

A3

Basic Categorical Grammar

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1. Introduction

In this chapter, we present the salient features of basic categorical grammar, which is employed throughout this book to do syntactic analysis.

2. Basic Ideas – Names, Sentences, Functors

Categorical grammar classifies grammatical expressions according to the following scheme. First, the grammatical categories consist of primitive and derivative categories. In basic categorical grammar, there are two primitive categories.

N	name
S	sentence

In addition to these two primitive categories, there are also derivative categories, which classify the huge variety of *functors* (officially defined below).

In basic categorical grammar, the *categories* are inductively defined as follows.

N is a category;
 S is a category;
 if K_0, K_1, \dots, K_m are categories, where $m \geq 1$,
 then the following is also a category: $((K_1 \times \dots \times K_m) \rightarrow K_0)$;
 nothing else is a category.

Note that parentheses are often dropped, the rules of omission paralleling those for sentential connectives.

Associated with derivative categories are the class of grammatical expressions called *functors*, which are defined as follows.

A ***functor*** is a grammatical expression with blanks (gaps, places, slots), which yields a grammatical expression of a particular category, when its blanks are filled with grammatical expressions of the appropriate categories.

The basic idea is this.

Suppose functor ϕ has category $(K_1 \times \dots \times K_m) \rightarrow K_0$. Then ϕ has m -many blanks, which require fill-in expressions of category K_1, \dots, K_m , respectively, and when so filled, ϕ yields an expression of category K_0 .

3. Monadic, Polyadic, Anadic Functors

Where k is any natural number (0, 1, 2, ...), a *k-place functor* is a functor with k blanks (places).

Alternative names:

monadic	:	1-place
dyadic	:	2-place
triadic	:	3-place
polyadic	:	2-place or 3-place or ...

Some functors are not specifically 1-place, 2-place, or any particular place. These are called *anadic* functors. The prefix ‘an’ means ‘without’, so an anadic functor is a functor without “adic”. It is arguable that conjunction and disjunction (especially exclusive disjunction) are examples of anadic connectives. The following is our official definition.

Df

An *anadic* functor is a syntactic expression with an open-ended blank that, when filled with any number (≥ 0) of expressions, all of a single particular category, results in an expression of a particular category.

A simple example of an anadic functor is anadic conjunction, which takes any number of sentences and yields a sentence. This is categorially depicted as follows.

$$S^* \rightarrow S$$

Here, the symbol ‘*’ indicates that the functor takes any number of sentences and produces a sentence.

4. Simple Functors

A *homogeneous* functor is a functor whose input must all be of the same category.

Homogeneous functors allow for a simplification of notation as follows.

$$\begin{aligned} S^1 \rightarrow S &=_{df} S \rightarrow S \\ S^2 \rightarrow S &=_{df} (S \times S) \rightarrow S \\ S^3 \rightarrow S &=_{df} (S \times S \times S) \rightarrow S \\ \text{etc.} & \end{aligned}$$

For example, a functor of category $[S^2 \rightarrow S]$ takes two sentences and produces a sentence.

An *primary* functor is a functor whose input *and* output are all one of the primitive categories – N and S.

A *simple* functor is a homogeneous primary functor.

Simple functors come in exactly four general varieties.

function-signs (N-operators)	$N^* \rightarrow N; N^k \rightarrow N$	takes one or more nouns, and yields a noun
connectives (S-operators)	$S^* \rightarrow S; S^k \rightarrow S$	takes one or more sentences, and yields a sentence
predicates	$N^* \rightarrow S; N^k \rightarrow S$	takes one or more nouns, and yields a sentence
subnectives	$S^* \rightarrow N; S^k \rightarrow N$	takes one or more sentences, and yields a noun

5. Examples of Complex Functors

Category	Grammatical Function
$N \rightarrow (N \rightarrow S)$	takes an expression of category N, and yields an expression of category $N \rightarrow S$; in other words, it takes a noun phrase and yields a 1-place predicate.
$(N \rightarrow S) \rightarrow (N \rightarrow S)$	takes an expression of category $N \rightarrow S$, and yields an expression of category $N \rightarrow S$; in other words, it takes a predicate and yields a predicate; these are sometimes called predicate adverbs.
$N \rightarrow [(N \rightarrow S) \rightarrow (N \rightarrow S)]$	takes something of category N, and yields an expression of category $(N \rightarrow S) \rightarrow (N \rightarrow S)$; in other words, it takes a noun phrase and yields a predicate adverb.
$[(N \rightarrow S) \rightarrow (N \rightarrow S)] \rightarrow [(N \rightarrow S) \rightarrow (N \rightarrow S)]$	takes an expression of category $(N \rightarrow S) \rightarrow (N \rightarrow S)$, and yields an expression of category $(N \rightarrow S) \rightarrow (N \rightarrow S)$; in other words, it takes a predicate adverb and yields a predicate adverb.

6. The Simplest Functors – Sentential Logic

It is customary for elementary symbolic logic to begin with Sentential Logic (SL) – also called *Propositional Logic*.

What is grammatically peculiar about SL is that it considers only one primitive grammatical type – namely, sentences – and it considers only one *group* of derivative types – namely *sentential-operators* (S-operators), also called *connectives*. A sentential-operator (connective) is a functor that takes one or more sentences as input and delivers a sentence as output. In beginning logic, five sentential-operators are examined.

negation	it is not true that __
conjunction	__ and __
disjunction	__ or __
conditional	if __ then __
bi-conditional	__ if and only if __

In more advanced courses, many other sentential-operators are examined, including the following.

necessity	it is necessary that __
possibility	it is possible that __
obligation	it is obligatory that __
permission	it is permissible that __
subjunctive	if it were the case that __ , then it would be the case that __

Although the various phrases mentioned above may not seem to form a "natural kind" from the standpoint of conventional grammar, they do according to categorical grammar. In particular, each one takes a sentence (or two) as input and yields a sentence as output.

Note that each example above comes with one or more blanks, which correspond to places (slots) that must be filled in order to "complete" the functor in question. For example, negation is a one-place functor, whereas conjunction is a two-place functor. In principle, for each natural number k , there are k -place functors, and in particular, k -place sentential-operators. We can depict this as follows.

$S^1 \rightarrow S$	1-place S-operator (connective)
$S^2 \rightarrow S$	2-place S-operator (connective)
$S^3 \rightarrow S$	3-place S-operator (connective)
etc.	

In general, by categorizing a functor ϕ as $S^k \rightarrow S$, we say that ϕ takes k -many sentences (S^k) as input, and generates (\rightarrow) a sentence (S) as output.

7. Syntactic Trees in SL

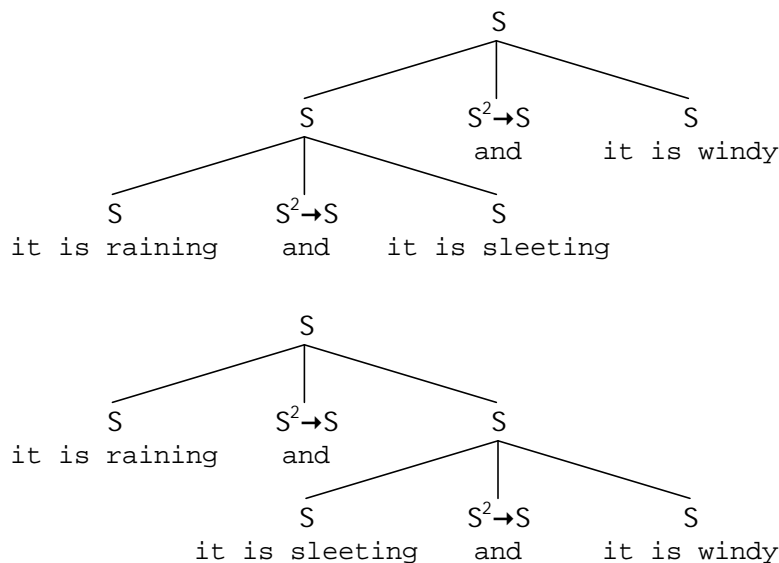
It is commonplace for syntactic analyses to be diagrammed by phrase-structure trees, which display the part-whole relation among the various sub-phrases of any given phrase. In the case of SL, the phrase structures are fairly simple. For example, consider the following sentence.

it is raining, and it is sleeting, and it is windy

Officially in SL, this sentence is structurally ambiguous, insofar as it allows the following two symbolizations.

$(R \ \& \ S) \ \& \ W$
 $R \ \& \ (S \ \& \ W)$

Fortunately, this is a harmless ambiguity, since the two formulas are logically equivalent. Nevertheless, there are two syntactic analyses available, which are diagrammed as follows.



Later (Section **Error! Reference source not found.**) we consider an alternative analysis according to which 'and' is not a dyadic (two-place) functor, but an *anadic* (any-place) functor.

8. Other Simple Functors – Predicate Logic

A logic, such as SL, that treats most ordinary English sentences as lexical (atomic), is not even minimally adequate for logical analysis, let alone linguistic analysis. Accordingly, logic and linguistics consider a wider class of primitive types and functor types. In elementary logic at least, the next step is *Predicate Logic* (PL), which expands the syntax of elementary logic by adding the following syntactic items.¹

- (1) (singular²) proper-noun phrases
- (2) predicates

¹ Predicate logic also adds quantification, but we ignore that in this chapter.

² Singular NPs include 'Jay', 'Kay', 'the dog', 'my cat', whereas plural NPs include 'Jay and Kay', 'the dogs', 'my cats'. Elementary logic studiously avoids plural NPs.

As with sentential-operators, predicates can be displayed categorially.

$N^1 \rightarrow S$	1-place predicate
$N^2 \rightarrow S$	2-place predicate
$N^3 \rightarrow S$	3-place predicate
etc.	

As mentioned before, N is the type of proper-noun phrases, and S is the type of sentences. The notation is read straightforwardly, as before. For example, a 2-place predicate [$N^2 \rightarrow S$] takes 2 proper-noun phrases (N^2) as input, and generates (\rightarrow) a sentence (S) as output.

In the next few subsections, we consider various examples of predicates from English, starting with 1-place predicates.

1. 1-Place Predicates

English abounds with expressions that can be plausibly analyzed as 1-place predicates. For example, the paradigm formation-procedure is to take a singular common noun, for example,

dog cat mouse fish

and prefix the phrase 'is a', thus yielding the following 1-place predicates.

_ is a dog _ is a cat _ is a mouse _ is a fish

An alternative formation-procedure takes an adjective, for example,

friendly smart happy beautiful

and prefixes the *copula* 'is' to form the following 1-place predicates.

_ is friendly _ is smart _ is happy _ is beautiful

Notice each of the above expressions has a single blank, which can be filled by a proper-noun phrase to construct a sentence. The following are examples of sentences so constructed.

Lady Penny	is a dog
my dog	is friendly
that	is a cat
this cat	is beautiful

2. 2-Place Predicates

Examples of 2-place predicates in English are easy to conjure up. The simplest examples are transitive verbs, such as

respects, admires, loves, enjoys

which produce the following corresponding 2-place predicates.

__ respects __
 __ admires __
 __ loves __
 __ enjoys __

Another class of examples are comparatives, including

taller, longer, wider, heavier, etc.

In particular, combining a comparative with the *copula* 'is' and the *conjunction*³ 'than', produces a 2-place predicate. For example:

__ is taller than __
 __ is longer than __
 __ is wider than __
 __ is heavier than __
 etc.

3. 3-Place Predicates

3-place predicates are less common in English than 1-place predicates and 2-place predicates, but they are still abundant. Probably the simplest examples are di-transitive verbs. One class of such verbs are those that take both a direct object (DO) and an indirect object (IO). The following are examples.

give, take, buy, sell, borrow, lend, recommend, explain

as in

Jay gave Fluffy to Kay (for her birthday)

Another, considerably less common, class of di-transitive verbs are the ones that take symmetrical direct objects – for example,

marries

as in

Fay married Jay and Kay⁴

Similar to di-transitive verbs are di-transitive prepositions, of which there may only be one example – namely,

³ So called because it combines two clauses, although the second one is usually abbreviated, as in

Jay is heavier than Kay (is), but Kay is taller than Jay (is).

⁴ These are, of course, structurally ambiguous, given the categorial ambiguity of 'and'. Here, we have in mind the "mereological" (plural) reading, rather than the "logical" (singular) reading, of 'and'. This suggests that a verb with symmetrical direct-objects is probably better understood as taking a plural NP as a single direct object.

between
as in
New York is between Boston and Philadelphia

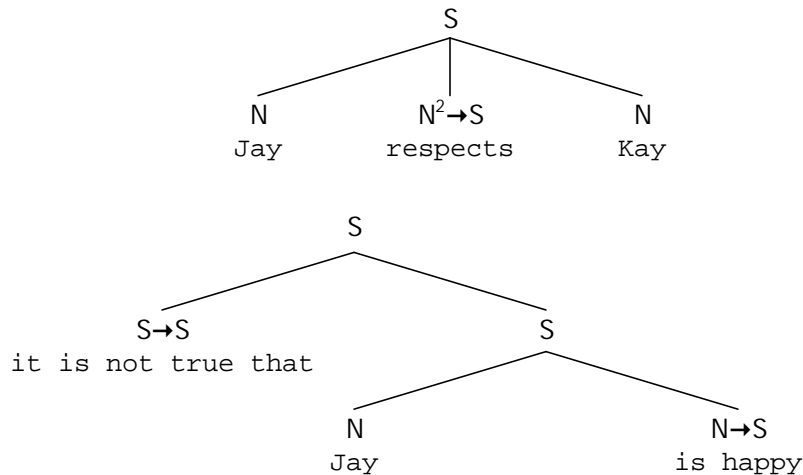
4. 4-Place Predicates

Natural examples of 4-place predicates are extremely hard to find in English, although there are plenty of artificially manufactured ones, such as the following.⁵

both ___ and ___ respect both ___ and ___
___ likes ___ more than ___ likes ___

5. Examples of Trees in Predicate Logic

By way of illustration, we offer the following tree diagrams.



9. Function-Signs

In elementary logic, after Predicate Logic comes Function Logic (FL), which expands the syntax of elementary logic by adding a class of functors called *function-signs* (also called *N-operators*)⁶. Whereas an S-operator (sentential operator) takes one or more sentences as input, and delivers a sentence as output, an N-operator (function-sign) takes one or more proper-noun phrases as input, and delivers a proper-noun phrase as output. Function-signs are easy to depict categorially.

$N^1 \rightarrow N$	1-place function-sign
$N^2 \rightarrow N$	2-place function-sign
$N^3 \rightarrow N$	3-place function-sign
etc.	

As usual, they are straightforwardly read; for example, a 2-place function-sign takes 2 proper-noun phrases (N^2) as input and generates (\rightarrow) a proper-noun phrase (N) as output.

⁵ The artificial-natural distinction is largely intuitive, although it can be fleshed out by reference to logical entailment.

⁶ Sometimes the word ‘operator’ is used unmodified to mean ‘N-operator’, sometimes to mean ‘S-operator’, and sometimes to mean ‘functor’. It is a much-used term.

Function-signs are widespread in the language of mathematics, in which many domains (e.g., natural numbers, real numbers, vector spaces) are analyzed using familiar looking function-signs, including the following.

+	__ plus __
×	__ times __
–	__ minus __
÷	__ divided by __
$\sqrt{\quad}$	the square root of __
\quad^2	__ squared

Notice that the first four expressions are 2-place operators, whereas the last two are one-place operators. The following are examples of compound proper-noun phrases formed using these operators.

2 + 3	two plus three
3 × 4	three times four
4 – 5	four minus five
5 ÷ 6	five divided by six
$\sqrt{6}$	the square root of six
2^2	two squared

Next, although it may not seem very natural to linguists, there are many quite ordinary expressions in English that may be fruitfully analyzed as function-signs, including the following.

the mother of __	__'s mother
the father of __	__'s father
the best friend of __	__'s best friend
the favorite song of __	__'s favorite song

In particular, each one of these may be understood as a 1-place N-operator (function-sign); each one takes a proper-noun phrase as input, and delivers a proper-noun phrase as output. For example, the following are all NP's obtained by substituting a proper-noun phrase into one of the blanks in the above examples.

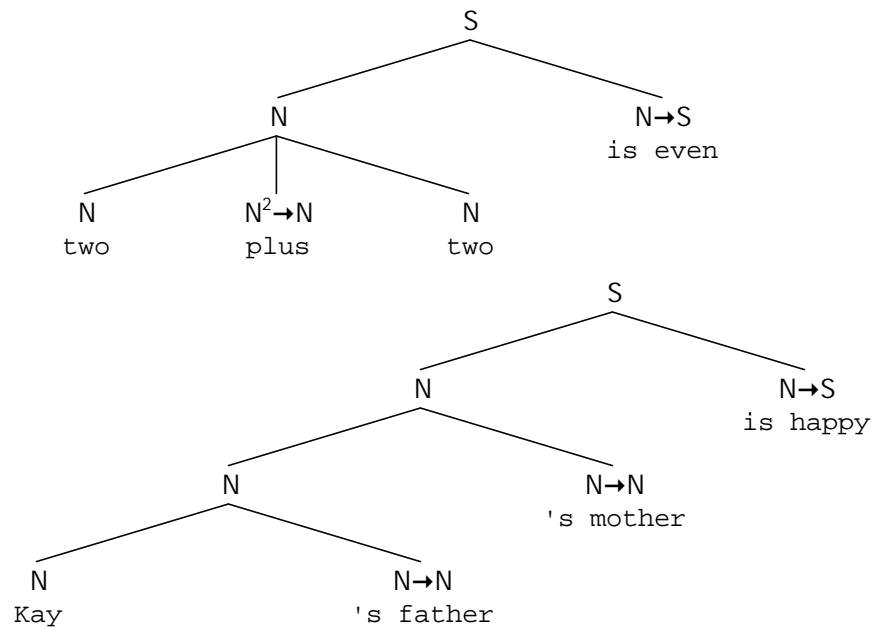
the mother of Jay	Kay's mother
the father of Kay	my father
the best friend of Fay	your best friend
the favorite song of Ray	the President's favorite song

Notice that N-operators (function-signs) are *recursive*; by this we mean that they produce output that can in turn be fed back in as further input.⁷ Thus, the following are all perfectly well-formed.

my mother
 my mother's mother
 my mother's mother's mother
 my mother's mother's mother's mother
 etc.

The following are simple examples of trees in Function Logic.

⁷ In general, syntactic construction-rules are *collectively* recursive – each rule produces output that can be input for other rules. On the other hand, N-operators and S-operators are *individually* recursive.



10. Subnectives

We have now examined three functor groups, given as follows.

$S^* \rightarrow S$	connectives (S-operators)
$N^* \rightarrow S$	predicates
$N^* \rightarrow N$	function-signs (N-operators)

Here, the asterisk is the wild-card symbol borrowed from computer science. From an abstract point of view, at least, this list is incomplete. By way of correcting this mathematical inelegance, we now add the following group of functors.

$S^* \rightarrow N$	subnectives
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As the type-group name suggests, subnectives takes sentences as input and generate proper-noun phrases as output. As with the other functors, a subnective can be monadic, dyadic, etc.

Are there any such functors in natural languages, and in particular in English? Or, are we just being mathematically obsessive? The chief candidate is so prevalent, and yet so stealthy, that we hardly notice it. I have in mind the expression⁸

that

as used in the following excerpt from a rather famous document written by Thomas Jefferson (*my* punctuation).

⁸ This is hardly to suggest that every use of 'that' is a subnective-use. Consider the following passage.

that mouse used to live in the house **that** Jack built;
 she hated it so much **that** she left;
 at least **that** is what I've been told.

I believe this passage uses 'that' in *four* different ways, none subnective.

We hold these truths to be self-evident -
that all men are created equal;
that they are endowed by their creator with certain unalienable⁹
rights;
that among these are life, liberty, and the pursuit of happiness;
...

I understand this passage as presenting a list of self-evident truths, which is to say a list of propositions that are self-evidently true. Lists generally consist of nouns and noun-like phrases, which include gerunds and infinitives (more about this later). For example, a list of invitees might go as follows.

Thomas Jefferson
John Adams
Benjamin Franklin

Similarly, a to-do list might go as follows.

have dinner with John Adams
write a letter to wife (back in Virginia)
write the declaration of independence

That-phrases and infinitive-phrases are examples of *nominalized sentences*. Basically,¹⁰ a nominalized sentence is a proper-noun phrase whose immediate constituent is a sentence.¹¹ The standard method of doing this in logic is by prefixing the word 'that'. Accordingly, categorially-speaking, the prefix 'that' takes a sentence (S) as input and delivers a proper-noun phrase (N) as output.

There are many examples of that-phrases in object position, as illustrated in the following examples.

Rumsfeld believes that war with Iraq is inevitable
Powell fears that war with Iraq is inevitable
Bush hopes that war with Iraq is inevitable

In each case, we have an SVO (subject-verb-object) sentence in which the direct object is a nominalized sentence (the underlined material). The following is a partial tree analysis.

⁹ I think that most modern speakers of English would say 'inalienable'. In fact, we often "quote" the famous document this way.

¹⁰ There are some syntactic indicators of noun-hood that nominalized sentences fail to satisfy, but we ignore these problems at the moment.

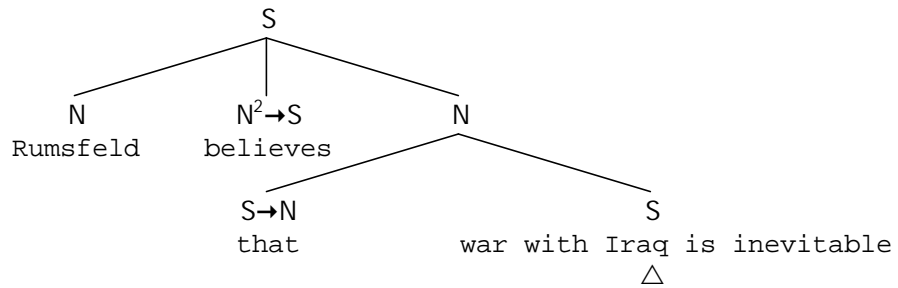
¹¹ Infinitive phrases have subjects, which are explicit or implicit. For example, in

Jay wants Kay to be president

the infinitive phrase 'Kay to be president' has 'Kay' as its explicit subject, but in

Jay wants to be president

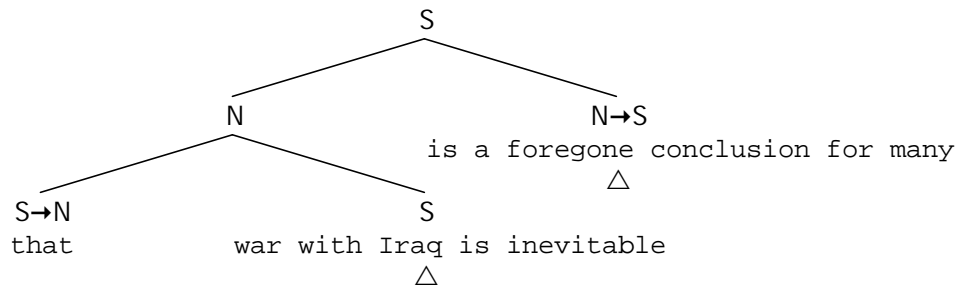
the infinitive phrase 'to be president' has 'Jay' as its implicit subject.



Note, as before, that the triangle (Δ) indicates that the phrase has further structure which is pertinent, but which we choose at this moment to ignore.

There are not many examples of that-phrases in subject position, at least in ordinary SVO form, although they are grammatically permitted, as in the following example.

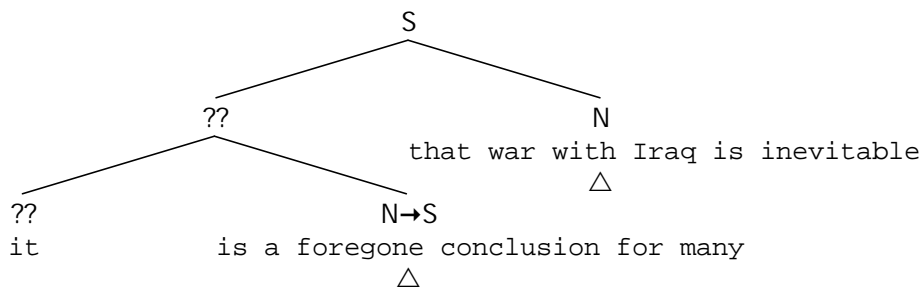
that war with Iraq is inevitable is a foregone conclusion for many



It is more common, however, to *extrapose* the subject when it is a nominalized sentence to produce examples such as the following.

it is a foregone conclusion for many that war with Iraq is inevitable
it is generally believed that war with Iraq is inevitable
it has always been thought by the administration that war with Iraq is inevitable

Note the appearance of 'it' in the *usual* subject position. The following is a plausible phrase structure analysis, which is not a species of elementary logic, even with subnective-that added.



The grammatical oddity of extraposition is remarked by Lewis Carroll in the following brilliant passage in *Alice in Wonderland*.¹²

¹² Note carefully the presence of extra quote marks, which I bold-face. This is because the mouse is reading from English history. Accordingly, Carroll is quoting the mouse, who is quoting the historian.

(The group is soaking wet, so they need to dry off, so the Mouse proposes reciting the driest topic he can think of – namely, English history!)

“I proceed. “Edwin and Morcar, the earls of Mercia and Northumbria, declared for him: and even Stigand, the patriotic archbishop of Canterbury, found it advisable – ” ”

“Found *WHAT?*” said the Duck.

“Found **IT**,” the Mouse replied rather crossly: “of course you know what *“it”* means.”

“I know what *“it”* means well enough, when I find a thing,” said the Duck: “it’s generally a frog or a worm. The question is, what did the archbishop find?”

The Mouse did not notice this question, but hurriedly went on, “--found it advisable to go with Edgar Atheling to meet William and offer him the crown. William’s conduct at first was moderate. But the insolence of his Normans -- ”

So the question remains to this day – what did the archbishop find? The answer, I think, is quite complicated, but it goes something like this.

the archbishop found **that** something was advisable, namely – **that** he go with Edgar Atheling to meet William and offer him the crown.¹³

Before continuing, we note that the it-that construction is ubiquitous in logic, and even appears immediately in intro logic in the form of the “standard” form of negation – namely:

it is not true **that** ____

¹³ And thus, in 1066, began the Norman ascendancy in England, which among other things ultimately lead to a near doubling of the vocabulary of English. After that, English had both a Germanic (old English) word, and a Romanic (old French) word, for nearly everything.