,max} = \frac{m_\mu^2 + m_e^2}{2 m_\mu}$

$\simeq m_\mu/2$

$\frac{d\Gamma}{dE_e} \propto E^2 \left(1 - \frac{4E_e}{3m_\mu}\right) \Rightarrow$ Decay spectrum.

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3} \Rightarrow \tau = \frac{192\pi^3}{G_F m_\mu^5} \frac{\hbar c}{c} \left\{ \frac{197 \text{ MeV fm}}{3 \times 10^8 \text{ m/s}} \right\}$$

$$= 2.2 \ \mu s.$$

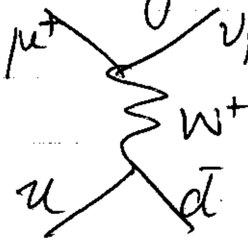
LECTURE IXX

Most general decay formula: Replace ~~m~~m_μ with m_{f_1}

So long as $m_{f_1} >> m_{f_2}$: $f_1 \to f_2 + \ell + \bar{\nu}_\ell$

Examples: $\tau^- \to \mu^- + \bar{\nu}_\mu + \nu_\tau \qquad b \to c + e^- + \bar{\nu}_e$

π^- decay. $\xleftarrow[\bar{\nu}_\ell]{} \bullet \xrightarrow[m_\ell]{}$ $\quad \pi^- \to \mu^- \bar{\nu}_\mu \,,\ \pi^- \to e^- \bar{\nu}_e$



$$M = \frac{G_F}{\sqrt{2}} \bar{\mu}\, \gamma_\mu (1-\gamma^5) \nu_\mu F^\mu$$

Spin 0 $\Longrightarrow$ $F^\mu = f_\pi p^\mu$

2 Body decay : $\Longrightarrow$ f_π is a constant.

$$\langle |M|^2 \rangle = 8 G_F^2 f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2)$$

$$|P_f| \Longrightarrow \ P_\nu^2 = P_\pi^2 + P_\ell^2 - 2 m_\pi E_\ell \Longrightarrow E_\ell = \frac{m_\pi^2 + m_\ell^2}{2m_\pi}$$

$$\Longrightarrow |P_\ell| = \frac{m_\pi^2 - m_\ell^2}{2m_\pi}$$

$$\Gamma = \frac{|P_f|}{8\pi m_\pi^2} \langle |M|^2 \rangle = \frac{G_F^2 f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2)^2}{2\pi\ m_\pi^3}$$

$$\frac{\Gamma(\pi^- \to e^-)}{\Gamma(\pi^- \to \mu^-)} = \frac{m_e^2}{m_\mu^2} \frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2} = 1.28 \times 10^{-4}$$

Why?

Helicity Suppression: $\bar{\nu}_\ell \Leftarrow \underset{0}{\bullet} \Rightarrow \ell^-$

If $m_\ell = 0$, decay is forbidden.

The heavier ~~is the~~ the particle (ℓ^-), the easier it is to make a right-handed state.

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LECTURE IXX

Neutron Decay: $n \to p + e^- + \bar{\nu}_e$

$m_n \simeq m_p \Rightarrow$ phase space integral small $\Rightarrow \Gamma_n$ small.

$\Rightarrow \tau_n \sim 900\,s$

Explicit calculation with $G_F \sim 1300\,s$

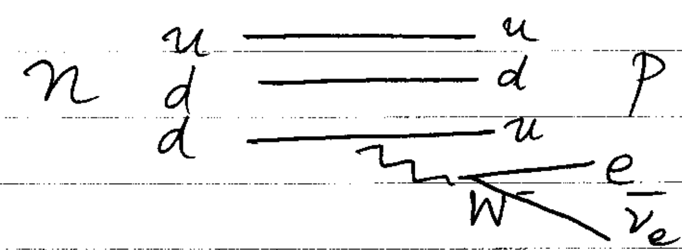
Neutron & Proton aren't elementary $\Rightarrow$ form factors.
However, in e-p scattering as $q^2 \sim 0 \Rightarrow$ correct charge.
$\Rightarrow$ QCD conserves charge.
But in the weak interactions, we have axial charge.
$\Rightarrow$ QCD does not conserve axial charge

$\mathcal{L} \sim \bar{p}(C_V - C_A\gamma^5)n \Rightarrow C_V = 1$ but $C_A \neq -1$

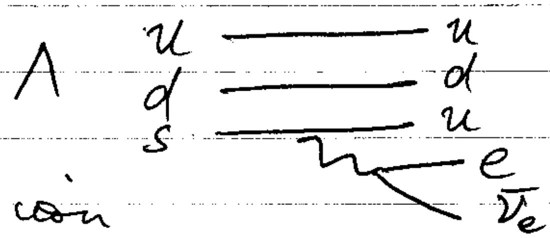
Experiments measures $C_A = -1.26$.

$\Gamma \propto \frac{1}{4}(C_V^2 + 3C_A^2) \Rightarrow \tau \simeq 800\,s$

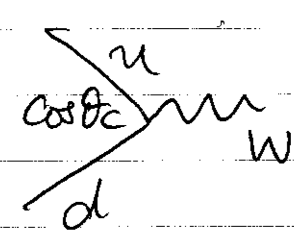
$d \to u + W^-$

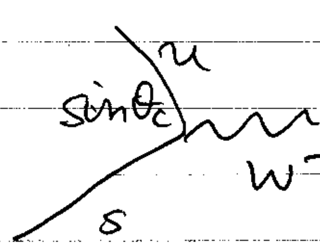


$s \to u + W^-$



Lifetime prediction
wrong for $\Lambda \to p + e^- + \bar{\nu}_e$ unless.





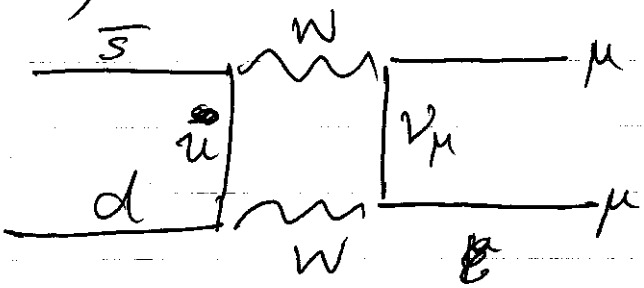
LECTURE IXX

Introduction of θ_c explains all of u d s decays.

Test: $$\frac{\Gamma(K\to\mu)}{\Gamma(\pi\to\mu)} = \tan^2\theta_c \left(\frac{m_\pi}{m_K}\right)^3 \left(\frac{m_K^2-m_\mu^2}{m_\pi^2-m_\mu^2}\right)^2$$

This works for Kaon leptonic decays.

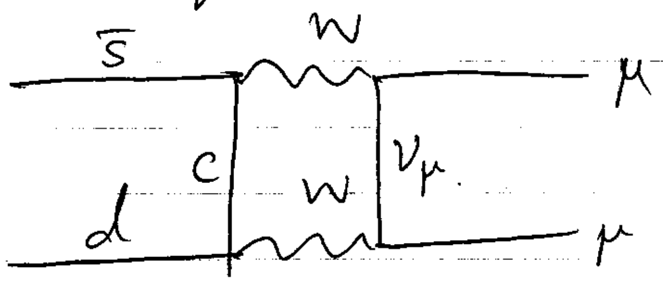
However, consider $K^0 \longrightarrow \mu^+\mu^-$



$\propto \sin\theta_c \cos\theta_c$.

was not observed.

Glashow, Iliopoulos & Maiani introduced a 4^{th} quark.



$\propto -\sin\theta_c \cos\theta_c$.

cancels out!

Prediction was made befor the discovery of c-quark.

Now we know there are 6 quarks.

This has important implications for CP violation