

Notes on Knowledge

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1. Introduction

The word ‘science’ derives from the Latin word ‘sciens’, which is the present participle of ‘scire’ [*to know*]. In this connection, also note that other words in English contain ‘science’ as a meaningful constituent, including:

conscience	moral/ethical awareness
prescience	knowledge of actions or events <i>before</i> they occur
omniscience	having <i>total</i> knowledge or knowledge of everything

although no one of them is pronounced in a way suggestive of ‘science’. Interestingly, whereas the latter two have simple adjective forms – ‘prescient’ and ‘omniscient’ – the first one does not; ‘conscient’ is not a word of English, although ‘conscious’ and ‘conscientious’ are.

Given that knowledge is central to the meaning of the word ‘science’, it seems worthwhile in discussing science to begin with a brief discussion of knowledge. Not only does this get us started in our discussion, it also provides an example of scientific/philosophic method.

2. Epistemology

Philosophy, which is traditionally thought of as nourishing and supporting all the liberal arts and sciences, begins by asking some seemingly simple questions, including the following, just as a sample.

what is reality?
 what is knowledge?
 what is good?
 what is beauty?
 what is reason?

These simple questions are respectively associated with five major branches of philosophy:

metaphysics
 epistemology
 ethics
 aesthetics
 logic

The question ‘what is knowledge?’ is the basic question in epistemology, which is also called ‘theory of knowledge’.

3. The Verb ‘Know’

The goal of epistemology is to discover a systematic [scientific] account of knowledge. Since the beginning of the 20th Century, philosophy has been interested in language, since language is thought to be a "window to the mind". Also, an analysis of concepts must invariably rely on language, since concepts are conveyed by language. In keeping with that idea, no account of knowledge can get started without at least a short account of the word ‘knowledge’.

The word ‘knowledge’, which is a noun, derives from the verb ‘to know’, which is used in a variety of sentence types, illustrated as follows.

- | | | | |
|------|-----|-------|--|
| (1) | Jay | knows | Kay |
| (2) | Kay | knows | the <i>Declaration of Independence</i> |
| (3) | Jay | knows | of Kay |
| (4) | Kay | knows | how to swim |
| | | | where to swim |
| | | | when to swim |
| (5) | Jay | knows | to call for help when he is in trouble |
| (6) | Jay | knows | who wrote the <i>Declaration of Independence</i> |
| (7) | Kay | knows | where the <i>Declaration of Independence</i> was signed |
| (8) | Jay | knows | when we celebrate the signing of the <i>Declaration of Independence</i> |
| (9) | Kay | knows | why we celebrate the signing of the <i>Declaration of Independence</i> |
| (10) | Kay | knows | what an arthropod is |
| (11) | Jay | knows | whether he will be here tomorrow |
| (12) | Jay | knows | which shoe to put on which foot |
| (13) | Jay | knows | that his shoes are untied |

Science begins with **data collecting**, and the above list is a simple example of that. After data collecting, science often proceeds to **data organization** and **classification**. The uses of the verb ‘know’

seem quite varied; for example, what does Jay knowing Kay have to do with Jay knowing that his shoes are untied? It has been commonly proposed that knowledge divides into three broad categories, which we list as follows.

personal knowledge (acquaintance)	Jay knows Kay (personally)
procedural knowledge	Jay knows how to tie his shoes
propositional knowledge	Jay knows that his shoes are untied

This is a **proposal** to be considered, or a **hypothesis**.¹ The precise hypothesis is that every instance of knowledge fits into exactly one of the above categories.

Next, in judging an hypothesis, or proposal, we need to **test it against data**, both data we have already collected, and future data. Let us examine some test-cases; for example, we have 13 data already collected; how do we classify these?

In this connection, we make a few observations. First, propositional knowledge is characterized by the expression ‘knows that’, so in classifying the examples above one should ask whether the example can be rephrased using ‘that’.² The following are examples of rephrasing.

- (1) Jay knows **to** call for help when he is in trouble
that he should call for help when he is in trouble
- (2) Jay knows **who** wrote the *Declaration of Independence*
that so-and-so wrote the *Declaration of Independence*
- (3) Kay knows **where** the *Declaration of Independence* is located
that the *D of I* is located at such-and-such-location
- (4) Jay knows **when** we celebrate the signing of the *Declaration of Independence*
that we celebrate the signing of the *D of I* on such-and-such date
- (5) Kay knows **why** we celebrate the signing of the *Declaration of Independence*
that we celebrate the signing of the *D of I*
because of such and such reasons
- (6) Kay knows **what** an arthropod is
that an arthropod is a kind with such-and-such characteristics
- (7) Jay knows **whether** he will be here tomorrow
that he will be here tomorrow
OR he knows **that** he won’t be here tomorrow

¹ One of the issues in philosophy of science is whether classification systems, such as the one just proposed, are to be judged in terms of **truth-and-falsity**, or should rather be judged in terms of **usefulness** and **convenience**. As in so many situations, the answer is not so straightforward; for example, it seems that it would be difficult to explain why a classification system is useful if it isn’t true (to some degree of approximation).

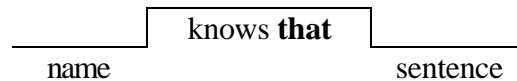
² Specifically as a “complementizer” to form subordinate clauses. In addition to this usage, the word ‘that’ is also used as a demonstrative pronoun, as in ‘**that** darn cat’ or ‘**that** is what I am talking about’. Demonstrative pronouns are usually employed to point out a particular object. The word ‘that’ is also used as a relative pronoun as in ‘the fish **that** got away’ or ‘the mouse **that** lived in the house **that** Jack built’. Finally, the word ‘that’ is also used as a complementizer, which is special modifier that prefixes a declarative sentence to form a subordinate clause, as in ‘Kay believes **that** snow is white’.

4. Propositional Attitudes and Propositional Adjectives

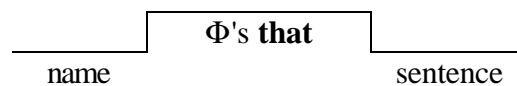
The sort of knowledge we are primarily concerned with is *propositional knowledge*, exemplified by sentences like

Kay knows **that** $2+2=4$

which have the form.



Propositional knowledge is an instance of a more general class of relations called **propositional attitudes**, which are all exemplified by sentences of the form.



where the Greek letter ' Φ ' (phi) serves as a blank to filled in by a special propositional-attitude verb. There are many verbs that can replace ' Φ ', including the following

says, knows, believes, doubts, expects, fears, hopes, desires, regrets, demands

Propositional attitudes, in turn, are a special case of **propositional properties**, which are exemplified by sentences of the form.



where ' Φ ' is a special propositional-adjective. Probably the simplest propositional adjectives are 'true' and 'false'. The following are simple examples.³

it is **true** that $2+2=4$

it is **false** that $2+2=5$

There are numerous other examples of propositional adjectives, including the following.

it is **believed** that ...

it is **necessary** that...

it is **possible** that...

it is **probable** that...

In this connection, notice that propositional relations are a special case of propositional properties. For example,

Kay knows that $2+2=4$

may be rewritten as:

³ The expressions 'it is false that' and 'it is not true that' are often presented as the official expressions of logical-negation. In particular, 'not-P' is understood as short for 'it is not true that P'.

it is known by Kay **that** $2+2=4$

5. Propositions

The term ‘proposition’ has a number of related uses in English. At one end of the spectrum, its use as a verb – ‘to proposition’ – has an almost exclusively sexual connotation. At the other end of the spectrum, ‘proposition’ conveys a very high-brow concept that is central to philosophy.

The noun-usage of ‘proposition’ divides into two uses, which we will call "Jeffersonian" and "Corleonean". First, Thomas Jefferson wrote one of the great documents in the English language – *The Declaration of Independence*.⁴ Its second paragraph begins with the now-famous line.

We hold these truths to be self-evident, ...

He could have also used the following words, but didn’t for poetic/rhetorical reasons.

We hold these **propositions** to be self-evidently true, ...

Second, in one of the great films of the English language, *The Godfather* (1972), the character Don Corleone (played by Marlon Brando) says the now famous line.

We made him an **offer** he couldn’t refuse...

Once again, the author could have used alternative language, but didn’t for poetic reasons. In particular, Don Corleone could have said:

We made him a **proposition** he couldn’t refuse.

Here the word ‘proposition’ is close in meaning to the word ‘proposal’.

The verb ‘propose’ is another example of a propositional-attitude verb. In particular, one can say something like the following.

we proposed to him **that** he sell us his casino

We are interested in proposals, to be sure, since science advances by making and testing proposals (i.e., hypotheses). There is a potential confusion, however, since a proposal, like every propositional attitude, has three components.

- | | | | |
|-----|----------------------------|--|------------|
| (1) | the agent of the attitude | the one who proposes, believes, desires, etc. | "the who" |
| (2) | the attitude | the act of proposing, believing, desiring, etc. | "the how" |
| (3) | the object of the attitude | what is proposed, believed, desired, etc. | "the what" |

It’s easy to get (2) and (3) confused, especially when the act is proposing. The word ‘belief’ refers both to the *mental act* of believing, and to the *object/content* of the belief (the proposition). Some adjectives apply to the mental act, and others apply to the proposition. For example, a belief can be sincere or unshakable; these apply to the mental act of believing. On the other hand, a belief can be true or false; these apply to the content of the belief.

⁴ <http://lcweb2.loc.gov/const/declar.html>.

When we consider the verb ‘propose’, things are even more complicated. There is the **act** of proposing, which may be called a proposition, and there is **what** is proposed, which is also called a proposition. The philosophical use of ‘proposition’ primarily pertains to the latter meaning.

6. The Distinction between Sentences and Propositions

The critical distinction between **sentences** and **propositions** is dealt with extensively elsewhere in this book.⁵ Let us make a few remarks by way of amplifying that discussion. First, the following summarizes the relation and the distinction.

sentences **express** propositions

The distinction between a sentence and the proposition it expresses is precisely analogous to the distinction between a name and the individual it denotes (refers to). No one is apt to confuse the name ‘George Bush’ with the man George Bush, by which I mean the individual who is currently U.S. president. Whereas words are made of letters/sounds, humans (or at least their bodies) are made of molecules. A similar distinction applies to sentences and propositions; whereas sentences have linguistic objects (words and phrases) as constituents, propositions have non-linguistic objects as constituents.

Unfortunately, the precise nature of these constituents is a subject of ongoing debate. For example, Bertrand Russell proposed that the proposition that-two-is-even consists of precisely two constituents.

- | | |
|--------------------------------|--------------------------------|
| (1) the actual number two | the thing denoted by ‘two’ |
| (2) the property of being even | the thing denoted by ‘is even’ |

More complicated propositions then have a more complicated array of constituents.

7. The Ubiquitous ‘That’

The single biggest clue that a proposition is being mentioned is the appearance of the word ‘that’.⁶ It is prominent in *The Declaration of Independence*. After Jefferson says “We hold these truths to be self-evident,” he goes on to *name* a few such propositions, including the following.

that all Men are created equal,

that they are endowed by their Creator with certain unalienable Rights,

⁵ Cf. “Philosophical Distinctions”.

⁶ Not all propositional references explicitly use ‘that’. Sometimes it is omitted for conciseness/laziness. For example, the following sentence

I know you know $2+2=4$

is officially an abbreviation of

I know **that** you know **that** $2+2=4$.

Other propositional references use other grammatical constructions, the most prominent alternative being infinitive phrases. For example, the following sentence

Jay wants Kay to be the president

involves the proposition that-Kay-is-the-president. Generally, propositional references can be paraphrased (although awkwardly sometimes!) using a ‘that’ phrase. For example:

Jay wants it to be the case **that** Kay is the president

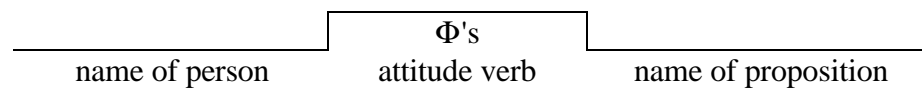
that among these are Life, Liberty and the Pursuit of Happiness –

That to secure these Rights, Governments are instituted among Men, deriving their just Powers from the Consent of the Governed,

that whenever any Form of Government becomes destructive of these Ends, it is the Right of the People to alter or to abolish it, and to institute new Government, laying its Foundation on such Principles, and organizing its Powers in such Form, as to them shall seem most likely to effect their Safety and Happiness.

The basic idea is that the word ‘that’ is a sentence-prefix with the following grammatical function. It takes a *sentence* – e.g., ‘2+2=4’ – and forms a *noun phrase* – in this case ‘that 2+2=4’ – which names the proposition that the original sentence expresses.

This allows us to re-work the logical forms of attitude sentences, as follows.



This in turn allows us to consider alternative methods of naming propositions, including the following

Jay believes **the proposition that all humans are created equal**

Kay wants **all humans to be equally treated**

8. Direct versus Indirect Quotation

Suppose I report an exchange between two politicians, Smith and Jones. I might use **direct quotation**, in which case I provide a more or less *verbatim* report of their exchange. It might go something like this.

First, Jones said to Smith, **“You are an idiot.”**

To which Smith replied, **“You are an idiot, and so is your mother.”**

Notice that the words of the exchange are placed in double-quotes. This indicates that these are the very words used by the two politicians; for example, they were speaking English.

Now if I choose to provide the content of the exchange, rather than the words, I will employ **indirect quotation**, in which case I have a lot more latitude in reporting, but most likely it would go something like this.

First, Jones said to Smith **that he (i.e., Smith) is an idiot**.

To which Smith replied **that he (i.e., Jones) is an idiot and so is his mother**.

First notice that, when we use indirect quotation, we are not required to convey the exact words of the quoted discourse, but only their combined content. For example, from an indirect quotation, we cannot even deduce what language was originally spoken. In this connection, also notice that the move from direct to indirect quotation requires grammatical adjustments. For example, the following would be inaccurate.

First, Jones said to Smith **that you are an idiot**.

To which Smith replied **that you are an idiot and so is your mother**.

This means that Jones told Smith that **you** (i.e., **the reader**) are an idiot. And Smith said that **you** (i.e., **the reader**) are an idiot, and that furthermore **your** (i.e., **the reader's**) mother is an idiot.

Not all direct quotations translate into indirect quotations. Consider the following transcript.

First, Jones said to Smith, “You are an idiot.”
To which Smith replied, “Ouch!”

It would be very odd indeed to report this as follows.

First, Jones said to Smith **that** he (i.e., Smith) is an idiot.
To which Smith replied **that** ouch.

Questions are sometimes difficult to convey also.

Jones said, “you are a liberal, aren’t you?”

Compare this with:

Jones said, “you aren’t a liberal, are you?”

How do we convey these two questions, which are somewhat different in meaning, using indirect quotation?

Things get even worse. Sometimes, intonation is critical in conveying the meaning of a question. For example, consider the following

Kay asked Jay, “you didn’t put the dog out, did you?”

where the intonation indicates that Jay was supposed to put the dog out, but he didn’t. Now compare that with the following

Kay asked Jay, “you didn’t put the dog out, did you?”

where the intonation indicates that Jay was supposed to **not** put the dog out, but he did. Once again, it is difficult to convey this interchange using indirect quotation.

9. Sentential Attitudes and Properties

Some adjectives and verbs pertain to propositions, and others pertain to sentences. There is considerable overlap, as we have already seen; for example, both of the following sentences are perfectly acceptable.

Jay said that two plus two is four
Jay said “two plus two is four”

These are best understood grammatically as follows.

Jay	said	that two plus two is four
subject	verb	object
Jay	said	“two plus two is four”
subject	verb	object

If we want to be more explicit about the sorts of noun (-phrases) that are grammatically required in the subject-positions and the object-positions, we can rewrite the forms as follows.

Jay	said	that two plus two is four
name of agent	verb	name of proposition
Jay	said	“two plus two is four”
name of agent	verb	name of sentence

Next, let us consider adjectives. The most prominent adjectives that apply to both propositions and sentences are ‘true’ and ‘false’. It makes sense to say a sentence, or a proposition, is true/false. However, the colloquial syntactic constructions are quite different.

(ok)	“two plus two is four” is true	(?)	it is true “two plus two is four”
(?)	that two plus two is four is true	(ok)	it is true that two plus two is four

10. The Ubiquitous ‘It’

Although they are officially noun phrases, complementary phrases (i.e., ‘that-S’ phrases) are not “happy” in subject position. For example, the following is grammatical in a purely technical sense, but it is not colloquially grammatical.⁷

that 2+2=4	is true
subject	predicate
name of proposition	

Rather, English grammar insists that the complementizer be placed after the predicate, and its original location be filled by a “dummy” subject ‘it’, as follows.

it	is true	that 2+2=4
dummy subject	predicate	complementary phrase
name of ???		name of proposition

Complementary phrases often go happily in object position, as in

Jay	said	that two plus two is four
name of agent	verb	name of proposition

But not always! For example, in a rather brilliant passage from Lewis Carroll’s *Alice in Wonderland*, we have the following interchange.

(the mouse is speaking⁸...) “I proceed. “Edwin and Morcar, the earls of Mercia and Northumbria, declared for him: and even Stigand, the patriotic archbishop of Canterbury, found it advisable – ” ”

“Found *WHAT?*” said the Duck.

⁷ In the technical language of Linguistics, they are grammatical at deep-structure, but not at surface-structure. Rather, the surface-structure is derived from the deep-structure by a transformation.

⁸ Note carefully the presence of extra quote marks. This is because the mouse is reading from English history. Accordingly, Carroll is quoting the mouse, who is quoting the historian.

“Found **IT**,” the Mouse replied rather crossly: “of course you know what *“it”* means.”

“I know what *“it”* means well enough, when I find a thing,” said the Duck: “it’s generally a frog or a worm. The question is, what did the archbishop find?”

The Mouse did not notice this question, but hurriedly went on, “--found it advisable to go with Edgar Atheling to meet William and offer him the crown. William’s conduct at first was moderate. But the insolence of his Normans -- ”

So the question remains to this day – what did the archbishop find? The answer, I think, is quite complicated, but it goes something like this.

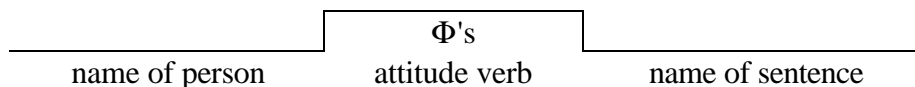
the archbishop found **that** something was advisable, namely **that** he (should) go with Edgar Atheling to meet William and offer him the crown.⁹

11. Sentential Attitudes can be Weird

Some propositional-concepts give rise to counterpart sentential-concepts. We have already seen the verb ‘says’ and the adjectives ‘true’ and ‘false’. Are there any others? In many cases, the corresponding sentential attitude is ludicrous. For example.

- (1) Jay demands ‘ $2+2=4$ ’
- (2) Kay fears ‘ $2+2=4$ ’
- (3) Ray expects ‘ $2+2=4$ ’
- (4) Fay knows ‘ $2+2=4$ ’
- (5) May believes ‘ $2+2=4$ ’

These have the form of sentential-attitude sentences. In particular, they all have the following form.¹⁰



The above examples are technically-speaking grammatical, but they seem a bit weird. For example, although it is highly unlikely, Kay may in fact be afraid of the **sentence** ‘ $2+2=4$ ’ and maybe many other sentences of Arithmetic. Likewise, Fay may in fact be personally acquainted with that same **sentence**. And perhaps Jay may in fact demand, and Ray may expect, the **sentence** ‘ $2+2=4$ ’; for example, they may demand/expect us to write that sentence.

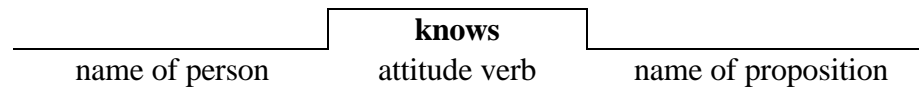
Example (5) above is not entirely ludicrous. For one to believe a person is for one to believe what that person says (on the given occasion). So correspondingly, for one to believe a sentence is for one to believe what that sentence says; but what a sentence says is simply the proposition that it expresses (on the given occasion). So, in the fifth example above, for May to believe the sentence ‘ $2+2=4$ ’ is simply for May to believe the proposition **that** $2+2=4$.

⁹ And thus, in 1066, began the Norman ascendancy in England, which among other things ultimately lead to a near doubling of the vocabulary of English. After that, English had both a Germanic (old English) word, and a Romanic (old French) word, for nearly everything.

¹⁰ Notice once again that, instead of a name of a proposition, we insert a name of a sentence. Names of sentences are obtained by placing sentences inside quotes, whereas names of propositions are obtained by prefixing sentences by the word ‘that’.

12. Characterizing Knowledge

As already indicated, philosophy is primarily concerned with propositional knowledge, and propositional knowledge is conveyed in *canonical form*¹¹ by sentences of the form.



where that-clauses are the canonical names of propositions. This is illustrated in the following simple example.



The “holy grail” of Epistemology (Theory of Knowledge) is to provide a simple definition of knowledge. The following passage illustrates the usual formulation of the problem.¹²

The objective of the analysis of knowledge is to state the **conditions** that are **individually necessary** and **jointly sufficient** for propositional knowledge...

13. Necessary and Sufficient Conditions

Let us consider some of the technical vocabulary in the above passage. Let us review a little elementary logic. Consider the following sentence.

in order for me to graduate
it is **necessary** for me to pass intro logic

This may be paraphrased using ‘that’, as follows.

in order *that* I graduate
it is **necessary** *that* I pass intro logic

It may also be paraphrased using ‘ing’ verbs.

(me) *passing* intro logic is **necessary** for (me) *graduating*

All of these may be paraphrased as follows.

I will graduate **only if** I pass intro logic

This in turn may be paraphrased by converting the word ‘only’ into two negations, as follows.

I will **not** graduate **if** I do **not** pass intro logic

¹¹ The word ‘canonical’ is a heavy-duty word, both in mathematics and philosophy. First, the root word ‘canon’ should not be confused with ‘cannon’ (which derives from an old Italian word *cannone* [tube]), or with the Spanish word ‘cañon’ (which means *canyon*). The word ‘canon’ derives from Latin *can*¹² [rule], which derives from Greek *kan*¹² [measuring rod, rule]. The word basically refers to orthodoxy or orthodox criteria, whether it be it in religion (original meaning), or in general.

¹² This passage, which comes from the *Stanford Encyclopedia of Philosophy*, is fairly typical of the statement of the problem.

This in turn may be paraphrased by inverting the constituents, as follows.

if I do **not** pass intro logic, **then** I will **not** graduate

Let us next consider the word 'sufficient'. Consider the following equivalent sentences

in order for me to get an A in intro logic
it is **sufficient** for me to get 100% on every exam

in order that I get an A in intro logic
it is **sufficient** that I get 100% on every exam

(me) getting 100% on every exam is **sufficient** for (me) getting an A in intro logic

Sentences involving 'sufficient' may be paraphrased using 'if', as follows.

I will get an A in intro logic **if** I get 100% on every exam

This in turn may be paraphrased by inverting the constituents, as follows.

if I get 100% on every exam, **then** I will get an A in intro logic

Next, these two ideas can be combined in various ways.

- (1) necessary **and** sufficient
- (2) necessary **but not** sufficient
- (3) sufficient **but not** necessary
- (4) **neither** necessary **nor** sufficient

The simplest combination is the first one, which is the concept of **necessary and sufficient condition**. This may be paraphrased using the phrase 'if and only if'; for example, the sentence

in order for me to get an A in intro logic
it is **both necessary and sufficient** for me to get 375 total points

may be paraphrased as follows.

I will get an A in intro logic **if and only if** I get 375 total points

This in turn is a conjunction (using 'and') of the following two sentences.

- (1) I will get an A in intro logic **if** I get 375 total points
- and**
- (2) I will get an A in intro logic **only if** I get 375 total points

Where the latter sentence is equivalent to:

- (2') I will **not** get an A in intro logic **if** I do **not** get 375 total points

which is equivalent to:

- (2'') **if** I do **not** get 375 total points, **then** I will **not** get an A in intro logic

Now, recall the passage we are explicating.

The objective of the analysis of knowledge is to state the **conditions** that are **individually necessary** and **jointly sufficient** for propositional knowledge...

Consider a list of conditions,

condition 1, condition 2, condition 3, ...

which is syntactically just a list of declarative sentences.

S_1, S_2, S_3, \dots

Now consider a single sentence S_0 that expresses a situation/condition/state-of-affairs that interests us (for example, knowledge). Then to say that conditions $\{S_1, S_2, S_3, \dots\}$ are individually necessary for S_0 is just to say that each one of $\{S_1, S_2, S_3, \dots\}$ is necessary for S_0 , which is to say:

S_1 is necessary for S_0	in order that S_0 it is necessary that S_1
S_2 is necessary for S_0	in order that S_0 it is necessary that S_2
S_3 is necessary for S_0	in order that S_0 it is necessary that S_3
etc.	etc.

Paraphrasing these using standard logical terminology, we have

if not S_1 , then not S_0
 if not S_2 , then not S_0
 if not S_3 , then not S_0
 etc.

As the intro logic student can readily demonstrate, the above list of conditionals is equivalent to the following single conditional.

S_0 **only if** (S_1 and S_2 and S_3 and ...)

or:

if not (S_1 and S_2 and S_3 and ...) **then not** S_0

Next, to say that $\{S_1, S_2, S_3, \dots\}$ are *jointly sufficient* for S_0 is just to say that *combined together* they are sufficient for S_0 , which is to say:

(S_1 **and** S_2 **and** S_3 **and** ...) is **sufficient** for S_0

or:

in order that S_0 it is **sufficient** that (S_1 **and** S_2 **and** S_3 **and** ...)

Paraphrasing this using standard logical words, we have

S_0 **if** (S_1 and S_2 and S_3 and ...)

or:

if (S_1 and S_2 and S_3 and ...) **then** S_0

Thus, when we combine these two ideas we have the following. To say that $\{S_1, S_2, S_3, \dots\}$ are individually necessary and jointly sufficient for S_0 is just to say:

S_0 **only if** (S_1 and S_2 and S_3 and ...) **and**
 S_0 **if** (S_1 and S_2 and S_3 and ...)

or:

if not (S_1 and S_2 and S_3 and ...) **then not** S_0
and
if (S_1 and S_2 and S_3 and ...) **then** S_0

The most succinct formulation of this is given as follows.

S_0 **if and only if** (S_1 and S_2 and S_3 and ...)

14. A Restatement of the Objective of Epistemology

Having clarified some of the terms, we can now reformulate the objective of the analysis knowledge. Specifically, the goal of Epistemology is to provide a simple account of knowledge of the following form.

person p knows that- S
if and only if
 (1) condition 1, **and**
 (2) condition 2, **and**
 ... **and**
 (n) condition n

15. A Notational Problem (Warning!)

Before continuing, the reader is warned about a potential notational problem. In our presentation, we write propositional knowledge claims as follows.

p knows that- S

Here, the letter ' p ' stands in place of any proper noun – for example, 'Kay' – and the letter ' S ' stands in place of any sentence. Unfortunately, this conflicts with the *usual* presentation of the form, which goes as follows.

S knows that p

Here, the mnemonics are

S : subject
 p : proposition

Unfortunately, this runs the risk of confusing sentences and propositions. If we want to use the propositional mnemonic, then the form should be.

S knows p

not

(*) S knows that p

The 'that' expression in combination with a **sentence** produces a name of a proposition, which could be abbreviated by ' p '.

Henceforth, we will use the following abbreviations.

\mathcal{S}	:	allows the substitution of a sentence
\mathbb{P}	:	allows the substitution of a name of a proposition
p	:	allows the substitution of a name of a person

In any case, we want to come up with **necessary and sufficient conditions** for the following class of sentences.

p knows \mathbb{P}
 p knows that \mathcal{S}

16. Necessary Conditions for Knowledge

We begin considering *necessary* conditions for knowledge. Suppose that

Jay knows that his shoes are untied.

What must also be true if this statement is true. Alternatively, what can we logically deduce from this statement? It seems that at least the following two statements can be deduced.

Jay **believes** that his shoes are untied

it is true that Jay's shoes are untied

This is summarized in the following two principles about knowledge.

- (1) knowledge **entails** belief
- (2) knowledge **entails** truth

Alternatively

- (1) in order to know that \mathcal{S} , it is necessary to believe that \mathcal{S}
- (2) in order to know that \mathcal{S} , it is necessary that it is true that \mathcal{S}

Writing these more succinctly, using logical terms, we have:

- (1) **if** p does **not** believe that \mathcal{S} , **then** p does **not** know that \mathcal{S}
- (2) **if** it is **not** true that \mathcal{S} , **then** p does **not** know that \mathcal{S}

Or even more succinctly, we can rewrite these without the negations.

- (1) **if** p knows that \mathcal{S} , **then** p believes that \mathcal{S}
- (2) **if** p knows that \mathcal{S} , **then** it is true that \mathcal{S}

Or even more succinctly.

- (2) **if** p knows that \mathcal{S} , **then** \mathcal{S}

17. Belief

Since knowledge entails belief, let us briefly consider belief. Like ‘knowledge’, ‘belief’ has a number of uses, but oddly there are many constructions that allow the verb ‘know’ but do not allow the verb ‘believe’. For example, compare the following pairs, and also come up with examples of your own.

- | | | |
|-----|----------------------------------|-------------------------------------|
| (1) | ✓ Jay knows where Kay is | ✗ Jay believes where Kay is |
| (2) | ✓ Jay knows who Kay is | ✗ Jay believes who Kay is |
| (3) | ✓ Jay knows when spring break is | ✗ Jay believes when spring break is |
| (4) | ✓ Jay knows whether it will rain | ✗ Jay believes whether it will rain |
| (5) | ✓ Jay knows how to swim | ✗ Jay believes how to swim |

There are other constructions in which both ‘know’ and ‘believe’ are permitted, but they are quite different in meaning. Compare the following.

- | | | |
|-----|--------------------|-----------------|
| (5) | ✓ Jay believes Kay | ✓ Jay knows Kay |
|-----|--------------------|-----------------|

Whereas the former means that Jay believes what Kay says (on the specified occasion), the latter does *not* mean that Jay *knows* what Kay says. Rather it means that Jay is acquainted with Kay.

If propositional knowledge entails propositional belief, as we have proposed, then in order to understand propositional knowledge we have to understand propositional belief. So, the question is how to understand sentences of the following form:

p believes that S

Here, ‘ p ’ stands in place of a proper name – like ‘Kay’ – and ‘ S ’ stands in place of a sentence – like ‘snow is white’.

Let us propose the following initial account.

p believes that S

if and only if:

if p were asked whether S ,
then p would say ‘yes’.

According to this account, in order to ascertain a person's beliefs, we could give that person a (rather long!) multiple-choice questionnaire, which would have questions such as the following.

- | | |
|-----|--------------------------------|
| (1) | snow is white |
| | ___ true |
| | ___ false |
| | ___ don’t know (or no opinion) |
| (2) | $2+2 = 5$ |
| | ___ true |
| | ___ false |
| | ___ don’t know (or no opinion) |

Notice that we allow the answer ‘don’t know’, since the experimental subject is not being *graded*, but is instead being *measured*.¹³

The reader will probably recognize some serious problems in the above experimental design. As a method to ascertain a person's beliefs, this method depends upon the following conditions obtaining.

- (1) p understands the question;
- (2) p can readily access his/her beliefs;
- (3) p can speak or otherwise communicate;
- (4) p is benevolent towards the questioner, and in particular does not lie.

This suggests that we need a more idealized account of belief like the following.

p believes that \mathcal{S}

if and only if:

if p were asked whether \mathcal{S} ,
then p would say ‘yes’.

supposing the following:

- (1) p understands the question;
- (2) p can readily access his/her beliefs;
- (3) p can speak or otherwise communicate;
- (4) p is benevolent towards the questioner.

18. Proposal 1 – Knowledge is True Belief

Let us *suppose* we have belief under control. That still leaves knowledge. So far, we have two necessary conditions for knowledge. In particular.

- (1) knowledge entails belief; i.e., if p knows that \mathcal{S} , then p believes that \mathcal{S}
- (2) knowledge entails truth; i.e., if p knows that \mathcal{S} , then (it is true that) \mathcal{S}

One proposed account of knowledge is the following.

Proposal 1: knowledge **is** true belief

This is written as a slogan. It is officially rewritten as follows.

Proposal 1:
a person p knows that \mathcal{S} **if and only if:**

- (1) p believes that \mathcal{S} **and**
- (2) (it is true that) \mathcal{S}

¹³ The difference is actually *very* subtle. Exams are given to people in order to measure their mastery of a subject; guessing is permitted. On the other hand, questionnaires are given to people measure their beliefs (knowledge and opinions); we would rather the subject not guess.

19. Counterexamples

The word ‘counterexample’ plays a central role in philosophy. In intro logic, the word arises in connection with arguments. In particular, it is claimed that *invalid* arguments *do* admit counterexamples, whereas *valid* arguments *do not* admit (any) counterexamples. In this context, ‘counterexample’ is defined as follows.

- (d1) A **counterexample to an argument** \mathcal{A}_1 is an argument \mathcal{A}_2 such that
- (1) \mathcal{A}_1 and \mathcal{A}_2 have the **same logical form**, and
 - (2) \mathcal{A}_2 is obviously invalid, which is to say:
 - (2a) \mathcal{A}_2 has all true premises;
 - (2b) \mathcal{A}_2 has a false conclusion.

The basic idea is that finding a counterexample to an argument refutes it *as an argument*.

This usage of ‘counterexample’ is a highly customized usage of the word. It also has a more general usage in philosophy as follows.

- (d2) A **counterexample to a claim/proposition** \mathbb{P} is a *situation* that *refutes* \mathbb{P} .

Two terms bear scrutiny. The word ‘refutes’ means ‘demonstrates/shows the falsity of \mathbb{P} ’. The term ‘situation’ is a bit vague. The best thing is simply to consider a simple example.

- (e) let \mathbb{P} be the claim/proposition
that all swans are white;
then a counterexample to \mathbb{P} would be any situation containing a non-white swan;

Notice that the existence of a non-white swan demonstrates the falsity of this claim, and accordingly serves as a counterexample.

Let us next look at a further customization of the term ‘counterexample’.

- (d3) A **counterexample to a proposition** \mathbb{P} expressible as a *biconditional* of the form
 \mathbb{S}_1 if and only if \mathbb{S}_2
is any situation that verifies \mathbb{S}_1 but refutes \mathbb{S}_2 , or verifies \mathbb{S}_2 but refutes \mathbb{S}_1

This is not really a new definition, but merely an application of (d2) to biconditionals. In order for ‘ \mathbb{S}_1 if and only if \mathbb{S}_2 ’ to be refuted, it is sufficient to find a situation in which they don’t have the same truth-value [i.e., \mathbb{S}_1 true, and \mathbb{S}_2 false, or \mathbb{S}_2 true and \mathbb{S}_1 false].

20. Counterexamples to Proposal 1

Proposal 1 is that knowledge is (merely) true belief. Many people use the word ‘know’ in this manner, it seems. They conjecture that such and such will happen, and when it does happen, they say ‘I knew it!’. For example, imagine a sports fan watching a football game; imagine that he says ‘they are going to score’, and imagine that the team in fact scores shortly thereafter; it is not hard to imagine the sports fan saying at this point ‘I knew it!’.

I suspect that this **is** knowledge in some *minimal* way, but it does not seem completely satisfying. There seems to be something missing. The following example illustrates the problem with the claim that knowledge is (merely) true belief.

Suppose that Freddy has been asleep, or in a coma, for the last 12 years, and suppose he finally wakes up. Suppose we are attending him and we ask him the following simple question.

what is the last name of the president?

Suppose that Freddy says the following.

the president's last name is 'Bush'

Let us suppose that Freddy is speaking sincerely, is self-cognizant, etc., so it natural to surmise from this that

Freddy **believes** that the president's last name is 'Bush'.

As of this moment, of course, the president's last name is in fact 'Bush'. So we have a case of true belief; i.e.,

- (1) Freddy **believes that** the president's last name is 'Bush';
- (2) **it is true that** the president's last name is 'Bush'.

Now, let us suppose that Freddy is very disoriented, which is very likely considering the situation. He went to sleep back in 1990, and he has only just awakened. He probably thinks it is still 1990, so his beliefs are a bit skewed. For example, he thinks that the president is George Bush **the elder**, and has never even heard of George Bush **the younger**. If we quizzed him further, we could easily figure out just how skewed his beliefs are.

Now it doesn't seem that Freddy *knows* the last name of the president; rather, it seems that he has made a "lucky guess".

In this second example, we have a person, call him Jim Bob, who plays Lotto every week, and furthermore firmly believes that he will win each week he plays. What is worse, he loses every week, but this does not deter him from playing the next week. In fact, he believes every week that his odds are getting better, so every week he is even more confident that he will win.

Now, suppose that finally one day Jim Bob does in fact win Lotto. Once again, we have a case of true belief. However, it seems implausible to call this a case of knowledge. Once again, it is merely a "lucky guess".

What is common to these examples is that the belief in question turns out to be true as a matter of pure luck, not as a matter of epistemic "virtue". It seems similar to a person winning a game by luck rather than by skill or hard work, or a person becoming rich by luck rather than by skill or hard work. In some sense, the person does not "deserve" his reward.

21. Proposal 2 – Knowledge is Justified True Belief

We have seen that although knowledge entails true belief, knowledge is not *merely* true belief. There is more to it, although it is not obvious at the moment what that "more" is, although we have suggested that there is something like a "moral" component.

Whether knowledge has a moral component or not, we have logically that the set of individually necessary and jointly sufficient conditions must include.

- (1) p believes that \mathcal{S} ;
 (2) it is true that \mathcal{S} ;
plus
 (3) **one or more** additional conditions.

What are the additional conditions?

The most famous, and most discussed, proposal is the following further condition.

- (3) p is **justified** in believing that \mathcal{S}

This gives rise to Proposal 2.

Proposal 2 (JTB):

a person p knows that \mathcal{S}

if and only if:

- (1) p believes that \mathcal{S} **and**
 (2) (it is true that) \mathcal{S} **and**
 (3) p is **justified** in believing that \mathcal{S}

The word ‘justify’ comes from Latin ‘iustificare’ which means ‘to act justly towards’.¹⁴ This reminds us of the moral connotations of the word. It seems that we don’t want to ascribe knowledge to someone who doesn’t deserve it in some sense.

22. Gettier's Counterexample

Rather than get too involved in what justification amounts to, we go on to consider whether JTB admits any counter-examples. As it turns out, it does. Specifically, in 1963, Edmund Gettier offered a counter-example to JTB, which is now referred to as "the Gettier Problem".¹⁵ The following is an adaptation of Gettier's original formulation.

Suppose

- (1) Jones believes
 (a) that the German Chancellor owns a Mercedes

Suppose further that:

- (2) Jones is justified in believing (a)

How this justification arises is a matter of some detail, to be sure, but let us simply say that Jones has a number of beliefs, which are all true, and which confer upon (a) a fairly high probability, but not probability 1. For example, let us suppose that *most* top German officials drive Mercedes, and have as

¹⁴ Notice that one use of ‘justified’ – in printing – is quite odd. In particular, we speak of text being *left-justified* and *right-justified*. For example the text in many paragraphs in this work are both left-justified (the left margins all line up) and right-justified (the right margins all line up). The connection with justice is *via* the concepts of rectitude, straightness, and uprightness.

¹⁵ Gettier, Edmund L. "Is Justified True Belief Knowledge?" *Analysis*, Vol. 23, 1963, 121-123

long as there have been cars in Germany, which information Jones has. We may further suppose that the Chancellor has owned a Mercedes all his adult life, which information Jones also has. All this makes it a very good bet for Jones that the German Chancellor still owns a Mercedes.

Nevertheless, it is still *possible* that he does not! So, let us further suppose that:

- (3) (a) is false [i.e., the German Chancellor does *not in fact* own a Mercedes]

For example, he may have just recently become a classic James Bond fan, and accordingly has just sold his Mercedes, and bought an Aston-Martin.

First, it is clear that, at a minimum we must conclude that:

- (4) Jones *does not know*
(a) that the German Chancellor owns a Mercedes

This is because (a) is not true. So far, so good.

Now consider the following proposition.

- (b) that the German Chancellor owns a Mercedes, **or** he is in Berlin right now

There are two widely accepted principles of belief and justification, which may be stated as follows.

- (E1) **if** $\lceil S_1 \rceil$ logically entails $\lceil S_2 \rceil$, **then**
if one believes that S_1 ,
then one also believes that S_2
- (E2) **if** $\lceil S_1 \rceil$ logically entails $\lceil S_2 \rceil$, **then**
if one is justified in believing that S_1 ,
then one is also justified in believing that S_2

To say that one sentence *logically entails* another is to say that the latter can be *logically deduced* from the former.

Now, (a) does logically entail (b).¹⁶ Therefore, since Jones believes (a), it follows that he believes (b), and also since Jones is justified in believing (a), it follows that he is justified in believing (b).

Now, it is furthermore plausible to suppose that (b) is true on many occasions, given that Berlin is the capital of Germany. In particular, it is true at least on every occasion in which the German Chancellor is in Berlin. So, at least on each such occasion, we have the following.

- (1) Jones believes (b);
(2) (b) is true;
(3) Jones is justified in believing (b);

So, according to JTB, we have all the ingredients of knowledge, so we are to conclude:

- (4) Jones knows (b).

¹⁶ Here, we understand that the prefix 'that' has been removed from (a) and (b) respectively, so that we have sentences.

By the logical method of universal generalization, we furthermore have the following.

- (5) whenever the German Chancellor **is** in Berlin, Jones **knows** (b), and
whenever the German Chancellor **is not** in Berlin, Jones **does not know** (b).

Now, it does not seem very plausible that Jones' epistemic state switches from knowledge to non-knowledge every time the German Chancellor leaves Berlin, and it miraculously switches back to knowledge when the German Chancellor returns to Berlin.

23. Proposal 3 – The Counterfactual Analysis

In confronting the Gettier problem, philosophers have spilled a great deal of ink. Various proposals have been made to save Proposal 2 by modification. More recently, however, other proposals have been put forth, according to which justification does not play a role in knowledge. According to one such proposal, knowledge is characterized as follows.

Proposal 3 (CA):

a person p knows that S

if and only if:

- (1) p believes that S **and**
- (2) (it is true that) S **and**
- (3) **if it were not** true that S ,
then p would **not** believe that S

This is called 'the counterfactual analysis of knowledge' because of the presence of the counterfactual conditional in (3). The difficulty with this proposal is that the analysis of counterfactual conditionals is fraught with peril. Nevertheless, due to the work of numerous philosophers, including most prominently Robert Stalnaker (M.I.T.) and David Lewis (Princeton, but recently deceased) we have a fairly good grasp of counterfactual conditionals now.

According to the common thread of the Stalnaker-Lewis account, in order to assess a sentence of the form

- (c) **if it were** the case that A , **then** it **would** be the case that C

we must consider "minimal alterations" in the world that make the antecedent A true, and we check to see whether the consequent C is true in the resulting alternative worlds. In particular, (c) is true precisely if every minimal alteration that makes A true also makes C true.

Now, how does this account square with our earlier examples. In the Gettier example, Jones justifiably believes

- (b) that the German Chancellor drives a Mercedes, **or** he is in Berlin

and (b) is true at the moment because the German chancellor happens to be in Berlin. But what if (b) were not true? It seems that the smallest world-alteration that makes (b) false is an alteration that moves the Chancellor just outside of Berlin. Now, given the hypothesis about Jones' epistemic state, Jones would *still* believe (b). So, according to (CA), Jones does not know (b), even when it is true.