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Official Syntax For Alethic First-Order Modal Logic with Actuality

1.	Introduction.....	2
2.	Vocabulary	2
	1. Predicate letters.....	2
	2. Function letters	2
	3. Proper Nouns a-s, a ₁ -s ₁ , etc.	2
	4. Individual Variables t-z, t ₁ -z ₁ , etc.	2
	5. Individual Constants * a-s, a ₁ -s ₁ , etc.	2
	6. SL Connectives &, ∨, →, ↔, ~	2
	7. Modal Operators □, ◇, ◻, ◊, ≺, =	2
	8. Quantifiers ∀, ∃, ∀', ∃'	2
	9. Logical Predicates =, E!, A!	2
	10. Description operator ι	2
	11. Scoped term operator (./).....	2
	12. Actuality operators ○, ⊙.....	2
	13. Punctuation symbols (,), [,], etc.....	2
3.	Rules of Formation	3
	1. Singular Terms	3
	2. Atomic Formulas	3
	3. Formulas.....	3
4.	Notational Conventions	4
	1. Subscripts and Superscripts.....	4
	2. Square Brackets and Round Parentheses	4
	3. Parentheses Associated with ○ and ⊙	4
	4. Identity and Negated Identity	4
	5. Repeated Quantifiers	4

1. Introduction

In this appendix, we give the official syntax for first-order modal logic with actuality. We do not consider non-alethic modalities.

2. Vocabulary

1. Predicate letters

0.	0-place:	$A-Z, A_1-Z_1, A_2-Z_2, \text{ etc.}$
1.	1-place:	${}^1A-{}^1Z, {}^1A_1-{}^1Z_1, {}^1A_2-{}^1Z_2, \text{ etc.}$
2.	2-place:	${}^2A-{}^2Z, {}^2A_1-{}^2Z_1, {}^2A_2-{}^2Z_2, \text{ etc.}$
	etc.	

2. Function letters

0.	0-place:	$a-z, a_1-z_1, a_2-z_2, \text{ etc.}$
1.	1-place:	${}^1a-{}^1z, {}^1a_1-{}^1z_1, {}^1a_2-{}^1z_2, \text{ etc.}$
2.	2-place:	${}^2a-{}^2z, {}^2a_1-{}^2z_1, {}^2a_2-{}^2z_2, \text{ etc.}$
	etc.	

3. Proper Nouns $a-s, a_1-s_1, \text{ etc.}$

4. Individual Variables $t-z, t_1-z_1, \text{ etc.}$

5. Individual Constants * $a-s, a_1-s_1, \text{ etc.}$

6. SL Connectives $\&, \vee, \rightarrow, \leftrightarrow, \sim$

7. Modal Operators $\Box, \Diamond, \Box, \Diamond, \prec, =$

8. Quantifiers $\forall, \exists, \forall', \exists'$

9. Logical Predicates $=, E!, A!$

10. Description operator ι

11. Scoped term operator $(./)$

12. Actuality operators \bigcirc, \odot

13. Punctuation symbols $(,), [,], \text{ etc.}$

[*Note: we do not notationally distinguish proper nouns and constants in the formation rules. They are, however, critically different from the viewpoint of the rules of derivation. Constants are regarded as purely intra-derivational terms used in conjunction with UD and $\exists O$; they are simply variables minus their variable-binding operators. Their primary purpose is to make derivations easier to track visually.]

3. Rules of Formation

1. Singular Terms

1. Every proper noun is a singular term;
2. Every individual variable/constant is a singular term
3. If ϕ is an n -place function sign, and τ_1, \dots, τ_n are singular terms, then $\phi(\tau_1, \dots, \tau_n)$ is a singular term;
4. If τ is a singular term, then so are: $\bigcirc(\tau)$, $\odot(\tau)$;
4. If F is a formula, and v is a variable, then $\iota v F$ is a singular term;
5. Nothing else is a singular term.

2. Atomic Formulas

1. If \mathbb{P} is an n -place predicate, and τ_1, \dots, τ_n are singular terms, then $\mathbb{P}[\tau_1, \dots, \tau_n]$ is an atomic formula;
2. If τ is a singular term, then $A![\tau]$ and $E![\tau]$ are atomic formulas;
3. If τ_1 and τ_2 are singular terms, then $[\tau_1 = \tau_2]$ is an atomic formula;
4. Nothing else is an atomic formula.

3. Formulas

1. Every atomic formula is a formula.
2. If F is a formula, then so are:
 - (a) $\sim F$
 - (b) $\Box F$
 - (c) $\Diamond F$
 - (d) $\Box F$
 - (e) $\Diamond F$
 - (f) $\bigcirc F$
 - (g) $\odot F$.
3. If F_1 and F_2 are formulas, then so are:
 - (a) $(F_1 \ \& \ F_2)$
 - (b) $(F_1 \ \vee \ F_2)$
 - (c) $(F_1 \ \rightarrow \ F_2)$
 - (d) $(F_1 \ \leftrightarrow \ F_2)$
 - (e) $(F_1 \ \prec \ F_2)$
 - (f) $(F_1 = F_2)$
4. If F is a formula and v is a variable, then the following are formulas:
 - (a) $\forall v F$
 - (b) $\exists v F$
 - (a) $\forall' v F$
 - (b) $\exists' v F$
5. If F is a formula, τ is a singular term, and v is a variable, then $(\tau/v)F$ a formula.
6. Nothing else is a formula.

4. Notational Conventions

1. Subscripts and Superscripts

Except under special circumstances, we drop the superscripts and subscripts on predicate letters and function letters.

2. Square Brackets and Round Parentheses

When a predicate applies *exclusively* to simple terms, we are allowed to drop the brackets. We *never* drop the parentheses associated with non-logical function signs (see next item, however).

Examples:

$P[j]$	becomes	Pj	Jay is a professor
$R[j,k]$	becomes	Rjk	Jay respects Kay

whereas the following retain their brackets.

$P[m(j)]$	Jay's mother is a professor
$R[m(j),k]$	Jay's mother respects Kay

3. Parentheses Associated with \circ and \odot

The parentheses associated with the special logical signs \circ and \odot can be dropped.

4. Identity and Negated Identity

As with other predicates, the brackets may be dropped; we *usually* write

$x=y$ in place of the official $[x=y]$.

and we write

$x \neq y$ in place of the official $\sim[x=y]$.

Example:

$\exists x \exists y \{x \neq y \ \& \ f(x)=f(y)\}$

5. Repeated Quantifiers

Any repetition of a quantifier functor can be simplified, in accordance with the following definitions.

$\forall v_1 v_2 \mathbb{F} =_{df} \forall v_1 \forall v_2 \mathbb{F}$; $\forall v_1 v_2 v_3 \mathbb{F} =_{df} \forall v_1 \forall v_2 \forall v_3 \mathbb{F}$; etc.
 $\exists v_1 v_2 \mathbb{F} =_{df} \exists v_1 \exists v_2 \mathbb{F}$; $\exists v_1 v_2 v_3 \mathbb{F} =_{df} \exists v_1 \exists v_2 \exists v_3 \mathbb{F}$; etc.

Examples:

$\forall xy Rxy =_{df} \forall x \forall y Rxy$
 $\exists xyz Bxyz =_{df} \exists x \exists y \exists z Bxyz$