Definition of the spatial resolution of X-ray microanalysis in thin foils

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The spatial resolution of X-ray microanalysis in thin foils is defined in terms of the incident electron beam diameter and the average beam broadening. The beam diameter is defined as the full width tenth maximum of a Gaussian intensity distribution. The spatial resolution is calculated by a convolution of the beam diameter and the average beam broadening. This definition of the spatial resolution can be related simply to experimental measurements of composition profiles across interphase interfaces. Monte Carlo calculations using a high-speed parallel supercomputer show good agreement with this definition of the spatial resolution and calculations based on this definition. The agreement is good over a range of specimen thicknesses and atomic number, but is poor when excessive beam tailing distorts the assumed Gaussian electron intensity distributions. Beam tailing occurs in low-Z materials because of fast secondary electrons and in high-Z materials because of plural scattering.

1. Introduction

The spatial resolution of X-ray microanalysis is a measure of the smallest distance (R) between the centers of two analysis volumes from which independent X-ray microanalyses can be obtained. The definition of R has evolved over the last fifteen years as the analytical electron microscope (AEM) has improved and smaller analysis volumes have become attainable. It has long been recognized that the analysis volume and hence R is governed by the beam–specimen interaction volume since the energy-dispersive spectrometer (EDS) in the AEM can detect X-rays generated anywhere within that volume. The interaction volume is a function of the beam diameter (d) and the beam broadening (b). The beam broadens primarily as a result of high-angle elastic scatter of electrons within the specimen. Therefore the measured spatial resolution is a function of the specimen itself and this fact has made it difficult to define R. Theoretical treatments of the spatial resolution have, until recently, emphasisized the beam broadening contribution and experimental investigations have used a variety of specimens. In this paper we will review the development of both the theoretical and experimental aspects of spatial resolution and attempt to justify a specific definition of R. We will also show how both experimental measurements and Monte Carlo simulations offer support for the definition.

1.1. Theoretical definition of spatial resolution

The earliest attempts to define spatial resolution appeared in 1977 in papers by Goldstein et al. [1] and Faulkner et al. [2]. Both papers considered the effects of beam broadening primarily and agreed that beam size was of secondary importance in “typical” thin foils. Goldstein et al. [1] assumed that each electron undergoes a single scattering event in the middle of the foil. This approach resulted in the “single-scattering” equation which is a conservative estimate of the broadening, giving a value for b (in cm) equal to 90% of the diameter of the electron distribution.
emerging from the specimen. The expression for $b$ is:

$$b = 6.25 \times 10^5 (Z/E_0)(\rho/A)^{1/2} t^{3/2},$$  \hspace{1cm} (1)

where $Z$ is the atomic number, $A$ is the atomic weight, $E_0$ is the beam energy in eV, $\rho$ is the specimen density in g/cm$^3$ and $t$ is the specimen thickness in cm. On the basis of this equation, Goldstein [3] proposed the first definition of $R$ as simply the sum of the beam diameter and the beam broadening:

$$R = d + b. \hspace{1cm} (2)$$

The inverse relationship between $b$ and $E_0$ inherent in eq. (1) was also demonstrated by Faulkner et al. [2] who were the first to publish Monte Carlo simulations of the beam broadening in a thin foil. The dominant role of $t$ in eq. (1) and the Monte Carlo calculations of Faulkner et al. both lead to the conclusion (that was held for many years) that beam broadening in the specimen was the dominant factor in determining spatial resolution and that generally the beam size effects were secondary. This remains a valid conclusion for thick foils, but for thin foils, the beam size can have a significant effect on $R$ and field emission guns are the best source for the highest-resolution microanalysis.

An important aspect of defining $R$ is defining the fraction of the beam electrons that are included in definitions of $b$ and $d$. Several different definitions were used initially by different workers. But as already stated, eq. (1) assumed that $b$ contains 90% of the scattered electrons. This is now the generally accepted figure which has been used in other treatments of the beam broadening such as the Monte Carlo calculations of Newbury and Myklebust [4], the plural-scattering approach of Cliff and Lorimer [5] and the modified single-scattering model of Reed [6]. In this last paper Reed adjusted the expression for $b$ in eq. (1) by integrating the effects of single-scattering events throughout the foil thickness to give:

$$b = 7.21 \times 10^5 (Z/E_0)(\rho/A)^{1/2} t^{3/2}. \hspace{1cm} (3)$$

Reed then proposed that if the electron intensity distributions in both the incident and emergent beams are Gaussian, then:

$$R = (d^2 + b^2)^{1/2}. \hspace{1cm} (4)$$

This equation remained the standard definition of $R$ for almost a decade. However, it is worth noting that no set of experiments has yet investigated the effects of all the variables in eq. (3), despite the general agreement for many years that this is a reasonable explanation of the major factors controlling $R$. About the same time as Reed’s definition of $R$ was proposed, Gaussian beam-broadening models were introduced by Doig and Flewitt [7,8] and Hall et al. [9]. Michael and Williams [10] showed that these models were effectively identical to the single-scattering model of Reed [6]. Michael and Williams also made the observation that both $b$ and $d$ must be similarly defined if a consistent definition of $R$ was to be obtained. In other words, since $b$ is defined in terms of 90% of the electrons emerging through the foil, $d$ must also contain 90% of the incident electrons. This definition of $d$ is equivalent to the full width tenth maximum (FWTM) of a Gaussian distribution. In earlier studies, estimates of $d$ usually assumed a FWHM beam diameter which contains only 50% of the electrons. The Gaussian model of Doig and Flewitt [8] described the electron intensity as a function of the radial distance from the beam center at any depth in the specimen. This model allowed the complete description of $R$ to be calculated in terms of the effects of the incident beam intensity distribution and the beam broadening.

Based on the Gaussian model and experimental measurements, Michael et al. [11] proposed that the definition of $R$ be modified so as not to present the worst case (given by the exit beam diameter) but to define $R$ at a plane midway through the thin foil as shown in fig. 1:

$$R = (d + R_{\text{max}})/2, \hspace{1cm} (5)$$

where $R_{\text{max}}$ is given by eq. (4).

It is this equation which we propose as the best current definition of the X-ray spatial resolution.

There is no fundamental justification for the choice of various factors such as the FWTM
The proposed definition of the spatial resolution $R$ of X-ray microanalysis: the incident beam size $d$ corresponding to the FWTM Gaussian beam diameter is broadened as the electrons are scattered through the thin foil, thickness $t$. The traditional definition of $R$ is the exit diameter shown here as $R_{max}$. The average broadening $R$ at the mid-plane of the foil is the spatial resolution defined in this paper (eq. (5)).

1.2. Calculation of spatial resolution

Since the original work of Faulkner et al. [2], the Monte Carlo technique has continued to be a major tool for determining $R$. It has the advantage of being able to simulate experimental data that would otherwise be difficult to obtain, but it is very dependent on the user's choice of the many variables such as ionization cross section and electron energy loss parameters. Faulkner et al. used only 100 trajectories (because of the "high computational time involved"), but the speed and power of computers has progressed to the point where parallel computing [13] permits the generation of $10^5$ trajectories in less than 5 seconds. In this paper we use parallel-computer Monte Carlo simulations to test the definition of $R$ which we propose.

1.3. Experimental measurement of spatial resolution

Experimental measurements of $R$ appeared slightly before the first theoretical treatments. In 1976 composition profiles measured across atomically sharp interphase interfaces were presented by Lorimer et al. [14] and Goldstein et al. [15]. Since then, several other kinds of specimens have been proposed such as spherical particles in a foil of known thickness [2], artificial samples of Au lines deposited on a Si foil [16], grain boundary films [9] and quantum well structures [17] amongst others. We believe the first method retains its validity since there are fewer unknowns than for the other samples. If we assume local thermodynamic equilibrium exists at the interphase interface then the solute content of each phase at the interface is well defined. At true equilibrium the solute content is constant throughout each phase, not just at the interface. Choice of a specimen such that average atomic number changes between the two phases are minimal makes it easier to calculate the expected spatial resolution. Even if there are large differences in $Z$ the effects are still amenable to Monte Carlo simulation. In this investigation we have confined our experimental results to transition metal alloys containing Fe, Ni and Cr, because previous studies, notably by Zhang et al. [18], as well as refs. [10] and [11], have generated data from these systems.

In order to compare experimental and calculated measurements of $R$ it is necessary to understand how the measured composition profile across the interface is related to the actual discrete shape, shown schematically in fig. 2. This relationship can be determined by deconvolution.
of the beam shape from the measured profile, as described by Michael and Williams [10]. The finite beam size and the effect of \( b \) degrade the sharp profile to a width \( L \), shown on fig. 2 and related to \( R \) by the following equation [11]:

\[
R = 1.414L. \tag{6}
\]

Using this relationship, \( R \) can be obtained by measuring the distance \( L \) between the 2\% and 98\% points on the experimental profile, as shown in fig. 2. This distance, \( L \), contains 90\% of the beam electrons, consistent with the assumption of a 90\% (FWTM) incident beam diameter. In practice the 2\% and 98\% points are difficult to measure experimentally because of the errors in the experimental data. Therefore it is easier to measure the distance from the 10\% to the 90\% points on the profile, corresponding to the distance containing 50\% of the electrons (FWHM), then multiply by 1.8 to give the FWTM. Thus, this approach of Michael et al. [11] gives a profile width measurement via a simple equation (6). In contrast, the Doig and Flewitt approach [7,8], while the first to permit extraction of the experimental composition profile, required numerical integration of a rather daunting triple integral.

In summary, we now have a theoretical definition of \( R \), a way to measure it experimentally and the computer power to simulate the effects of many experimental variables. Our aim in this paper and several preceding papers [10,11,18] has been to develop a simple definition of \( R \) such that, from a measured composition profile width \( L \), we can back-calculate the true profile (which is the relevant information for the materials scientist). From this approach comes the definition of \( R \) given in eq. (5) and measured via eq. (6). This definition, although influenced by the choice of the specimen, has the advantages that it is simple to define and easy to measure. In all cases in this study, the value of \( R \) obtained from the Monte Carlo simulations was compared with the value of \( R \) calculated using eq. (5). Finally, comparison was made between the Monte Carlo simulations and experimental composition profile data available in the literature. To the authors' knowledge these simulations represent the first time that all the variables in the single scattering equation (1) have been modeled and compared with experimental data where available.

2. Experiments

Monte Carlo simulations were carried out on an nCUBE 2 hypercube parallel supercomputer. The nCUBE 2 is a multiple-instruction, multiple-data (MIMD) machine with 1024 nodes, connected in a hypercube topology. MIMD machines are well suited to Monte Carlo simulations since each processor (or node) executes its own copy of the simulation program independently with its own set of random numbers. Therefore, 1024 trajectories can be simulated simultaneously, which dramatically reduces the amount of time required to run the simulation. Messages can be passed between nodes at the end of the simulation to sum the results. The serial Monte Carlo simulation program developed at NIST (based on that described in ref. [4]) was adapted to run on the parallel supercomputer. Due to the
MIMD architecture of the nCUBE, very few changes to the program were required. Adapting serial programs to run on other types of supercomputers can be quite difficult as major software changes are often needed. Typically between $10^3$ and $10^6$ electron trajectories were simulated for each datum point in this study. In a 50 nm thin film of Fe, $10^5$ trajectories were simulated in about 3 seconds of computer time.

In all simulations, a beam diameter ($d$) of 1.8 nm FWTM was assumed. This value was typical of previous probe size measurements in a VG HB501 cold field emission gun STEM [10]. (This probe diameter, although large compared to the smallest size that can be used for imaging, contains about 1 nA of current. This current is necessary to produce the minimum X-ray count rate for quantitative analysis.) Simulations were carried out to determine the variation of $R$ with atomic number, using carbon ($Z=6$), aluminum ($Z=13$), copper ($Z=29$), molybdenum ($Z=42$) and gold ($Z=79$) as examples. The effect of $E_0$ was simulated by choice of 100, 200 and 300 keV electrons; specimen thickness was varied from 50 nm to 100 nm.

![Graphs showing the comparison of $R_{\text{simulated}}$ and $R_{\text{calculated}}$ as a function of atomic number, ignoring the effects of beam tailing.](image)

Fig. 3. Comparison of $R_{\text{simulated}}$ (■) and $R_{\text{calculated}}$ (□) as a function of atomic number, ignoring the effects of beam tailing: agreement is good at $t = 50$ nm over a range of beam energies from 100 keV (a) to 300 keV (b) and also at $t = 100$ nm for 100 keV (c) and 300 keV (d).
to 200 nm and simulations were performed both with and without considering the effects of fast secondary (FS) electrons.

3. Results and discussion

3.1. Comparison of Monte Carlo simulations and calculations of the spatial resolution

As explained in section 1.3, the experimental measurements of $R$ were obtained by measuring the composition profile width from the 10% and 90% points and then multiplying this width by 1.8 to give the FWTM width. The simulated value of $R$ from the Monte Carlo program was obtained in a similar manner by first creating a composition profile from the computer simulations of beam spreading. Secondly, the computer determined the 10%–90% profile width and then multiplied this width by 1.8. The simulated value of $R$ was then compared with calculations of $R$ using eqs. (5) and (6).

In figs. 3a and 3b the effect of $t_0$ at constant $t$ (50 nm) is shown, while in figs. 3c and 3d the experiment is repeated for 100 nm foils. (In all

Fig. 4. Comparison of $R_{\text{simulated}}$ (■) and $R_{\text{calculated}}$ (○) as a function of atomic number, including the effects of beam tailing: the values of $t$ and $E_0$ are the same as for the corresponding graphs in fig. 3. Agreement between simulation and calculation is good at low $Z$ (= 6 and 13) but generally gets worse as $t$ increases, $Z$ increases and $E_0$ decreases. Note the different scale for $R$ in (c).
cases the effect of FS electrons is ignored and will be discussed later.) The agreement between the simulation and the calculation is very good over the chosen ranges of $Z$, $E_0$ and $t$. The conclusion to be drawn from fig. 3 is that the simulation results support both the definition of $R$ given in eq. (5) and the measurement of $R$ given by eq. (6). However, this approach of measuring the 10% and 90% points ignores any beam tailing effects.

To see if beam tailing factors are a problem, the Monte Carlo simulations can provide a direct measure of $R$ determined from the distance between the 2% and 98% points on the simulated profiles. In contrast to any experimental profiles, which are noisy and have relatively large errors (see section 3.3 for example), the profiles simulated by the Monte Carlo techniques are relatively smooth and the computer can give a direct readout of the 2% and 98% points. The results are shown in fig. 4. Again in figs. 4a and 4b, the effect of $E_0$ at constant $t$ (50 nm) is shown and in figs. 4c and 4d the simulations were repeated for varying $E_0$ in 100 nm foils. The overall agreement between the simulation and the calculations is substantially poorer than for the 10%–90% case in fig. 3. Agreement is best for low-Z elements, thin specimens and high $E_0$ and becomes progressively worse as $Z$ and $t$ increase and $E_0$ decreases. The disagreement between simulation and calculation can be explained by non-Gaussian beam broadening, resulting in electron tailing at the extremes of the beam.

There are two possible reasons for non-Gaussian beam tailing: FS electrons and plural scattering effects. Fig. 5 shows the effect of FS electrons on the broadening of a composition profile across an interface in carbon ($Z = 6$). From fig. 5 it is obvious that if the profile width is measured between the 10% and 90% points any effects of FS electrons will be missed. However, measurement of the profile width between the 2% and 98% point will incorporate the beam tailing and show a significantly poorer spatial resolution. In order to see the range of this FS effect, the Monte Carlo simulations in fig. 4 were re-run for all elements, thicknesses and accelerating voltages. In fig. 6 we compare the data with and without FS electrons.

It is clear from fig. 6 that the simulated effect of FS electrons is greatest for the lightest elements, giving a much larger value of $R$ for carbon than that calculated. While there are no data points between carbon and aluminum, it is clear that the effect of FS electrons is negligible for aluminum and elements higher in the periodic table. The effect of FS on spatial resolution in elements between carbon and aluminum should be investigated separately. Clearly if measurement of beam broadening includes a large fraction of the beam electrons, then FS effects will dominate the value of $R$ at low $Z$. Because such comparisons of calculations and simulations of $R$ that we showed in fig. 4 did not incorporate the effects of FS electrons, it is not surprising that there is reasonable agreement for the light elements. An experimental measurement would be expected to show the effects of FS electrons, but this has not yet been investigated. On the other hand, there is considerable disagreement between calculation and simulation in fig. 4 for the heavier elements, but this cannot be ascribed to FS electrons, because their effects are negligible.

![Fig. 5. Monte Carlo simulations showing the extraordinary beam tailing that results from FS electrons in carbon: the dotted line shows the composition profile without FS effects and the full line with FS electrons. Measurements of $R$ made between the 10% and 90% points on a composition profile would not discern this tailing but measurements made between the 2% and 98% points would show a large degradation in spatial resolution.](image-url)
for the heavier elements. However, such disagreement may be expected given the single-scattering assumption in the equations used for the calculation. Obviously plural scattering increases as \( Z \) and \( t \) increase and as \( E_0 \) decreases. As a result, the Monte Carlo simulations, which are not confined to simulating a single scattering event, will give a higher value of \( R \).

### 3.2. Effects of specimen thickness and beam energy on spatial resolution

From the simulated and calculated data it is possible to model the effects of various experimental variables on \( R \). While the single-scattering approach implies many variables, \( Z \), \( A \) and \( \rho \) are all specimen dependent and you may not have control over them. The effects of \( t \) and \( E_0 \), however, are under the operator's control. We have examined the variation of \( R \) in Cu as a function of \( E_0 \) and \( t \) and compared the simulated and calculated values. On the basis of the results in fig. 6, we have assumed that FS electrons are not a problem for Cu.

The data in fig. 7 show the effects of thickness on \( R \) for various values of \( E_0 \). The basic single-scattering model ignores contributions of the beam diameter and plural scattering and predicts

![Graphs showing the effect of specimen thickness and beam energy on spatial resolution](image)

**Fig. 6.** The effect of FS electrons on \( R \): the Monte Carlo simulation data in fig. 4 are plotted as circles and compared with the same data incorporating FS electron scatter, shown as squares. There is a large increase in the value of \( R \) when FS electrons are considered at low \( Z \) but FS effects are negligible for Al (\( Z = 13 \)) and above. Note the different scale for \( R \) in (c).
a $t^{3/2}$ variation of $R$. This prediction is approximately obeyed at 200 keV. At 100 keV there is a much stronger effect of thickness probably due to plural scattering, while at 300 keV there is weaker, almost linear effect of $t$ on $R$. In fig. 8, the inverse relationship between $R$ and $E_0$ predicted by single-scattering theory appears as a linear decrease in $R$ with increase $E_0$ for the 200 nm data. However, thinner specimens show a much weaker dependence on $E_0$, and at 50 nm increasing $E_0$ makes almost no difference to $R$. This is an intriguing result given the current drive to intermediate-voltage AEMs. When $R$ is less than 5 nm the FWTM 1.8 nm FEG probe is clearly a major contributor to $R$ and partially suppresses the effects of $b$. If $d$ were much bigger than 1.8 nm, as would be the case for a thermionic source, it might well swamp the effects of $b$ entirely.

3.3. Comparison of Monte Carlo simulations and experimental measurements of the spatial resolution

The Monte Carlo simulations are only as good as the data used within them (e.g. ionization cross sections etc.). In order to test the validity of the simulations, comparison was made with experimental composition profiles measured across interphase interfaces. The first two comparisons were made with Cr profiles across $\alpha/\gamma$ interfaces in 40wt%Fe–30wt%Ni–Cr alloys aged 100 h at 1000°C to attain thermodynamic equilibrium [11]. The $\alpha$ phase contains 80%–90% Cr while the $\gamma$ phase contains 25%–29% Cr. Experimental composition profiles were generated with 100 keV electrons in a VG HB501. Data points were obtained at 1 nm intervals in 55 nm thick foils and 2 nm intervals in 150 nm thick foils. As shown in figs. 9a and 9b there is good agreement at 55 and 150 nm thicknesses. Similar agreement was obtained from calculated profiles [11] in which beam tailing effects were avoided by selecting the 10% and 90% points as described above.

Table 1 lists the values of $R_{\text{Experiment}}$ for four different specimen thicknesses of Fe–Ni–Cr. Two of the thicknesses are those in fig. 9 and the other two are intermediate thicknesses. These experimental values are compared with Monte Carlo simulations, calculations from eqs. (5) and (6) and calculations using the original single-scattering equation.

A more difficult comparison was made with experimental data taken across a small austenite (high Ni) precipitate in a thin foil of Fe–Ni, shown in figs. 10a and 10b. The experimental plot (fig. 10a) does not show a discrete change in
composition across the interface and the composition profile within the precipitate is skewed. The simulation (fig. 10b) shows a much poorer fit to the experimental data than that in figs. 9a and 9b but the profile is symmetrical. There are several possible reasons for this discrepancy. First the specimen may be thicker than determined (20 nm). However, while simulation of a thicker specimen (fig. 10c) shows a less discrete change in composition, as expected, the profile remains

![Graphs and Tables](https://example.com/graphs-tables)

**Table 1**

Comparison of experimental, simulated and calculated values of $R$ for different thicknesses of Fe-Ni-Cr

<table>
<thead>
<tr>
<th>Thickness [nm]</th>
<th>$R_{\text{Experiment}}$</th>
<th>$R_{\text{Monte Carlo}}$</th>
<th>$R_{\text{eq.}(5)}$</th>
<th>$R_{\text{single scattering}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>5.2 ± 0.5</td>
<td>3.6</td>
<td>3.2</td>
<td>9.1</td>
</tr>
<tr>
<td>95</td>
<td>6.6 ± 0.5</td>
<td>6.7</td>
<td>7.1</td>
<td>20.2</td>
</tr>
<tr>
<td>112</td>
<td>9.0 ± 0.5</td>
<td>7.2</td>
<td>9.1</td>
<td>25.9</td>
</tr>
<tr>
<td>150</td>
<td>12.0 ± 0.5</td>
<td>10.8</td>
<td>14.1</td>
<td>39.9</td>
</tr>
</tbody>
</table>

Fig. 9. Comparison of Monte Carlo simulations (full line) and experimental data (individual points with error bars) describing the change in Cr composition across the $\alpha/\gamma$ interface in Fe-Ni-Cr. (a) 55 nm thick specimen; (b) 150 nm thick specimen. In both cases $R$ was determined by measuring $L$ at the 10% and 90% Cr composition positions. Agreement is excellent at both thicknesses. The experimental data are from ref. [11], reproduced courtesy of Blackwells Scientific Publications, Ltd.

Fig. 10. Comparison of an experimental Ni composition profile (a) and a Monte Carlo simulation (b) across a narrow Ni-rich precipitate in a 20 nm thin foil of Fe-Ni. There is relatively poor agreement between the asymmetrical experimental plot and the symmetrical simulated plot. (c) Simulated data for a specimen thickness of 50 nm: agreement with the experimental data (a) is no better.
symmetrical. It is most likely, therefore, that the precipitate/matrix interface is not parallel to the beam or the precipitate is not through thickness. This result illustrates one of the major drawbacks to this method of defining the spatial resolution. When we depend on a specific specimen and we need to know its thickness \( t \), uncertainties in \( t \) may contribute more error to our estimate of \( R \) than any other factor. Until we have a reliable, reproducible and simple method of measuring the thickness, this approach to defining \( R \) will not be completely satisfactory.

It should be noted finally that all attempts to improve the spatial resolution of microanalysis that reduce the X-ray signal are achieved at the expense of the analytical detectability limits [19].

4. Summary and conclusions

We propose a definition of the spatial resolution of X-ray microanalysis in thin foils as that given by the average beam broadening \( R \) defined by eq. (5).

Calculated values of \( R \) are based on the single-scattering model and Gaussian convolution of the beam diameter and beam broadening.

The resolution may be determined experimentally from measurements of the width \( L \) of the composition profile measured as shown in fig. 2 and related to \( R \) by eq. (6).

Monte Carlo simulations show good agreement between the proposed definition of \( R \) and calculations based on eqs. (5) and (6) over a range of thicknesses (50–200 nm), atomic numbers (6–79) and beam energies (100–300 keV).

Discrepancies between the simulations and the calculations exist if beam tailing effects are considered.

At low \( Z \) (<13) FS electrons cause beam tailing and thus increase the value of \( R \).

At higher \( Z \) (>13) the simulations give a greater value of \( R \) than the calculations. This discrepancy increases with \( Z \), \( t \) and decreases with \( E_0 \) and thus is probably due to plural scattering effects contained in the Monte Carlo simulations but ignored in the single-scattering calculations.

This approach to defining \( R \) is limited by having to know the specimen thickness \( a \) priori.

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References