Indefinite Noun Phrases

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1. Introduction

In the previous chapter "A New Account of Quantifiers", we present an account of 'every', 'some', 'no', and 'any', based on a proposed class of objects called *junctions*, which are infinitary-operators. Specifically, we introduce three junctions – conjunction (⋀), disjunction (⋁), and subjunction (Л). In the present chapter, we expand our theory by proposing two more junctions – sum (Σ) and product (Π) – in an effort to account for indefinite noun phrases.

2. Basic Ideas

In English, and many other languages, a common-noun-phrase may be prefixed by an indefinite article, the resulting phrase being what we shall call an *indefinite noun phrase*. The following are example sentences from English, in which 'a' serves as an indefinite article.

Rex is **a** dog
Jay owns **a** dog
a dog is in the yard
there is **a** dog in the yard
if **a** dog is well-fed then it is happy
every man who owns **a** dog feeds it
if **a** man owns **a** dog, then he feeds it
a dog is **a** mammal
a dog can hear sounds **a** human can't
Jay is looking for **a** dog

Note in particular that, if we delete the word 'a', we obtain phrases that standard English rejects as syntactically ill-formed.[1] On the other hand, languages that lack indefinite articles – the biggest of which are Latin, Russian, and Mandarin – have no problem saying sentences corresponding to 'that is dog'. Furthermore, even English eschews indefinite articles when the common-nouns are plural-nouns or mass-nouns, as in the following examples.[2]

those are dogs	that is milk
Jay owns dogs	Jay has milk
dogs are in the yard	milk is in the refrigerator
there are dogs in the yard	there is milk in the refrigerator
if dogs are well-fed, they are happy	if milk is not refrigerated, it curdles
every man who owns dogs feeds them	every man who has milk drinks it
if men own dogs, they feed them	if men have milk, they drink it
dogs are mammals	milk is food
dogs can hear sounds humans can't	milk can be made into cheese
Jay is looking for dogs	Jay is looking for milk

[1] Supposing we reject the reading according to which 'dog' is a proper-name, and the reading according to which 'dog' is a mass-noun [referring presumably to dog-matter].

[2] For the sake of comparison, Spanish and French have plural indefinite articles – 'unos' (masculine), 'unas' (feminine), 'des' (masculine/feminine). Also, colloquial English employs unstressed 'some' ["səm"] as an indefinite article. For example, the following are colloquially interchangeable.

there is ∅ milk in the refrigerator
there is **səm** milk in the refrigerator

Given the strong structural similarities between these examples, we propose to use the term ‘indefinite noun phrase’ in reference to all such phrases, whether prefixed by an explicit indefinite article or not.

3. Initial Hypothesis

In accounting for ‘a’, the following is a fairly natural initial hypothesis.

(iH) ‘a’ is a variant of ‘some’

$[\![a]\!] = [\![\text{some}]\!] = \lambda P_0 \vee\{x \mid Px\}$

This hypothesis accounts for the following example.

1. Jay owns a dog

<table>
<tr><td>Jay</td><td>[+1]</td><td>owns</td><td>a</td><td>dog</td><td>[+2]</td></tr>
<tr><td>J</td><td>$\lambda x\{x_1\}$</td><td rowspan="2"></td><td>$\lambda P_0\vee\{x \mid Px\}$</td><td>$\mathbf{D}_0$</td><td></td></tr>
<tr><td colspan="2" rowspan="3">J_1</td><td colspan="2">$\vee\{ x \mid \mathbf{D}x \}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td>$\lambda y_2\lambda x_1\mathbf{O}xy$</td><td colspan="3">$\vee\{ x_2 \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="4">$\vee\{ \lambda y_1\mathbf{O}yx \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="6">$\vee\{ \mathbf{O}\text{J}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="6">$\exists x \{ \mathbf{D}x \;\&\; \mathbf{O}\text{J}x \}$</td></tr>
</table>

It also accounts for the following example,

2. Jay is a dog

provided we treat ‘is’ as a transitive verb, in which case this sentence has the same overall form as the previous one.

<table>
<tr><td>Jay</td><td>[+1]</td><td>is</td><td>a</td><td>dog</td><td>[+2]</td></tr>
<tr><td>J</td><td>$\lambda x\{x_1\}$</td><td rowspan="2"></td><td>$\lambda P_0\vee\{x \mid Px\}$</td><td>$\mathbf{D}_0$</td><td></td></tr>
<tr><td colspan="2" rowspan="3">J_1</td><td colspan="2">$\vee\{ x \mid \mathbf{D}x \}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td>$\lambda y_2\lambda x_1[x{=}y]$</td><td colspan="3">$\vee\{ x_2 \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="4">$\vee\{ \lambda y_1[y{=}x] \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="6">$\vee\{ \text{J}{=}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="6">$\exists x \{ \mathbf{D}x \;\&\; \text{J}{=}x \}$
$\mathbf{D}\text{J}$</td></tr>
</table>

It also accounts for the following examples.

3. Jay owns dogs

Jay	[+1]	owns	[s'm]	dog	s [+plural]	[+2]
$\mathbf{J}$	$\lambda x\{x_1\}$			$\mathbf{D}_0$	$\lambda P_0\lambda x_0\{Px \,\&\, \mathbb{P}x\}$	
			$\lambda P_0\vee\{x \mid Px\}$	$\lambda x_0\{\mathbf{D}x \,\&\, \mathbb{P}x\}$		
			$\vee\{\, x \mid \mathbf{D}x \,\&\, \mathbb{P}x \,\}$			$\lambda x\{x_2\}$
		$\lambda x_2\lambda y_1\mathbf{O}yx$	$\vee\{\, x_2 \mid \mathbf{D}x \,\&\, \mathbb{P}x \,\}$			
J_1		$\vee\{\, \lambda y_1\mathbf{O}yx \mid \mathbf{D}x \,\&\, \mathbb{P}x \,\}$				
$\vee\{\, \mathbf{O}Jx \mid \mathbf{D}x \,\&\, \mathbb{P}x \,\}$						
$\exists x\, \{\, \mathbf{D}x \,\&\, \mathbb{P}x \,\&\, \mathbf{O}Jx \,\}$						

4. Jay owns land

Jay	[+1]	owns	[s'm]	land	[+mass]	[+2]
$\mathbf{J}$	$\lambda x\{x_1\}$			$\mathbf{L}_0$	$\lambda P_0\lambda x_0\{Px \,\&\, \mathbb{M}x\}$	
			$\lambda P_0\vee\{x \mid Px\}$	$\lambda x_0\{\mathbf{L}x \,\&\, \mathbb{M}x\}$		
			$\vee\{\, x \mid \mathbf{L}x \,\&\, \mathbb{M}x \,\}$			$\lambda x\{x_2\}$
		$\lambda x_2\lambda y_1\mathbf{O}yx$	$\vee\{\, x_2 \mid \mathbf{L}x \,\&\, \mathbb{M}x \,\}$			
J_1		$\vee\{\, \lambda y_1\mathbf{O}yx \mid \mathbf{L}x \,\&\, \mathbb{M}x \,\}$				
$\vee\{\, \mathbf{O}Jx \mid \mathbf{L}x \,\&\, \mathbb{M}x \,\}$						
$\exists x\, \{\, \mathbf{L}x \,\&\, \mathbb{M}x \,\&\, \mathbf{O}Jx \,\}$						

Here, we have expanded the underlying domain of entities to include plural-entities (pluralities), signified by the predicate '$\mathbb{P}$', and mass-entities (quantities), signified by the predicate '$\mathbb{M}$'. Then the root-word 'dog' is understood to apply to dog entities, including dog-individuals, dog-pluralities, and dog-matter.[3]

We also hypothesize that, when 'a' attaches to a plural-noun or mass-noun, it is changed to unstressed 'some' [s'm], which is often deleted in the final presentation (pronunciation/spelling).

[3] In some cases, the very same phonetic form conveys all three morphemes, as in 'fish'.

4. Problems with the Initial Hypothesis

Thus far, the initial hypothesis is:

(iH) 'a' is a variant of 'some';
's'm' is a variant of 'a' which attaches to plural-nouns and mass-nouns, and may be deleted in the final form.

Although (iH) accounts for our initial data, it faces serious difficulties accounting for the following examples.

a dog is **a** mammal
if **a** dog is well-fed, then it is happy
every man who owns **a** dog feeds it
Jay is looking for **a** dog

Applying (iH) to these examples produces the following semantic-trees, which do not yield the intended readings of these sentences.

1. a dog is a mammal

<table>
<tr><td>a dog [+1]</td><td>is</td><td>a mammal [+2]</td></tr>
<tr><td rowspan="2">$\bigvee\{ x_1 \mid \mathbf{D}x \}$</td><td>$\lambda y_2 \lambda x_1[x=y]$</td><td>$\bigvee\{ y_2 \mid \mathbf{M}y \}$</td></tr>
<tr><td colspan="2">$\bigvee\{ \lambda x_1[x=y] \mid \mathbf{M}y \}$</td></tr>
<tr><td colspan="3">$\bigvee\{ \bigvee\{ x=y \mid \mathbf{D}x \} \mid \mathbf{M}y \}$</td></tr>
<tr><td colspan="3">$\exists y \{ \mathbf{M}y \;\&\; \exists x\{\mathbf{D}x \;\&\; x=y\} \}$
※ $\exists x\{\mathbf{M}x \;\&\; \mathbf{D}x\}$ ※</td></tr>
</table>

2. if a dog is well-fed, then it is happy

<table>
<tr><td>if</td><td>a dog</td><td>[+1,–1]</td><td>is well-fed</td><td>then</td><td>(–1) it [+1]</td><td>is happy</td></tr>
<tr><td rowspan="3">$\lambda P \lambda Q\{P \to Q\}$</td><td>$\bigvee\{ x \mid \mathbf{D}x \}$</td><td>$\lambda x\{x_1 \times x_{-1}\}$</td><td rowspan="2">$\lambda x_0 \mathbf{W}x$</td><td>$\varnothing$</td><td>$\lambda x_{-1}\{x_1\}$</td><td>$\lambda x_1 \mathbf{H}x$</td></tr>
<tr><td colspan="2">$\bigvee\{ x_1 \times x_{-1} \mid \mathbf{D}x \}$</td><td colspan="3" rowspan="3">$\lambda x_{-1}\mathbf{H}x$</td></tr>
<tr><td colspan="3">$\bigvee\{ \mathbf{W}x \times x_{-1} \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="4">$\bigvee\{ \lambda Q\{\mathbf{W}x \to Q\} \times x_{-1} \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">$\bigvee\{ \lambda Q\{\mathbf{W}x \to Q\} \times x_{-1} \times \lambda x_{-1}\mathbf{H}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">$\bigvee\{ \lambda Q\{\mathbf{W}x \to Q\} \times \mathbf{H}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">$\bigvee\{ \mathbf{W}x \to \mathbf{H}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">※ $\exists x \{ \mathbf{D}x \;\&\; (\mathbf{W}x \to \mathbf{H}x) \}$ ※</td></tr>
</table>

3. every man who owns a dog feeds it

<table>
<tr><th>every</th><th>man</th><th>who [+1]</th><th>owns</th><th>a dog</th><th>[+2, −1]</th><th>[+1]</th><th>feeds</th><th>(−1) it [+2]</th></tr>
<tr><td rowspan="5">$\lambda P_0 \wedge \{x \mid Px\}$</td><td rowspan="4">$\mathbf{M}_0$</td><td rowspan="3">$\lambda Q_1 \lambda P_0 \lambda x_0 \{Px \& Qx\}$</td><td rowspan="2">$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\vee\{ y \mid \mathbf{D}y \}$</td><td>$\lambda x\{x_2 \times x_{-1}\}$</td><td rowspan="6">$\lambda x\{x_1\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{F}xy$</td><td>$\lambda z_{-1}\{z_2\}$</td></tr>
<tr><td colspan="2">$\vee\{ y_2 \times y_{-1} \mid \mathbf{D}y \}$</td><td colspan="2" rowspan="6">$\lambda z_{-1} \lambda x_1 \mathbf{F}xz$</td></tr>
<tr><td colspan="3">$\vee\{ \lambda x_1 \mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="4">$\vee\{ \lambda P_0 \lambda x_0\{Px \& \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="5">$\vee\{ \lambda x_0\{\mathbf{M}x \& \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="6">$\vee\{ \wedge\{x \mid \mathbf{M}x \& \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="7">$\vee\{ \wedge\{x_1 \mid \mathbf{M}x \& \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="9">$\vee\{ \wedge\{x_1 \mid \mathbf{M}x \& \mathbf{O}xy\} \times \lambda x_1 \mathbf{F}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="9">$\vee\{ \wedge\{\mathbf{F}xy \mid \mathbf{M}x \& \mathbf{O}xy\} \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="9">※ $\exists y \{ \mathbf{D}y \& \forall x\{ \{\mathbf{M}x \& \mathbf{O}xy\} \rightarrow \mathbf{F}xy \} \}$ ※</td></tr>
</table>

The above construction grants ‘a dog’ wide-scope, which allows it to bind ‘it’, but the resulting reading is incorrect. If we pursue a construction that grants ‘every man’ wide-scope, then ‘a dog’ fails to bind ‘it’.

4. Jay is looking for a dog

<table>
<tr><th>Jay [+1]</th><th>is-looking-for</th><th>a dog [+2]</th></tr>
<tr><td rowspan="2">J_1</td><td>$\lambda y_2 \lambda x_1 \mathbf{L}xy$</td><td>$\vee\{ y_2 \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="2">$\vee\{ \lambda x_1 \mathbf{L}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="3">$\vee\{ \mathbf{L}\mathrm{J}y \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="3">$\exists y \{ \mathbf{D}y \& \mathbf{L}\mathrm{J}y \}$</td></tr>
</table>

According to this reading, there is a (particular) dog that Jay is looking for. Although this is an admissible reading, there is another, more plausible, reading according to which Jay is not looking for a particular dog; rather, ‘a dog’ is better thought of as indicating the *kind* of thing Jay is looking for. This is a reading that (iH) cannot generate.

5. The New Proposal

In order to account for indefinite noun phrases, we take a three-part approach.

(1) we propose a dual-pair of junctions – *product* and *sum*.
(2) we propose a type-logical extension of our account of common-noun-phrases.
(3) we propose to treat the article 'a', not as a quantifier, but as a common-noun modifier.

1. Product and Sum

We propose a dual-pair of junctions – product Π, and sum Σ. This involves expanding the type-formation rules so that if $\mathfrak{I}$ is a type, then so are $\Pi\mathfrak{I}$ and $\Sigma\mathfrak{I}$. It also involves expanding the syntax of type-theory to include all expressions of the following forms.

$\Pi_\nu\{\varepsilon \mid \Phi\}$ the product [over ν] of all ε such that Φ
$\Sigma_\nu\{\varepsilon \mid \Phi\}$ the sum [over ν] of all ε such that Φ

Here, ν is any variable [which is often omitted], ε is any expression of type $\mathfrak{I}$, Φ is any formula, and the resulting expression has type $\Pi\mathfrak{I}$ [respectively, $\Sigma\mathfrak{I}$].

The following are the associated simplification-rules.

$\Sigma D = D$
$\Sigma S = S$
$\Sigma_\nu\{\Phi \mid \Psi\}$ **/** $\exists\nu\{\Psi \,\&\, \Phi\}$
$\Pi S \neq S$
$\Pi_\nu\{\Phi \mid \Psi\}$ **/** $\forall\nu\{\Psi \to \Phi\}$ provided node is assertoric[4]

The following are the associated composition-rules.

α	α	α
$\{\alpha,\beta\} \vdash \gamma$	$\{\alpha,\beta\} \vdash \gamma$	$\{\alpha,\beta\} \vdash \gamma$
$\Pi\{\beta \mid \Phi\}$	$\Sigma\{\beta \mid \Phi\}$	$\Sigma\{\beta \mid \Phi\}$
$\Pi\{\gamma \mid \Phi\}$	$\Pi\{\gamma \mid \Phi\}$	$\Sigma\{\gamma \mid \Phi\}$
no admissibility-restrictions	if α is *any-promoting* (e.g., if α is anti-tonic)	if α is not *any-promoting*, and α is not a junction other than Σ

[4] As before, assertoric is ultimately complicated. For the moment, at least, a node is assertoric if and only if it is topmost and contains no free variables. This equivalence is then based on the plausible intuition that to assert a product of sentences is to assert all those sentences.

2. Common-Noun-Phrases Transform into Entity-Sums

As before, we propose that common-noun-phrases are fundamentally 0-marked predicates, which is to say they have type $D_0 \rightarrow S$. We further propose that every such phrase gives rise to an associated entity-sum, in accordance with the following *transformational rule*.

$$\lambda \nu_0 \Phi \quad // \quad \Sigma\{\nu \mid \Phi\}$$

This is not an identity, since the objects don't have the same type. Rather, it is a bi-directional rule that authorizes *deriving* an entity-sum (type ΣD) from a nullative-predicate (type $D_0 \rightarrow S$), and conversely.

3. The Indefinite Article 'a'

We propose that 'a' is fundamentally a number-word, basically equivalent to 'one', and semantically rendered as follows.[5]

$$[\![\text{a}]\!] \quad = \quad \lambda P_0 \lambda x_0\{Px \ \&\ \mathbf{1}x\}$$

Here, '**1**' is understood as follows.[6]

$$\mathbf{1}x \quad =_{df} \quad x \text{ is unital}$$

where 'unital' is a primitive notion.[7] If we do not admit plural-nouns or mass-nouns, as is common in elementary logic, then 'a' is redundant, but if we do admit these, then 'a' distinguishes (say) among the following.

being	**a** fish	(singular-entity)
being	fish	(plural-entity)
being	fish	(mass-entity)

6. Examples

In the following, we concentrate on singular-nouns, and accordingly treat 'a' as redundant; for example, 'a dog' is equivalent to 'dog'.

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[5] In many languages, including the Romance languages, the same surface form (spelling, pronunciation) is used for both the indefinite article "a" and the numeral "one". Latin has no articles, yet all its descendents do, which is an evolutionary curiosity. It is generally believed that the indefinite articles in Romance languages derive (independently?) from the Latin word 'unus' for 'one'.

[6] See chapter "Numerical Quantifiers". Note that 'one' can be used as a bare-adjective, whereas 'a' cannot.

[7] Also, its application is heavily context-dependent. Usually, "units " are simple individuals (another primitive notion), but other times they are more complex entities, as in the following,

a man and woman who are married

where 'a man and woman' means "one man-woman pair", which is not a simple individual.

1. every man owns a dog

<table>
<tr><th>every man [+1]</th><th>owns</th><th>a dog</th><th>[+2]</th></tr>
<tr><td></td><td></td><td>$\lambda y_0\mathbf{D}y$</td><td></td></tr>
<tr><td></td><td></td><td>$\Sigma\{y \mid \mathbf{D}y\}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td></td><td>$\lambda y_2\lambda x_1\mathbf{O}xy$</td><td colspan="2">$\Sigma\{\ y_2 \mid \mathbf{D}y\ \}$</td></tr>
<tr><td>$\bigwedge\{\ x_1 \mid \mathbf{M}x\ \}$</td><td colspan="3">$\Sigma\{\ \lambda x_1\mathbf{O}xy \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="4">$\bigwedge\{\ \Sigma\{\ \mathbf{O}xy \mid \mathbf{D}y\ \} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="4">$\forall x\ \{\ \mathbf{M}x \rightarrow \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\ \}$</td></tr>
</table>

<table>
<tr><th>every man [+1]</th><th>owns</th><th>a dog</th><th>[+2]</th></tr>
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<tr><td></td><td></td><td>$\Sigma\{y \mid \mathbf{D}y\}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td></td><td>$\lambda y_2\lambda x_1\mathbf{O}xy$</td><td colspan="2">$\Sigma\{\ y_2 \mid \mathbf{D}y\ \}$</td></tr>
<tr><td></td><td colspan="3">$\Sigma\{\ \lambda x_1\mathbf{O}xy \mid \mathbf{D}y\ \}$</td></tr>
<tr><td>$\bigwedge\{\ x_1 \mid \mathbf{M}x\ \}$</td><td colspan="3">$\lambda x_1\ \exists y\ \{\ \mathbf{D}y\ \&\ \mathbf{O}xy\ \ \}$</td></tr>
<tr><td colspan="4">$\bigwedge\{\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="4">$\forall x\ \{\ \mathbf{M}x \rightarrow \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\ \}$</td></tr>
</table>

2. every man who owns a dog is happy

<table>
<tr><th>every [+1]</th><th>man</th><th>who [+1]</th><th>owns</th><th>a dog [+2]</th><th>is happy</th></tr>
<tr><td></td><td></td><td></td><td>$\lambda y_2\lambda x_1\mathbf{O}xy$</td><td>$\Sigma\{y_2 \mid \mathbf{D}y\}$</td><td></td></tr>
<tr><td></td><td></td><td>$\lambda Q_1\lambda P_0\lambda x_0\{Px\&Qx\}$</td><td colspan="2">$\Sigma\{\ \lambda x_1\mathbf{O}xy \mid \mathbf{D}y\ \}$</td><td></td></tr>
<tr><td></td><td>$\mathbf{M}_0$</td><td colspan="3">$\Sigma\{\ \lambda P_0\lambda x_0\{Px\ \&\ \mathbf{O}xy\} \mid \mathbf{D}y\ \}$</td><td></td></tr>
<tr><td>$\lambda P_0\bigwedge\{x_1 \mid Px\}$</td><td colspan="4">$\Sigma\{\ \lambda x_0\{\mathbf{M}x\ \&\ \mathbf{O}xy\} \mid \mathbf{D}y\ \}$</td><td></td></tr>
<tr><td colspan="5">$\Pi\{\ \bigwedge\{x_1 \mid \mathbf{M}x\ \&\ \mathbf{O}xy\} \mid \mathbf{D}y\ \}$</td><td>$\lambda x_1\mathbf{H}x$</td></tr>
<tr><td colspan="6">$\Pi\{\ \bigwedge\{\mathbf{H}x \mid \mathbf{M}x\ \&\ \mathbf{O}xy\} \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="6">$\forall y\ \{\ \mathbf{D}y \rightarrow \forall x\{\{\mathbf{M}x\ \&\ \mathbf{O}xy\} \rightarrow \mathbf{H}x\ \}$</td></tr>
</table>

In this computation, ⟦every⟧ is anti-tonic and therefore converts Σ to Π, which ultimately grants ‘a dog’ wide-scope. There is another computation, given as follows, according to which ‘a dog’ has narrow-scope, although the resulting formula is logically-equivalent to the previous example.

3. every man who owns a dog is happy [alternative computation]

<table>
<tr><th>every [+1]</th><th>man</th><th>who [+1]</th><th>owns a dog</th><th>is happy</th></tr>
<tr><td></td><td></td><td></td><td>$\Sigma\{\ \lambda x_1\mathbf{O}xy \mid \mathbf{D}y\ \}$</td><td></td></tr>
<tr><td></td><td></td><td>$\lambda Q_1\lambda P_0\lambda x_0\{Px\&Qx\}$</td><td>$\lambda x_1\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}$</td><td></td></tr>
<tr><td></td><td>$\mathbf{M}_0$</td><td colspan="2">$\lambda P_0\ \lambda x_0\ \{\ Px\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\ \}$</td><td></td></tr>
<tr><td>$\lambda P_0\bigwedge\{x_1 \mid Px\}$</td><td colspan="3">$\lambda x_0\ \{\ \mathbf{M}x\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\ \}$</td><td></td></tr>
<tr><td colspan="4">$\bigwedge\{\ x_1 \mid \mathbf{M}x\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\ \}$</td><td>$\lambda x_1\mathbf{H}x$</td></tr>
<tr><td colspan="5">$\bigwedge\{\ \mathbf{H}x \mid \mathbf{M}x\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\ \}$</td></tr>
<tr><td colspan="5">$\forall x\ \{\ \{\mathbf{M}x\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\} \rightarrow \mathbf{H}x\ \}$</td></tr>
</table>

In the previous example, ‘a dog’ can be narrow-∃ or wide-∀, the resulting formulas being logically equivalent. The following example is superficially similar, but semantically quite different.

4. every man who owns a dog feeds **it**

<table>
<tr><td>every [+1]</td><td>man who own a dog</td><td>feeds</td><td>(–1) it [+2]</td></tr>
<tr><td>$\lambda P_0 \wedge\{x_1 \mid \mathbf{M}x\}$</td><td>$\Sigma\{ \lambda x_0\{\mathbf{M}x\&\mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y \}$</td><td>$\lambda y_2\lambda x_1\mathbf{F}xy$</td><td>$\lambda y_{-1}\{y_2\}$</td></tr>
<tr><td colspan="2">$\Pi\{ \wedge\{x_1 \mid \mathbf{M}x\&\mathbf{O}xy \} \times y_{-1} \mid \mathbf{D}y \}$</td><td colspan="2">$\lambda y_{-1}\, \lambda x_1\, \mathbf{F}xy$</td></tr>
<tr><td colspan="4">$\Pi\{ \wedge\{x_1 \mid \mathbf{M}x\&\mathbf{O}xy \} \times \lambda x_1\, \mathbf{F}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="4">$\Pi\{ \wedge\{ \mathbf{F}xy \mid \mathbf{M}x\&\mathbf{O}xy \} \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="4">$\forall y\ \{ \mathbf{D}y \rightarrow \forall x\ \{ (\mathbf{M}x\ \&\ \mathbf{O}xy) \rightarrow \mathbf{F}xy \} \}$</td></tr>
</table>

Notice, in particular, that a narrow-scope reading of ‘a dog’ is impossible because Σ-simplification is not available at any node, because the sum is not a sum of sentences. In order to accomplish the latter, we must remove the anaphoric marker [–1], but then ‘a dog’ does not bind ‘it’.

The following is another example in which ‘a’ must gain wide-scope in order to bind its pronoun.

5. if a dog is well-fed, then it is happy

<table>
<tr><td>if</td><td>a dog</td><td>[+1, –1]</td><td>is well-fed</td><td>then</td><td>(–1) it [+1]</td><td>is happy</td></tr>
<tr><td rowspan="3">$\lambda P\lambda Q\{P\rightarrow Q\}$</td><td>$\Sigma\{x \mid \mathbf{D}x\}$</td><td>$\lambda x\{x_1 \times x_{-1}\}$</td><td rowspan="2">$\lambda x_1\mathbf{W}x$</td><td>$\varnothing$</td><td>$\lambda x_{-1}\{x_1\}$</td><td>$\lambda x_1\mathbf{H}x$</td></tr>
<tr><td colspan="2">$\Sigma\{ x_1 \times x_{-1} \mid \mathbf{D}x \}$</td><td colspan="3" rowspan="3">$\lambda x_{-1}\mathbf{H}x$</td></tr>
<tr><td colspan="3">$\Sigma\{ \mathbf{W}x \times x_{-1} \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="4">$\Pi\{ \lambda Q\{\mathbf{W}x\rightarrow Q\} \times x_{-1} \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">$\Pi\{ \lambda Q\{\mathbf{W}x\rightarrow Q\} \times \mathbf{H}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">$\Pi\{ \mathbf{W}x\rightarrow\mathbf{H}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">$\forall x\ \{ \mathbf{D}x \rightarrow \{\mathbf{W}x \rightarrow \mathbf{H}x\} \}$</td></tr>
</table>

Notice that ⟦if⟧ is anti-tonic, and accordingly converts Σ to Π. The following is a variation in which ‘if’ is centrally placed.

6. a dog is happy if it is well-fed

<table>
<tr><td>a dog</td><td>[+1, –1]</td><td>is happy</td><td>if</td><td>(–1) it [+1]</td><td>is well-fed</td></tr>
<tr><td>$\Sigma\{x \mid \mathbf{D}x\}$</td><td>$\lambda x\{x_1 \times x_{-1}\}$</td><td rowspan="2">$\lambda x_1 \mathbf{H}x$</td><td rowspan="2">$\lambda P \lambda Q\{P \rightarrow Q\}$</td><td>$\lambda x_{-1}\{x_1\}$</td><td>$\lambda x_1 \mathbf{W}x$</td></tr>
<tr><td colspan="2">$\Sigma\{\ x_1 \times x_{-1} \mid \mathbf{D}x\ \}$</td><td colspan="2">$\lambda x_{-1} \mathbf{W}x$</td></tr>
<tr><td colspan="3">$\Sigma\{\ \mathbf{H}x \times x_{-1} \mid \mathbf{D}x\ \}$</td><td colspan="3">$\lambda x_{-1} \lambda Q\{\mathbf{W}x \rightarrow Q\}$</td></tr>
<tr><td colspan="6">$\Pi\{\ \mathbf{H}x \times \lambda Q\{\mathbf{W}x \rightarrow Q\} \mid \mathbf{D}x\ \}$</td></tr>
<tr><td colspan="6">$\Pi\{\ \mathbf{W}x \rightarrow \mathbf{H}x \mid \mathbf{D}x\ \}$</td></tr>
<tr><td colspan="6">$\forall x\ \{\ \mathbf{D}x \rightarrow \{\mathbf{W}x \rightarrow \mathbf{H}x\}\ \}$</td></tr>
</table>

Notice that ⟦if it is well-fed⟧ is anti-tonic, and accordingly converts Σ to Π.

The following is a variation with ‘if and only if’ instead of ‘if’.

7. a dog is happy if and only if it is well-fed

<table>
<tr><td>a dog</td><td>[+1, –1]</td><td>is happy</td><td>if-and-only-if</td><td>(–1) it [+1]</td><td>is well-fed</td></tr>
<tr><td>$\Sigma\{x \mid \mathbf{D}x\}$</td><td>$\lambda x\{x_1 \times x_{-1}\}$</td><td rowspan="2">$\lambda x_1 \mathbf{H}x$</td><td rowspan="2">$\lambda P \lambda Q\{P \leftrightarrow Q\}$</td><td>$\lambda x_{-1}\{x_1\}$</td><td>$\lambda x_1 \mathbf{W}x$</td></tr>
<tr><td colspan="2">$\Sigma\{\ x_1 \times x_{-1} \mid \mathbf{D}x\ \}$</td><td colspan="2">$\lambda x_{-1} \mathbf{W}x$</td></tr>
<tr><td colspan="3">$\Sigma\{\ \mathbf{H}x \times x_{-1} \mid \mathbf{D}x\ \}$</td><td colspan="3">$\lambda x_{-1} \lambda Q\{\mathbf{W}x \leftrightarrow Q\}$</td></tr>
<tr><td colspan="6">$\Pi\{\ \mathbf{H}x \times \lambda Q\{\mathbf{W}x \leftrightarrow Q\} \mid \mathbf{D}x\ \}$</td></tr>
<tr><td colspan="6">$\Pi\{\ \mathbf{W}x \leftrightarrow \mathbf{H}x \mid \mathbf{D}x\ \}$</td></tr>
<tr><td colspan="6">$\forall x\ \{\ \mathbf{D}x \rightarrow \{\mathbf{W}x \leftrightarrow \mathbf{H}x\}\ \}$</td></tr>
</table>

Notice that ⟦if and only if it is well-fed⟧ is not (obviously) anti-tonic; we nevertheless postulate that it is enough like ⟦if if it is well-fed⟧ that it also converts Σ to Π.

The following is a variation with two occurrences of ‘a’ with corresponding pronouns.

8. if a man owns a dog, then he feeds it

<table>
<tr><td>if</td><td>a man [+1,–1]</td><td>owns</td><td>a dog [+2, –2]</td><td>then</td><td>(–1) he [+1]</td><td>feeds</td><td>(–2) it [+2]</td></tr>
<tr><td rowspan="3">$\lambda P \lambda Q$
$\{P \rightarrow Q\}$</td><td rowspan="2">$\Sigma\{\ x_1 \times x_{-1} \mid \mathbf{M}x\ \}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{\ y_2 \times y_{-2} \mid \mathbf{D}y\ \}$</td><td rowspan="2">$\varnothing$</td><td rowspan="2">$\lambda x_{-1}\{x_1\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{F}xy$</td><td>$\lambda y_{-2}\{y_2\}$</td></tr>
<tr><td colspan="2">$\Sigma\{\ \lambda x_1 \mathbf{O}xy \times y_{-2} \mid \mathbf{D}y\ \}$</td><td colspan="2">$\lambda y_{-2} \lambda x_1 \mathbf{F}xy$</td></tr>
<tr><td colspan="3">$\Sigma\{\ \Sigma\{\ \mathbf{O}xy \times y_{-2} \mid \mathbf{D}y\ \} \times x_{-1} \mid \mathbf{M}x\ \}$</td><td colspan="4" rowspan="2">$\lambda y_{-2} \lambda x_{-1} \mathbf{F}xy$</td></tr>
<tr><td colspan="4">$\Pi\{\ \Pi\{\ \lambda Q\{\mathbf{O}xy \rightarrow Q\} \times y_{-2} \mid \mathbf{D}y\ \} \times x_{-1} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="8">$\Pi\{\ \Pi\{\ \lambda Q\{\mathbf{O}xy \rightarrow Q\} \times \lambda x_{-1} \mathbf{F}xy \mid \mathbf{D}y\ \} \times x_{-1} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="8">$\Pi\{\ \Pi\{\ \lambda Q\{\mathbf{O}xy \rightarrow Q\} \times \mathbf{F}xy \mid \mathbf{D}y\ \} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="8">$\Pi\{\ \Pi\{\ \lambda Q\{\mathbf{O}xy \rightarrow Q\} \times \mathbf{F}xy \mid \mathbf{D}y\ \} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="8">$\Pi\{\ \Pi\{\ \mathbf{O}xy \rightarrow \mathbf{F}xy \mid \mathbf{D}y\ \} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="8">$\forall x\ \{\ \mathbf{M}x \rightarrow \forall y\ \{\ \mathbf{D}y \rightarrow \{\mathbf{O}xy \rightarrow \mathbf{F}xy\}\ \}\ \}$</td></tr>
</table>

Notice that ⟦if⟧ is anti-tonic, and accordingly converts each Σ to Π.

9. if no person owns a dog, then it is free

<table>
<tr><td>if</td><td>no person [+1]</td><td>owns</td><td>a dog [+2, –1]</td><td>then</td><td>(–1) it [+1]</td><td>is free</td></tr>
<tr><td rowspan="4">$\lambda P \lambda Q\{P \rightarrow Q\}$</td><td rowspan="2">$\wedge\{\neg \times x_1 \mid \mathbf{P}x\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{y_2 \times y_{-1} \mid \mathbf{D}y\}$</td><td>$\varnothing$</td><td>$\lambda y_{-1}\{y_1\}$</td><td>$\lambda x_1 \mathbf{F}x$</td></tr>
<tr><td colspan="2">$\Sigma\{\ \lambda x_1 \mathbf{O}xy \times y_{-1} \mid \mathbf{D}y\ \}$</td><td colspan="3" rowspan="4">$\lambda y_{-1} \mathbf{F}y$</td></tr>
<tr><td colspan="3">$\Pi\{\ \wedge\{\sim \mathbf{O}xy \mid \mathbf{P}x\} \times y_{-1} \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="3">$\Pi\{\ \forall x\{\mathbf{P}x \rightarrow \sim \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="4">$\Pi\{\ \forall Q\{\forall x\{\mathbf{P}x \rightarrow \sim \mathbf{O}xy\} \rightarrow Q\} \times y_{-1} \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="7">$\Pi\{\ \forall Q\{\forall x\{\mathbf{P}x \rightarrow \sim \mathbf{O}xy\} \rightarrow Q\} \times \mathbf{F}y \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="7">$\Pi\{\ \forall x\{\mathbf{P}x \rightarrow \sim \mathbf{O}xy\} \rightarrow \mathbf{F}y \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="7">$\forall y\{\ \mathbf{D}y \rightarrow \{\forall x\{\mathbf{P}x \rightarrow \sim \mathbf{O}xy\} \rightarrow \mathbf{F}y\}\ \}$</td></tr>
</table>

Notice that ⟦no person⟧ is anti-tonic, and promotes Σ to Π, which admits ⟦if⟧. This is why Σ is not promote to ∧ – for then it could not out-scope ‘if’. See later section on difference between ‘a’ and ‘any’.

Also notice that the following alternative attempt to compose ⟦no person⟧ and ⟦owns a dog⟧ does not ultimately compute.

10. if no person owns a dog, then it is free [alternative attempt]

<table>
<tr><td>if</td><td>no person [+1]</td><td>owns</td><td>a dog [+2, −1]</td><td>then</td><td>(−1) it [+1]</td><td>is free</td></tr>
<tr><td rowspan="4">$\lambda P\lambda Q\{P\rightarrow Q\}$</td><td rowspan="2">$\wedge\{\neg \times x_1 \mid \mathbf{P}x\}$</td><td>$\lambda y_2\lambda x_1\mathbf{O}xy$</td><td>$\Sigma\{y_2 \times y_{-1} \mid \mathbf{D}y\}$</td><td>$\varnothing$</td><td>$\lambda y_{-1}\{y_1\}$</td><td>$\lambda x_1\mathbf{F}x$</td></tr>
<tr><td colspan="2">$\Sigma\{ \lambda x_1\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \}$</td><td colspan="3" rowspan="4">$\lambda y_{-1}\mathbf{F}y$</td></tr>
<tr><td colspan="3">$\wedge\{\neg \times \Sigma\{ \mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \} \mid \mathbf{P}x\}$</td></tr>
<tr><td colspan="3">$\wedge\{ \Pi\{ \sim\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \} \mid \mathbf{P}x\}$</td></tr>
<tr><td colspan="4">※</td></tr>
<tr><td colspan="7">※</td></tr>
</table>

The problem is that the ∧-expression does not admit ⟦if⟧ and ⟦if⟧ does not combine sensibly with the ∧-expression since it does not simplify to a sentence, unless the anaphoric-marker is removed, but then the resulting expression does not bind ‘it’.

One more example:

11. a man's mother respects him if he respects her

<table>
<tr><td>a man's [−1]</td><td>mother</td><td>[+1, −2]</td><td>respects</td><td>(−1) him [+2]</td><td>if</td><td>(−1) he [+1]</td><td>respects</td><td>(−2) her [+2]</td></tr>
<tr><td>$\Sigma\{x_6\times x_{-1} \mid \mathbf{M}x \}$</td><td>$\lambda x_6 \{\mathbf{m}(x)\}$</td><td rowspan="2">$\lambda x\{x_1 \times x_{-2}\}$</td><td>$\lambda y_2\lambda x_1 \mathbf{R}xy$</td><td>$\lambda z_{-1}\{z_2\}$</td><td rowspan="3">$\lambda P\lambda Q \{P\rightarrow Q\}$</td><td rowspan="2">$\lambda x_{-1}\{x_1\}$</td><td>$\lambda y_2\lambda x_1 \mathbf{R}xy$</td><td>$\lambda y_{-2}\{y_2\}$</td></tr>
<tr><td colspan="2">$\Sigma\{ \mathbf{m}(x) \times x_{-1} \mid \mathbf{M}x \}$</td><td colspan="2" rowspan="2">$\lambda z_{-1}\lambda x_1\mathbf{R}xz$</td><td colspan="2">$\lambda y_{-2}\lambda x_1\mathbf{R}xy$</td></tr>
<tr><td colspan="3">$\Sigma\{ \mathbf{m}(x)_1 \times \mathbf{m}(x)_{-2} \times x_{-1} \mid \mathbf{M}x \}$</td><td colspan="3">$\lambda y_{-2}\lambda x_{-1}\mathbf{R}xy$</td></tr>
<tr><td colspan="5">$\Sigma\{ \mathbf{m}(x)_1 \times \mathbf{m}(x)_{-2} \times \lambda y_1\mathbf{R}yx \times x_{-1} \mid \mathbf{M}x \}$</td><td colspan="4" rowspan="2">$\lambda y_{-2}\, \lambda x_{-1}\, \lambda Q\, \{\mathbf{R}xy \rightarrow Q\}$</td></tr>
<tr><td colspan="5">$\Sigma\{ \mathbf{R}[\mathbf{m}(x), x] \times \mathbf{m}(x)_{-2} \times x_{-1} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="9">$\Pi\{ \mathbf{R}[\mathbf{m}(x), x] \times \lambda Q \{ \mathbf{R}[x,\mathbf{m}(x)] \rightarrow Q \} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="9">$\Pi\{ \mathbf{R}[x,\mathbf{m}(x)] \rightarrow \mathbf{R}[\mathbf{m}(x), x] \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="9">$\forall x \{ \mathbf{M}x \rightarrow \{\mathbf{R}[x,\mathbf{m}(x)] \rightarrow \mathbf{R}[\mathbf{m}(x), x]\} \}$</td></tr>
</table>

7. Sometimes 'some' is Indefinite

When applied to special domains, quantifier phrases occasionally take on special forms, as in 'always', 'never', 'everywhere', and 'somewhere'. When the special domain is persons, we have the following forms.[8]

everyone	$=_{df}$	every person
anyone	$=_{df}$	any person
someone	$=_{df}$	some person
no one	$=_{df}$	no person

The morphological rule is clear. However, when we apply it to 'a person', we obtain 'a one', which is inadmissible.[9] What we have instead is the following abbreviation.

someone $=_{df}$ a person

Thus, 'someone' is ambiguous between an indefinite noun phrase and a quantifier phrase, which can be a source of semantic confusion. The following illustrates its use as an indefinite noun phrase.

1. if someone owns a dog, then he/she feeds it

if	someone [+1, –1]	owns	a dog [+2, –2]	then	(–1) he/she [+1]	feeds	(–2) it [+2]
$\lambda P \lambda Q \{P \to Q\}$	$\Sigma\{x_1 \times x_{-1} \mid \mathbf{P}x\}$	$\lambda y_2 \lambda x_1 \mathbf{O}xy$	$\Sigma\{y_2 \times y_{-2} \mid \mathbf{D}y\}$	$\varnothing$	$\lambda x_{-1}\{x_1\}$	$\lambda y_2 \lambda x_1 \mathbf{F}xy$	$\lambda y_{-2}\{y_2\}$
		$\Sigma\{ \lambda x_1 \mathbf{O}xy \times y_{-2} \mid \mathbf{D}y \}$				$\lambda y_{-2} \lambda x_1 \mathbf{F}xy$	
	$\Sigma\{ \Sigma\{ \mathbf{O}xy \times y_{-2} \mid \mathbf{D}y \} \times x_{-1} \mid \mathbf{P}x \}$			$\lambda y_{-2} \lambda x_{-1} \mathbf{F}xy$			
$\Pi\{ \Pi\{ \lambda Q\{\mathbf{O}xy \to Q\} \times y_{-2} \mid \mathbf{D}y \} \times x_{-1} \mid \mathbf{P}x \}$							
$\Pi\{ \Pi\{ \lambda Q\{\mathbf{O}xy \to Q\} \times y_{-2} \mid \mathbf{D}y \} \times x_{-1} \mid \mathbf{P}x \}$							
$\Pi\{ \Pi\{ \lambda Q\{\mathbf{O}xy \to Q\} \times \lambda x_{-1} \mathbf{F}xy \mid \mathbf{D}y \} \times x_{-1} \mid \mathbf{P}x \}$							
$\Pi\{ \Pi\{ \lambda Q\{\mathbf{O}xy \to Q\} \times \mathbf{F}xy \mid \mathbf{D}y \} \mid \mathbf{P}x \}$							
$\Pi\{ \Pi\{ \mathbf{O}xy \to \mathbf{F}xy \mid \mathbf{D}y \} \mid \mathbf{P}x \}$							
$\forall x \{ \mathbf{P}x \to \forall y\{\mathbf{D}y \to (\mathbf{O}xy \to \mathbf{F}xy)\} \}$							

[8] Note that the phrase 'every one' – with a slight pause between 'every' and 'one' – is analogous to 'this one', as in the following example.

I have many dogs; every **one** is smart; this **one** is very smart

[9] And the rule does not apply to plural quantifiers; for example, 'several persons' does not abbreviate as 'several ones'; also, the rule does not apply to numerical quantifiers; for example, 'exactly one person' does not abbreviate as 'exactly one one'.

8. Other Constructions that Convert Σ to Π

We have dealt with numerous examples in which 'a' starts out as Σ but converts to Π midway though the derivation, by inter-acting with an anti-tonic function. Recall that 'any' similarly converts from Л to ∧ when it interacts with an anti-tonic function. Also, recall that other constructions perform this conversion, which we call any-promoting.

We hereby propose that *any* construction that promotes Л to ∧ likewise converts Σ to Π. So, for example, the following sentence

a dog is a mammal

is nomic (i.e., law-like) in character, and accordingly supports counterfactual/subjunctive reasoning, and so Σ gets promoted to Π, in virtue of which it is a universal, not an existential. On the other hand, the following example,

a dog is in the yard

is presumably not nomic, and accordingly does not support counterfactual reasoning, so Σ does not get promoted, in virtue of which it is an existential, not a universal.[10] [11]

9. Mereological Sums

We still don't have an account of the reading of

Jay is looking for a dog

according to which 'a dog' does not indicate a particular individual Jay is not looking for, but rather indicates the *kind* of thing Jay is looking for.

To account for this *generic* reading of 'a dog', we take advantage of the following type-identity.

$$\Sigma D = D$$

In other words, any sum of entities is itself an entity. These compound entities are known in the philosophical literature variously as *mereological-sums* and *pluralities*.[12] Although such entities are built into our semantic theory, they have not been utilized so far, mostly because we have concentrated on predicates that apply to ordinary (simple, singular) entities.

---

[10] This offers an exception to 'a person'/'some one' conversion. In particular, the philosophical claim

a person is a moral agent

is presumably nomic. But if 'a person' is replaced by 'someone', we obtain

someone is a moral agent

which does not seem to be nomic.

[11] An alternative hypothesis is to introduce yet another species of 'be' which is applied in nomic cases, but not simple-indicative cases, according to which 'be' corresponds to class-inclusion. See Section 9 on mereological-sums.

[12] Note carefully, however, that mereological-sums (also called *fusions*) and pluralities are logically and ontologically distinct, although they are mathematically very similar, which can be a source of confusion. Given a fixed domain of individual-entities, there is a derivative domain of plural-entities, which have a part-whole structure, conveyed by the word 'among'. But the latter is quite distinct from the part-whole relations that individuals "naturally" bear to one another, and these are quite distinct from the part-whole relations born between count-entities and mass-entities For example, the blood in my right index finger is naturally part of my right index finger, which is naturally part of my right hand, but the former entities are not among my right hand (supposing the latter is even grammatical!). For a general overview of mereology [the logic of parts and wholes], consult the Stanford Encyclopedia entry [http://plato.stanford.edu/entries/mereology/#4.2].

The current example provides an opportunity to invoke compound entities. In particular, we can posit that ‘look for’ denotes a relation between cognitive-agents and entities, the latter of which may be simple or complex. This means in turn that ‘looking for a dog’ is inherently ambiguous, as seen in the following two constructions.

Jay [+1]	is-looking-for	a dog [+2]
	$\lambda y_2 \lambda x_1 \mathbf{L}xy$	$\Sigma\{\, y_2 \mid \mathbf{D}y \,\}$
J_1	$\Sigma\{\, \lambda x_1 \mathbf{L}xy \mid \mathbf{D}y \,\}$	
$\Sigma\{\, \mathbf{L}\mathrm{J}y \mid \mathbf{D}y \,\}$		
$\exists y \,\{\, \mathbf{D}y \;\&\; \mathbf{L}\mathrm{J}y \,\}$		

Jay [+1]	is-looking-for	a dog [+2]
	$\lambda y_2 \lambda x_1 \mathbf{L}xy$	$\Sigma\{y_2 \mid \mathbf{D}y\}$
J_1	$\lambda x_1 \mathbf{L}[x, \Sigma\{y \mid \mathbf{D}y\}]$	
$\mathbf{L}[\mathrm{J}, \Sigma\{y \mid \mathbf{D}y\}]$		

In the first reading, Jay stands in relation **L** to a particular dog. In the second reading, Jay stands in relation **L** to a complex entity – namely, the mereological-sum of all dogs.[13]

To see that this is not as exotic as it might sound at first, consider what it means to be looking for a spatially-complex entity such as Australia; one stands in relation **L**, not to any particular part of Australia, but to the (mereological) whole. We propose that looking for a dog, or dogs, can be similarly "holistic".

In the above example, ‘looking for’ is given a purely extensional interpretation; the relation **L** stands between *actual* entities. Sometimes, however, ‘looking for’ is *intensional* in nature. For example, looking for a unicorn is different from looking for a dragon, although the extensions of ‘unicorn’ and ‘dragon’ are identical, both being empty. In such cases, what we seek is not so much an entity, simple or complex, but a more abstract item – a state of affairs. For example, seeking a unicorn can be understood as seeking *to-behold-a-unicorn*.[14] Then looking for a unicorn is seeking a state of affairs in which one beholds a unicorn. But notice that one is thereby seeking *a* state of affairs, but no particular state of affairs, which means one stands in the *seek* relation to a mereological-sum, not of unicorns, but of states of affairs. So either way, we end up with mereological-sums.

Mereological-sums are also useful is in explaining *generic readings* of common nouns, as in the following examples

1. children like dogs
2. fruit-flies like a banana[15]

If one thinks there are covert quantifiers lurking in the deep-structure representations, one must say which ones; unfortunately, no *particular* quantifiers seem to work. On the other hand, if one takes the pertinent phrases to be indefinite noun phrases, and one takes the latter to be (promotable to) entity-sums, then one can interpret (1) as asserting a relation between children-as-a-whole and dogs-as-a-whole, and (2) as asserting a relation between fruit-flies-as-a-whole and bananas-as-a-whole.[16]

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[13] We do need a further (but completely plausible) algebraic principle, that case-marking distributes over Σ.

[14] More generally, one seeks to stand in some tacitly understood relation to a unicorn – for example, *owning*. Also, if I am seeking a spouse, I may be seeking to be related to someone who is a spouse (of mine, or of someone), or I may be seeking to be spousally-related to someone.

[15] This comes from Groucho Marx, which is a follow-up to ‘time flies like an arrow’. A variant joke might be for Groucho to pull out a very crooked and bent arrow, and say, “… but not this one”.

[16] Similarly, one can interpret sentences like

a dog is a mammal

(continued)

children [+1]	like	dogs [+2]
	$\lambda y_2 \lambda x_1 \mathbf{L}xy$	$\Sigma\{y_2 \mid \mathbf{D}y\ \&\ \}$
$\Sigma\{x_1 \mid \mathbf{C}x\}$	$\lambda x_1 \mathbf{L}[x, \Sigma\{y \mid \mathbf{D}y\}]$	
$\mathbf{L}[\ \Sigma\{x \mid \mathbf{C}x\}, \Sigma\{y \mid \mathbf{D}y\}\ ]$		

fruit-flies [+1]	like	a banana [+2]
	$\lambda y_2 \lambda x_1 \mathbf{L}xy$	$\Sigma\{y_2 \mid \mathbf{B}y\}$
$\Sigma\{x_1 \mid \mathbf{F}x\}$	$\lambda x_1 \mathbf{L}[x, \Sigma\{y \mid \mathbf{B}y\}]$	
$\mathbf{L}[\ \Sigma\{x \mid \mathbf{F}x\}, \Sigma\{y \mid \mathbf{B}y\}\ ]$		

10. The Difference between 'a' and 'any'

There is a striking similarity between 'a' and 'any'. In particular, both can be promoted to a junction that ultimately gets simplified to $\forall$. Because of this, the following sentences convey the same information.

if **a** wild animal[17] comes into the house, we put it back outside
if **any** wild animal comes into the house, we put it back outside

On the other hand, 'a' and 'any' are not equivalent, as immediately seen in the following contrast-pair.

☺ Rex is a dog
☹ Rex is any dog

There are also more complicated examples.

if Jay doesn't respect **a woman**, he doesn't talk to **her**
if Jay doesn't respect **any woman**, he doesn't talk to **her**

Note, in particular, that the former, but not the latter, succeeds in binding 'her' to the NP.

The following indicates a further difference between 'a' and 'any'.

if **a man** doesn't respect **a woman**, **he** doesn't talk to **her**
☹ if **any man** doesn't respect **any woman**, he doesn't talk to her

The former is sensible (grammatically); the latter is downright bizarre.

11. Scope-Issues involving Indefinite Noun Phrases

So far, we have adjudicated quantifier-scope by appealing to admissibility-restrictions on the relevant junctions. For example, $\wedge$ and $\vee$ admit each other, so 'every' and 'some' can each out-scope the other. On the other hand, $\wedge\neg$ admits $\wedge$, so 'no' out-scopes 'every', but not vice versa.

How does this idea develop when dealing with indefinite noun phrases and Σ? For example, the following sentence

1. Jay doesn't own a dog

does not have a reading according to which 'a dog' is a wide-scope existential. The following are two admissible calculations, which produce different (but logically-equivalent) formulas.

as asserting a relation (specifically species-genus) between dogs-as-a-whole and mammals-as-a-whole.

17 For example, a moth!

<table>
<tr><td>Jay [+1]</td><td>doesn't</td><td>own</td><td>a dog [+2]</td></tr>
<tr><td></td><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{y_2 \mid \mathbf{D}y\}$</td></tr>
<tr><td></td><td>$\lambda \mathrm{P}_1 \lambda x_1 {\sim}\mathrm{P}x$</td><td colspan="2">$\Sigma\{ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td>J_1</td><td colspan="3">$\Pi\{ \lambda x_1 {\sim}\mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="4">$\Pi\{ {\sim}\mathbf{O}\mathrm{J}y \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="4">$\forall y \{ \mathbf{D}y \rightarrow {\sim}\mathbf{O}\mathrm{J}y \}$</td></tr>
</table>

<table>
<tr><td>Jay [+1]</td><td>doesn't</td><td>own</td><td>a dog [+2]</td></tr>
<tr><td></td><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{y_2 \mid \mathbf{D}y\}$</td></tr>
<tr><td></td><td></td><td colspan="2">$\Sigma\{ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td></td><td>$\lambda \mathrm{P}_1 \lambda x_1 {\sim}\mathrm{P}x$</td><td colspan="2">$\lambda x_1\, \exists y\{\mathbf{D}y \,\&\, \mathbf{O}xy\}$</td></tr>
<tr><td>J_1</td><td colspan="3">$\lambda x_1\, {\sim}\exists y\{\mathbf{D}y \,\&\, \mathbf{O}xy\}$</td></tr>
<tr><td colspan="4">${\sim}\exists y\{\mathbf{D}y \,\&\, \mathbf{O}\mathrm{J}y\}$</td></tr>
</table>

The following is similar.

2. every man owns a dog

<table>
<tr><td>every man [+1]</td><td>owns</td><td>a dog [+2]</td></tr>
<tr><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{y_2 \mid \mathbf{D}y\}$</td></tr>
<tr><td>$\bigwedge\{x_1 \mid \mathbf{M}x\}$</td><td colspan="2">$\Sigma\{ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="3">$\bigwedge\{ \Sigma\{ \mathbf{O}xy \mid \mathbf{D}y \} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="3">$\forall x \{ \mathbf{M}x \rightarrow \exists y\{\mathbf{D}y \,\&\, \mathbf{O}xy\} \}$</td></tr>
</table>

<table>
<tr><td>every man [+1]</td><td>owns</td><td>a dog [+2]</td></tr>
<tr><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{y_2 \mid \mathbf{D}y\}$</td></tr>
<tr><td></td><td colspan="2">$\Sigma\{ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td>$\bigwedge\{x_1 \mid \mathbf{M}x\}$</td><td colspan="2">$\lambda x_1\, \exists y\{\mathbf{D}y \,\&\, \mathbf{O}xy\}$</td></tr>
<tr><td colspan="3">$\bigwedge\{ \exists y\{\mathbf{D}y \,\&\, \mathbf{O}xy\} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="3">$\forall x \{ \mathbf{M}x \rightarrow \exists y\{\mathbf{D}y \,\&\, \mathbf{O}xy\} \}$</td></tr>
</table>

In contrast to this reading, the following reading is illegitimate.

<table>
<tr><td>every man [+1]</td><td>owns</td><td>a dog [+2]</td></tr>
<tr><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{y_2 \mid \mathbf{D}y\}$</td></tr>
<tr><td>$\bigwedge\{x_1 \mid \mathbf{M}x\}$</td><td colspan="2">$\Sigma\{ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="3">※ $\Sigma\{ \bigwedge\{ \mathbf{O}xy \mid \mathbf{M}x \} \mid \mathbf{D}y \}$ ※</td></tr>
<tr><td colspan="3">$\exists y \{ \mathbf{D}y \,\&\, \forall x\{\mathbf{M}x \rightarrow \mathbf{O}xy\} \}$</td></tr>
</table>

To block this reading, we postulate that Σ does not admit any junction except Σ, so it does not admit ⋀.[18]

But then, how do we explain the following Rodney Dangerfield joke?

> I have **a** suit for **every** occasion… unfortunately, this is it.

Or the following variation?

> I have **a** friend in **every** city… he is very big.

If Σ does not admit ⋀, how does 'a' manage to out-scope 'every', as it does in these examples?

[18] This is not to say that it does not admit expressions that involve junctions; so Σ admits ⟦every⟧, but not ⟦every man⟧.

Central to solving this problem is the idea that an indefinite noun phrase starts life as a common-noun phrase, which sometimes metamorphoses into an entity-sum. Moreover, between inception and metamorphosis, it can undergo other transformations – in particular, it can be adjectively-modified, as illustrated in the following example.

3. Rodney has a suit for every occasion

Rodney [+1]	has	a suit	[mod]	for	every occasion [+2]	[+2]
				$\lambda z_2 \lambda y_0 \mathbf{F} yz$	$\wedge\{z_2 \mid \mathbf{O}z\}$	
				$\wedge\{ \lambda y_0 \mathbf{F} yz \mid \mathbf{O}z \}$		
			$\lambda Q_0 \lambda P_0 \lambda x_0\{Px \& Qx\}$	$\lambda y_0\ \forall z\{\mathbf{O}z \rightarrow \mathbf{F}yz\}$		
		$\mathbf{S}_0$	$\lambda P_0\ \lambda y_0\ \{ Py \ \&\ \forall z\{\mathbf{O}z \rightarrow \mathbf{F}yz\} \}$			
		$\lambda y_0\ \{ \mathbf{S}y \ \&\ \forall z\{\mathbf{O}z \rightarrow \mathbf{F}yz\} \}$				
		$\Sigma\{ y \mid \mathbf{S}y \ \&\ \forall z\{\mathbf{O}z \rightarrow \mathbf{F}yz\} \}$				$\lambda x\{x_2\}$
	$\lambda y_2 \lambda x_1 \mathbf{H}xy$	$\Sigma\{ y_2 \mid \mathbf{S}y \ \&\ \forall z\{\mathbf{O}z \rightarrow \mathbf{F}yz\} \}$				
	$\lambda x_1\ \Sigma\{ \mathbf{H}xy \mid \mathbf{S}y \ \&\ \forall z\{\mathbf{O}z \rightarrow \mathbf{F}yz\} \}$					
R_1	$\lambda x_1\ \exists y\ \{ \mathbf{S}y \ \&\ \forall z(\mathbf{O}z \rightarrow \mathbf{F}yz) \ \&\ \mathbf{H}xy \}$					
$\exists y\ \{ \mathbf{S}y \ \&\ \forall z(\mathbf{O}z \rightarrow \mathbf{F}yz) \ \&\ \mathbf{H}Ry \}$						

As usual, we treat 'a' as redundant, being semantically absorbed by 'suit'.

Finally, compare the above tree with the following, according to which 'every' gains wide scope.

4. Rodney has a suit for every occasion [with 'every occasion' having wide scope]

Rodney [+1]	has	a suit	[mod]	for	every occasion [+2]	[+2]
				$\lambda z_2 \lambda y_0 \mathbf{F} yz$	$\wedge\{z_2 \mid \mathbf{O}z\}$	
			$\lambda Q_0 \lambda P_0 \lambda x_0\{Px \& Qx\}$	$\wedge\{ \lambda y_0 \mathbf{F} yz \mid \mathbf{O}z \}$		
		$\mathbf{S}_0$	$\wedge\{ \lambda P_0 \lambda y_0\{Py \ \&\ \mathbf{F}yz\} \mid \mathbf{O}z \}$			
		$\wedge\{ \lambda y_0\{\mathbf{S}y \ \&\ \mathbf{F}yz\} \mid \mathbf{O}z \}$				
		$\wedge\{ \lambda y_0\{\mathbf{S}y \ \&\ \mathbf{F}yz\} \mid \mathbf{O}z \}$				
		$\wedge\{ \Sigma\{y \mid \mathbf{S}y \ \&\ \mathbf{F}yz\} \mid \mathbf{O}z \}$				$\lambda x\{x_2\}$
	$\lambda y_2 \lambda x_1 \mathbf{H}xy$	$\wedge\{ \Sigma\{y_2 \mid \mathbf{S}y \ \&\ \mathbf{F}yz\} \mid \mathbf{O}z \}$				
R_1	$\wedge\{ \Sigma\{\lambda x_1 \mathbf{H}xy \mid \mathbf{S}y \ \&\ \mathbf{F}yz\} \mid \mathbf{O}z \}$					
$\wedge\{ \Sigma\{\mathbf{H}Ry \mid \mathbf{S}y \ \&\ \mathbf{F}yz\} \mid \mathbf{O}z \}$						
$\forall z\ \{ \mathbf{O}z \rightarrow \exists y\ \{ \mathbf{S}y \ \&\ \mathbf{F}yz \ \&\ \mathbf{H}Ry \} \}$						