# UNIT 4: DERIVATIONS IN IDENTITY LOGIC

## 1. EXERCISES

**Directions:** For each of the following, construct a formal derivation of the conclusion (indicated by "/") from the premises (if any). In cases in which two formulas are separated by '//', construct a derivation of each formula from the other.

## 2. EXERCISE SET A

1. $/\ \forall x \forall y\{[Fx \,\&\, {\sim}Fy] \rightarrow x{\neq}y\}$
2. $/\ \forall x \forall y\{[{\sim}Fx \,\&\, Fy] \rightarrow x{\neq}y\}$
3. $/\ \forall x \forall y\{x{=}y \rightarrow [Fx \leftrightarrow Fy]\}$
4. $/\ \forall x \forall y\{x{=}y \rightarrow \forall z\{Rzx \leftrightarrow Rzy\}\}$
5. $/\ \forall x \forall y\{x{=}y \rightarrow f(x){=}f(y)\}$
6. $/\ \forall w \forall x \forall y \forall z\{[w{=}x \,\&\, y{=}z] \rightarrow s(x,z){=}s(w,y)\}$
7. $a{=}b\ //\ \forall x\{x{=}a \rightarrow x{=}b\}$
8. $a{=}b\ //\ \exists x\{x{=}a \,\&\, x{=}b\}$
9. $/\ \forall x\{Fx \leftrightarrow \forall y[y{=}x \rightarrow Fy]\}$
10. $/\ \forall x\{Fx \leftrightarrow \exists y[y{=}x \,\&\, Fy]\}$

## 3. EXERCISE SET B

11. $\forall x(Fx \rightarrow Gx)\ /\ \forall x(Fx \rightarrow \exists y[Gx \,\&\, (h(x){=}h(y))])$
12. $\exists x \forall y\{Fy \rightarrow y{=}x\}\ ;\ \exists x\{Fx \,\&\, Gx\}\ /\ \forall x\{Fx \rightarrow Gx\}$
13. $\forall x R[x,b(i)]\ ;\ \forall x\{R[b(i),x] \leftrightarrow x{=}i\}\ /\ i{=}b(i)$
14. $\exists x \forall y[x{=}y]\ //\ \forall x \forall y[x{=}y]$
15. $\exists x \exists y[x{\neq}y]\ //\ \forall x \exists y[x{\neq}y]$
16. ${\sim}\exists x \forall y[y{=}x]\ /\ \exists x \exists y[x{\neq}y]$
17. ${\sim}\exists x \exists y[x{\neq}y]\ /\ \exists x Fx \leftrightarrow \forall x Fx$
18. $/\ \forall x \exists y \forall z\{z{=}y \leftrightarrow z{=}x\}$
19. $\forall x(Fx \rightarrow Gx)\ ;\ \exists x \forall y\{Gy \rightarrow y{=}x\}\ /\ \exists x \forall y\{Fy \rightarrow y{=}x\}$
20. $\forall x\{Fx \leftrightarrow x{\neq}a\}\ ;\ \forall x\{Gx \leftrightarrow x{=}a\}\ /\ {\sim}\exists x(Fx \,\&\, Gx)$
21. $\exists x \forall y\{Fy \rightarrow y{=}x\} \vee \exists x \forall y\{Gy \rightarrow y{=}x\}\ /\ \exists x \forall y\{[Fy \,\&\, Gy] \rightarrow y{=}x\}$

22: ${\sim}\exists x \exists y[f(x){\neq}f(y)]\ ;\ \exists x \forall y[f(y){=}x]$

23: $\exists x \forall y\{Fy \rightarrow y{=}x\}\ /\ \forall x\{Fx \rightarrow \forall y(Fy \leftrightarrow y{=}x)\}$

## 4. EXERCISE SET C

24. $\exists x \forall y\{[Fy \,\&\, Gy] \leftrightarrow y{=}x\}$ / $\exists x(Fx \,\&\, Gx)$

25. $\exists x \forall y\{Fy \leftrightarrow y{=}x\}$ // $\exists x\{Fx \,\&\, \forall y[Fy \rightarrow y{=}x]\}$

26. $\exists xFx$ ; $\forall x \forall y\{(Fx \,\&\, Fy) \rightarrow x{=}y\}$ / $\exists x \forall y\{Fy \leftrightarrow y{=}x\}$

27. $\exists xFx$ ; $\sim\exists x \exists y(x{\neq}y \,\&\, [Fx \,\&\, Fy])$ / $\exists x\{Fx \,\&\, \sim\exists y(y{\neq}x \,\&\, Fy)\}$

28. $\exists xFx$ ; $\sim\exists x \exists y(x{\neq}y \,\&\, [Fx \,\&\, Fy])$ / $\exists x \forall y\{Fy \leftrightarrow y{=}x\}$

29. $\sim\exists x \forall y\{Fy \leftrightarrow y{=}x\}$ // $\exists xFx \rightarrow \exists x \exists y\{x{\neq}y \,\&\, Fx \,\&\, Fy\}$

30. $\exists x \forall y\{Fy \leftrightarrow y{=}x\}$ / $\forall x\{Fx \rightarrow Gx\} \leftrightarrow \exists x\{Fx \,\&\, Gx\}$

31. $\forall x(Fx \rightarrow Gx)$ ; $\exists x \forall y\{Gy \leftrightarrow y{=}x\}$ / $\exists xFx \rightarrow \exists x \forall y\{Fy \leftrightarrow y{=}x\}$

32. $\forall x\{Fx \rightarrow x{=}a\}$ // $\exists xFx \rightarrow \forall x\{Fx \leftrightarrow x{=}a\}$

33. $\forall x \forall y\{[Fx \,\&\, Fy] \rightarrow x{=}y\}$ // $\exists x \forall y\{Fy \rightarrow y{=}x\}$

34. $\exists xFx$ ; $\sim\exists x \exists y(x{\neq}y \,\&\, [Fx \,\&\, Fy])$ / $\exists x \forall y\{Fy \leftrightarrow y{=}x\}$

35. $\exists x \forall y\{Fy \leftrightarrow y{=}x\}$ / $\exists x\{Fx \,\&\, \sim\exists y[y{\neq}x \,\&\, Fy]\}$

36. $\exists x\{\forall y(Gy \leftrightarrow y{=}x) \,\&\, \forall y(Fy \rightarrow Ryx)\}$ / $\forall x \forall y([Fx \,\&\, Gy] \rightarrow Rxy)$

37. $\exists x \forall y\{Fy \leftrightarrow y{=}x\}$ ; $\forall x\{Fx \rightarrow Gx\}$ / $\exists x \forall y\{[Fy \,\&\, Gy] \leftrightarrow y{=}x\}$

## 5. EXERCISE SET D

38. $\exists xFx \,\&\, \forall x \forall y\{[Fx \,\&\, Fy] \rightarrow x{=}y\}$ // $\exists x \forall y\{Fy \leftrightarrow y{=}x\}$

39. $\forall x \exists y\{Fx \leftrightarrow y{=}x\}$ / $\exists x \exists y[x{\neq}y] \vee \forall xFx$

40 $\exists x\{\forall y(Fy \leftrightarrow y{=}x) \,\&\, Gx\}$ / $\exists x \forall y\{[Fy \,\&\, Gy] \leftrightarrow y{=}x\}$

41 $\exists x \forall y\{Fy \leftrightarrow y{=}x\}$ ; $\forall x(Fx \rightarrow Gx)$ / $\exists x \forall y\{[Fy \,\&\, Gy] \leftrightarrow y{=}x\}$

42. $\exists x \forall y\{[Gy \,\&\, Hy] \leftrightarrow y{=}x\}$ ; $\forall x(Fx \rightarrow Gx)$ / $\exists x \forall y\{[Fy \,\&\, Hy] \rightarrow y{=}x\}$

43. $\exists x \forall y(Fy \leftrightarrow y{=}x\}$ / $\sim\exists x \exists y\{x{\neq}y \,\&\, Fx \,\&\, Fy\}$

44. $\exists x\{\forall y(Fy \leftrightarrow y{=}x) \,\&\, \forall y(Gy \rightarrow Rxy)\}$
/ $\forall x(Gx \rightarrow \exists y \forall z\{[Fz \,\&\, Rzx] \leftrightarrow z{=}y\})$

45. $\exists x \forall y\{Fy \leftrightarrow y{=}x\}$ // $\exists x\{Fx \,\&\, \sim\exists y[y{\neq}x \,\&\, Fy]\}$

46. $\exists x \exists y\{x{\neq}y \,\&\, \forall z(Fz \leftrightarrow [z{=}x \vee z{=}y])\}$
/ $\sim\exists x \exists y \exists z\{x{\neq}y \,\&\, x{\neq}z \,\&\, y{\neq}z \,\&\, Fx \,\&\, Fy \,\&\, Fz\}$

47. $\exists x \exists y\{x{\neq}y \,\&\, Fx \,\&\, Fy\} \,\&\, \forall x \forall y \forall z\{[Fx \,\&\, Fy \,\&\, Fz] \rightarrow [x{=}y \vee x{=}z \vee y{=}z]\}$
// $\exists x \exists y\{x{\neq}y \,\&\, \forall z[Fz \leftrightarrow (z{=}x \vee z{=}y)]\}$

# 6. ANSWERS TO UNIT 4 EXERCISES

**#1:**

| | | |
|---|---|---|
| (1) | ~~SHOW~~: $\forall x \forall y\{[Fx \,\&\, {\sim}Fy] \to x{\neq}y\}$ | UD2 |
| (2) | ~~SHOW~~: $[Fa \,\&\, {\sim}Fb] \to a{\neq}b$ | CD |
| (3) | $Fa \,\&\, {\sim}Fb$ | As |
| (4) | ~~SHOW~~: $a{\neq}b$ | ID |
| (5) | $a{=}b$ | As |
| (6) | ~~SHOW~~: ※ | DD |
| (7) | $Fb$ | 3a,5,LL |
| (8) | ※ | 3b,7,SL |

**#2:**

| | | |
|---|---|---|
| (1) | ~~SHOW~~: $\forall x \forall y\{[{\sim}Fx \,\&\, Fy] \to x{\neq}y\}$ | UD2 |
| (2) | ~~SHOW~~: $({\sim}Fa \,\&\, Fb) \to a{\neq}b$ | CD |
| (3) | ${\sim}Fa \,\&\, Fb$ | As |
| (4) | ~~SHOW~~: $a{\neq}b$ | ID |
| (5) | $a{=}b$ | As |
| (6) | ~~SHOW~~: ※ | DD |
| (7) | $Fa$ | 3b,5,LL |
| (8) | ※ | 3a,7,SL |

**#3:**

| | | |
|---|---|---|
| (1) | ~~SHOW~~: ∀x∀y{x=y → [Fx ↔ Fy]} | UD2 |
| (2) | ~~SHOW~~: a=b → (Fa ↔ Fb) | CD |
| (3) | a=b | As |
| (4) | ~~SHOW~~: Fa ↔ Fb | 5,9↔I |
| (5) | ~~SHOW~~: Fa → Fb | CD |
| (6) | Fa | As |
| (7) | ~~SHOW~~: Fb | DD |
| (8) | Fb | 3,6,LL |
| (9) | ~~SHOW~~: Fb → Fa | CD |
| (10) | Fb | As |
| (11) | ~~SHOW~~: Fa | DD |
| (12) | Fa | 3,10,LL |

**#4:**

| | | |
|---|---|---|
| (1) | ~~SHOW~~: ∀x∀y{x=y → ∀z{Rzx ↔ Rzy}} | UD2 |
| (2) | ~~SHOW~~: a=b → ∀z{Rza ↔ Rzb} | CD |
| (3) | a=b | As |
| (4) | ~~SHOW~~: ∀z{Rza ↔ Rzb} | UD |
| (5) | ~~SHOW~~: Rca ↔ Rcb | 6,10,↔I |
| (6) | ~~SHOW~~: Rca → Rcb | CD |
| (7) | Rca | As |
| (8) | ~~SHOW~~: Rcb | DD |
| (9) | Rcb | 3,7,LL |
| (10) | ~~SHOW~~: Rcb → Rca | CD |
| (11) | Rcb | As |
| (12) | ~~SHOW~~: Rca | DD |
| (13) | Rca | 3,11,LL |

**#5:**

| | | |
|---|---|---|
| (1) | ~~SHOW~~: $\forall x\forall y\{x=y\rightarrow f(x)=f(y)\}$ | UD2 |
| (2) | \| ~~SHOW~~: $a=b\rightarrow f(a)=f(b)$ | CD |
| (3) | \| \| $a=b$ | As |
| (4) | \| \| ~~SHOW~~: $f(a)=f(b)$ | DD |
| (5) | \| \| $f(a)=f(a)$ | REF= |
| (6) | \| \| $f(a)=f(b)$3,5,LL | |

**#6:**

| | | |
|---|---|---|
| (1) | ~~SHOW~~: $\forall w\forall x\forall y\forall z\{[w=x\&y=z]\rightarrow f(w,y)=f(x,z)\}$ | UD4 |
| (2) | \| ~~SHOW~~: $[a=p\ \&\ b=q]\rightarrow f(a,b)=f(p,q)$ | CD |
| (3) | \| \| $a=p\ \&\ b=q$ | As |
| (4) | \| \| ~~SHOW~~: $f(a,b)=f(p,q)$ | DD |
| (5) | \| \| \| $f(a,b)=f(a,b)$ | REF= |
| (6) | \| \| \| $a=p$ | 3,SL |
| (7) | \| \| \| $f(a,b)=f(p,b)$ | 5,6,LL |
| (8) | \| \| \| $b=q$ | 3,SL |
| (9) | \| \| \| $f(a,b)=f(p,q)$ | 7,8,LL |

**#7a:**

| | | |
|---|---|---|
| (1) | $a=b$ | Pr |
| (2) | ~~SHOW~~: $\forall x\{x=a\rightarrow x=b\}$ | UD |
| (3) | \| ~~SHOW~~: $c=a\rightarrow c=b$ | CD |
| (4) | \| \| $c=a$ | As |
| (5) | \| \| ~~SHOW~~: $c=b$ | DD |
| (6) | \| \| \| $c=b$ | 1,4,LL |

**#7b:**

| | | |
|---|---|---|
| (1) | $\forall x(x=a\rightarrow x=b)$ | Pr |
| (2) | ~~SHOW~~: $a=b$ | DD |
| (3) | \| $a=a\rightarrow a=b$ | 1,∀O |
| (4) | \| $a=a$ | REF= |
| (5) | \| $a=b$ | 3,4,SL |

**#8a:**

| | | |
|---|---|---|
| (1) | $a=b$ | Pr |
| (2) | ~~SHOW~~: $\exists x(x=a\ \&\ x=b)$ | 4,∃I |
| (3) | \| $a=a$ | REF= |
| (4) | \| $a=a\ \&\ a=b$ | 1,3,SL |

**#8b:**

| | | |
|---|---|---|
| (1) | $\exists x(x=a\ \&\ x=b)$ | Pr |
| (2) | ~~SHOW~~: $a=b$ | DD |
| (3) | \| $c=a\ \&\ c=b$ | 1,∃O |
| (4) | \| $a=b$ | 3a,3b,LL |

**#9:**

| | | |
|---|---|---|
| (1) | ~~SHOW~~: $\forall x\{Fx \leftrightarrow \forall y[y=x \rightarrow Fy]\}$ | UD |
| (2) | ~~SHOW~~: $Fa \leftrightarrow \forall y(y=a \rightarrow Fy)$ | 3,10,↔I |
| (3) | ~~SHOW~~: $Fa \rightarrow \forall y(y=a \rightarrow Fy)$ | CD |
| (4) | $Fa$ | As |
| (5) | ~~SHOW~~: $\forall y(y=a \rightarrow Fy)$ | UD |
| (6) | ~~SHOW~~: $b=a \rightarrow Fb$ | CD |
| (7) | $b=a$ | As |
| (8) | ~~SHOW~~: $Fb$ | DD |
| (9) | $Fb$ | 4,7,LL |
| (10) | ~~SHOW~~: $\forall y(y=a \rightarrow Fy) \rightarrow Fa$ | CD |
| (11) | $\forall y(y=a \rightarrow Fy)$ | As |
| (12) | ~~SHOW~~: $Fa$ | DD |
| (13) | $a=a \rightarrow Fa$ | 11,∀O |
| (14) | $a=a$ | REF= |
| (15) | $Fa$ | 13,14,SL |

**#10:**

| | | |
|---|---|---|
| (1) | ~~SHOW~~: $\forall x\{Fx \leftrightarrow \exists y[y=x \;\&\; Fy]\}$ | UD |
| (2) | ~~SHOW~~: $Fa \leftrightarrow \exists y(y=a \;\&\; Fy)$ | 3,8,↔I |
| (3) | ~~SHOW~~: $Fa \rightarrow \exists y(y=a \;\&\; Fy)$ | CD |
| (4) | $Fa$ | As |
| (5) | ~~SHOW~~: $\exists y(y=a \;\&\; Fy)$ | ∃D |
| (6) | $a=a$ | REF= |
| (7) | $a=a \;\&\; Fa$ | 4,6,SL |
| (8) | ~~SHOW~~: $\exists y(y=a \;\&\; Fy) \rightarrow Fa$ | CD |
| (9) | $\exists y(y=a \;\&\; Fy)$ | As |
| (10) | ~~SHOW~~: $Fa$ | DD |
| (11) | $b=a \;\&\; Fb$ | 10,∃O |
| (12) | $Fa$ | 11a,11b,SL |

**#11:**

| | | |
|---|---|---|
| (1) | $\forall x(Fx \rightarrow Gx)$ | Pr |
| (2) | ~~SHOW~~: $\forall x(Fx \rightarrow \exists y[Gx \;\&\; (h(x)=h(y))])$ | UD |
| (3) | ~~SHOW~~: $Fa \rightarrow \exists y[Ga \;\&\; (h(a)=h(y))]$ | CD |
| (4) | $Fa$ | As |
| (5) | ~~SHOW~~: $\exists y[Ga \;\&\; (h(a)=h(y))]$ | DD |
| (6) | $Fa \rightarrow Ga$ | 1,∀O |
| (7) | $Ga$ | 4,6,SL |
| (8) | $h(a)=h(a)$ | REF= |
| (9) | $Ga \;\&\; (h(a)=h(a))$ | 7,8,SL |
| (10) | $\exists y[Ga \;\&\; (h(a)=h(y))]$ | 9,∃I |

**#12:**

| | | |
|---|---|---|
| (1) | $\exists x \forall y\{Fy \rightarrow y=x\}$ | Pr |
| (2) | $\exists x\{Fx \,\&\, Gx\}$ | Pr |
| (3) | ~~SHOW~~: $\forall x\{Fx \rightarrow Gx\}$ | UD |
| (4) | ~~SHOW~~: $Fa \rightarrow Ga$ | CD |
| (5) | $Fa$ | As |
| (6) | ~~SHOW~~: $Ga$ | DD |
| (7) | $\forall y(Fy \rightarrow y=b)$ | 1,∃O |
| (8) | $Fc \,\&\, Gc$ | 2,∃O |
| (9) | $Fc$ | 8,SL |
| (10) | $Gc$ | 8,SL |
| (11) | $Fa \rightarrow a=b$ | 7,∀O |
| (12) | $Fc \rightarrow c=b$ | 7,∀O |
| (13) | $a=b$ | 5,11,SL |
| (14) | $c=b$ | 9,12,SL |
| (15) | $a=c$ | 13,14,LL |
| (16) | $Ga$ | 10,15,LL |

**#13:**

| | | |
|---|---|---|
| (1) | $\forall x R[x,b(i)]$ | Pr |
| (2) | $\forall x\{R[b(i),x] \leftrightarrow x=i\}$ | Pr |
| (3) | ~~SHOW~~: $i=b(i)$ | DD |
| (4) | $R[b(i),b(i)] \leftrightarrow b(i)=i$ | ∀O |
| (5) | $R[b(i),b(i)]$ | 1,∀O |
| (6) | $b(i)=i$ | 4,5,SL |
| (7) | $i=b(i)$ | SYM= |

**#14a:**

| | | |
|---|---|---|
| (1) | $\exists x \forall y[x=y]$ | Pr |
| (2) | ~~SHOW~~: $\forall x \forall y[x=y]$ | UD2 |
| (3) | ~~SHOW~~: $a=b$ | DD |
| (4) | $\forall y[c=y]$ | 1,∃O |
| (5) | $c=a$ | 4,∀O |
| (6) | $c=b$ | 4,∀O |
| (7) | $a=b$ | 5,6,LL |

**#14b:**

| | | |
|---|---|---|
| (1) | $\forall x \forall y[x=y]$ | Pr |
| (2) | ~~SHOW~~: $\exists x \forall y[x=y]$ | 3,∃I |
| (3) | ~~SHOW~~: $\forall y[a=y]$ | UD |
| (4) | ~~SHOW~~: $a=b$ | DD |
| (5) | $a=b$ | 1,∀O2 |

**#15a:**

| | | |
|---|---|---|
| (1) | $\exists x\exists y[x\neq y]$ | Pr |
| (2) | ~~SHOW~~: $\forall x\exists y[x\neq y]$ | UD |
| (3) | │ ~~SHOW~~: $\exists y[a\neq y]$ | ID |
| (4) | │ │ $\sim\exists y[a\neq y]$ | As |
| (5) | │ │ ~~SHOW~~: ※ | DD |
| (6) | │ │ │ $\forall y\sim[a\neq y]$ | 4,QN |
| (7) | │ │ │ $b\neq c$ | 1,∃O2 |
| (8) | │ │ │ $\sim[a\neq b]$ | 6,∀O |
| (9) | │ │ │ $a=b$ | 8,SL |
| (10) | │ │ │ $\sim[a\neq c]$ | 6,∀O |
| (11) | │ │ │ $a=c$ | 10,SL |
| (12) | │ │ │ $b=a$ | 9,SYM= |
| (13) | │ │ │ $b=c$ | 11,12,LL |
| (14) | │ │ │ ※ | 7,13,SL |

**#15b:**

| | | |
|---|---|---|
| (1) | $\forall x\exists y[x\neq y]$ | Pr |
| (2) | ~~SHOW~~: $\exists x\exists y[x\neq y]$ | DD |
| (3) | │ $\exists y[a\neq y]$ | 1,∀O |
| (4) | │ $\exists x\exists y[x\neq y]$ | 3,∃I |

**#16:**

| | | |
|---|---|---|
| (1) | $\sim\exists x\forall y[y=x]$ | Pr |
| (2) | ~~SHOW~~: $\exists x\exists y[x\neq y]$ | ID |
| (3) | │ $\sim\exists x\exists y[x\neq y]$ | As |
| (4) | │ ~~SHOW~~: ※ | 6,7,SL |
| (5) | │ │ $\sim\forall y[y=a]$ | 1,~∃O |
| (6) | │ │ $b\neq a$ | 5,~∀O |
| (7) | │ │ $\sim[b\neq a]$ | 3,~∃O2 |

**#17:**

| | | |
|---|---|---|
| (1) | $\sim\exists x\exists y[x\neq y]$ | Pr |
| (2) | ~~SHOW~~: $\exists xFx\leftrightarrow\forall xFx$ | 3,14,↔I |
| (3) | │ ~~SHOW~~: $\exists xFx\rightarrow\forall xFx$ | CD |
| (4) | │ │ $\exists xFx$ | As |
| (5) | │ │ ~~SHOW~~: $\forall xFx$ | UD |
| (6) | │ │ │ ~~SHOW~~: $Fa$ | DD |
| (7) | │ │ │ │ $\forall x\sim\exists y[x\neq y]$ | 1,~∃O |
| (8) | │ │ │ │ $\sim\exists y[a\neq y]$ | 7,∀O |
| (9) | │ │ │ │ $Fb$ | 4.∃O |
| (10) | │ │ │ │ $\forall y\sim[a\neq y]$ | 8,~∃O |
| (11) | │ │ │ │ $\sim[a\neq b]$ | 10,∀O |
| (12) | │ │ │ │ $a=b$ | 11,SL |
| (13) | │ │ │ │ $Fa$ | 9,12,LL |
| (14) | │ ~~SHOW~~: $\forall xFx\rightarrow\exists xFx$ | CD |
| (15) | │ │ $\forall xFx$ | As |
| (16) | │ │ ~~SHOW~~: $\exists xFx$ | 17,∃I |
| (17) | │ │ │ $Fa$ | 15,∀O |

**#18:**

| | | |
|---|---|---|
| (1) | ~~SHOW~~: $\forall x\exists y\forall z(z=y\leftrightarrow z=x)$ | UD |
| (2) | │ ~~SHOW~~: $\exists y\forall z(z=y\leftrightarrow z=a)$ | 3,∃I |
| (3) | │ │ ~~SHOW~~: $\forall z(z=a\leftrightarrow z=a)$ | UD |
| (4) | │ │ │ ~~SHOW~~: $b=a\leftrightarrow b=a$ | SL |

**#19:**

| | | |
|---|---|---|
| (1) | $\forall x(Fx \to Gx)$ | Pr |
| (2) | $\exists x \forall y(Gy \to y{=}x)$ | Pr |
| (3) | ~~SHOW~~: $\exists x \forall y(Fy \to y{=}x)$ | 5,∃I |
| (4) | $\vert\ \forall y(Gy \to y{=}a)$ | 2,∃O |
| (5) | $\vert\ $ ~~SHOW~~: $\forall y(Fy \to y{=}a)$ | UD |
| (6) | $\vert\ \vert\ $ ~~SHOW~~: $Fb \to b{=}a$ | CD |
| (7) | $\vert\ \vert\ \vert\ Fb$ | As |
| (8) | $\vert\ \vert\ \vert\ $ ~~SHOW~~: $b{=}a$ | DD |
| (9) | $\vert\ \vert\ \vert\ \vert\ Gb \to b{=}a$ | 4,∀O |
| (10) | $\vert\ \vert\ \vert\ \vert\ Fb \to Gb$ | 1,∀O |
| (11) | $\vert\ \vert\ \vert\ \vert\ b{=}a$ | 7,9,10,SL |

**#20:**

| | | |
|---|---|---|
| (1) | $\forall x(Fx \leftrightarrow x{\neq}a]$ | Pr |
| (2) | $\forall x(Gx \leftrightarrow x{=}a)$ | Pr |
| (3) | ~~SHOW~~: $\sim\exists x(Fx \,\&\, Gx)$ | ID |
| (4) | $\vert\ \exists x(Fx \,\&\, Gx)$ | As |
| (5) | $\vert\ $ ~~SHOW~~: ※ | DD |
| (6) | $\vert\ \vert\ Fb \,\&\, Gb$ | 4,∃O |
| (7) | $\vert\ \vert\ Fb \leftrightarrow x{\neq}a$ | 1,∀O |
| (8) | $\vert\ \vert\ Gb \leftrightarrow x{=}a$ | 2,∀O |
| (9) | $\vert\ \vert\ x{\neq}a$ | 6a,7,SL |
| (10) | $\vert\ \vert\ x{=}a$ | 6b,8,SL |
| (11) | $\vert\ \vert\ $ ※ | 9,10,SL |

**#21:**

| | | |
|---|---|---|
| (1) | $\exists x \forall y(Fy \to y{=}x) \vee \exists x \forall y(Gy \to y{=}x)$ | Pr |
| (2) | ~~SHOW~~: $\exists x \forall y([Fy \,\&\, Gy] \to y{=}x)$ | SC |
| (3) | $\vert\ c1: \exists x \forall y(Fy \to y{=}x)$ | As |
| (4) | $\vert\ $ ~~SHOW~~: $\exists x \forall y([Fy \,\&\, Gy] \to y{=}x)$ | 6,∃I |
| (5) | $\vert\ \vert\ \forall y(Fy \to y{=}a)$ | 3,∃O |
| (6) | $\vert\ \vert\ $ ~~SHOW~~: $\forall y([Fy \,\&\, Gy] \to y{=}a)$ | UD |
| (7) | $\vert\ \vert\ \vert\ $ ~~SHOW~~: $[Fb \,\&\, Gb] \to b{=}a$ | CD |
| (8) | $\vert\ \vert\ \vert\ \vert\ Fb \,\&\, Gb$ | As |
| (9) | $\vert\ \vert\ \vert\ \vert\ $ ~~SHOW~~: $b{=}a$ | DD |
| (10) | $\vert\ \vert\ \vert\ \vert\ \vert\ Fb \to b{=}a$ | 5,∀O |
| (11) | $\vert\ \vert\ \vert\ \vert\ \vert\ b{=}a$ | 8a,10,SL |
| (12) | $\vert\ c2: \exists x \forall y(Gy \to y{=}x)$ | As |
| (13) | $\vert\ $ ~~SHOW~~: $\exists x \forall y([Fy \,\&\, Gy] \to y{=}x)$ | 15,∃I |
| (14) | $\vert\ \vert\ \forall y(Gy \to y{=}a)$ | 12,∃O |
| (15) | $\vert\ \vert\ $ ~~SHOW~~: $\forall y([Fy \,\&\, Gy] \to y{=}a)$ | UD |
| (16) | $\vert\ \vert\ \vert\ $ ~~SHOW~~: $[Fb \,\&\, Gb] \to b{=}a$ | CD |
| (17) | $\vert\ \vert\ \vert\ \vert\ Fb \,\&\, Gb$ | As |
| (18) | $\vert\ \vert\ \vert\ \vert\ $ ~~SHOW~~: $b{=}a$ | DD |
| (19) | $\vert\ \vert\ \vert\ \vert\ \vert\ Gb \to b{=}a$ | 14,∀O |
| (20) | $\vert\ \vert\ \vert\ \vert\ \vert\ b{=}a$ | 17b,19,SL |

**#22:**

| | | |
|---|---|---|
| (1) | $\sim\exists x\exists y[f(x)\neq f(y)]$ | Pr |
| (2) | ~~SHOW~~: $\exists x\forall y[f(y)=x]$ | 3,∃I |
| (3) | │ ~~SHOW~~: $\forall y[f(y)=f(a)]$ | UD |
| (4) | │ │ ~~SHOW~~: $f(b)=f(a)$ | ID |
| (5) | │ │ │ $f(b)\neq f(a)$ | As |
| (6) | │ │ │ ~~SHOW~~: ※ | DD |
| (7) | │ │ │ │ $\sim[f(b)\neq f(a)]$ | 1,~∃O2 |
| (8) | │ │ │ │ ※ | 5,7,SL |

**#23:**

| | | |
|---|---|---|
| (1) | $\exists x\forall y\{Fy\rightarrow y=x\}$ | Pr |
| (2) | ~~SHOW~~: $\forall x\{Fx\rightarrow\forall y(Fy\leftrightarrow y=x)\}$ | UD |
| (3) | │ ~~SHOW~~: $Fa\rightarrow\forall y(Fy\leftrightarrow y=a)$ | CD |
| (4) | │ │ $Fa$ | As |
| (5) | │ │ ~~SHOW~~: $\forall y(Fy\leftrightarrow y=a)$ | UD |
| (6) | │ │ │ ~~SHOW~~: $Fb\leftrightarrow b=a$ | 7,14,↔I |
| (7) | │ │ │ │ ~~SHOW~~: $Fb\rightarrow b=a$ | CD |
| (8) | │ │ │ │ │ $Fb$ | As |
| (9) | │ │ │ │ │ ~~SHOW~~: $b=a$ | DD |
| (10) | │ │ │ │ │ │ $\forall y\{Fy\rightarrow y=c\}$ | 1,∃O |
| (11) | │ │ │ │ │ │ $a=c$ | 4,10,∀→O |
| (12) | │ │ │ │ │ │ $b=c$ | 10,11,∀→O |
| (13) | │ │ │ │ │ │ $b=a$ | 11,12,LL |
| (14) | │ │ │ │ ~~SHOW~~: $b=a\rightarrow Fb$ | CD |
| (15) | │ │ │ │ │ $b=a$ | As |
| (16) | │ │ │ │ │ ~~SHOW~~: $Fb$ | DD |
| (17) | │ │ │ │ │ │ $Fb$ | 4,15,LL |

**#24:**

| | | |
|---|---|---|
| (1) | $\exists x\forall y\{[Fy\,\&\,Gy]\leftrightarrow y=x\}$ | Pr |
| (2) | ~~SHOW~~: $\exists x(Fx\,\&\,Gx)$ | 6,∃I |
| (3) | │ $\forall y\{[Fy\,\&\,Gy]\leftrightarrow y=a\}$ | 1,∃O |
| (4) | │ $[Fa\,\&\,Ga]\leftrightarrow a=a$ | 3,∀O |
| (5) | │ $a=a$ | REF= |
| (6) | │ $Fa\,\&\,Ga$ | 4,5,SL |

**#25a:**

| | | |
|---|---|---|
| (1) | $\exists x\forall y(Fy\leftrightarrow y=x)$ | Pr |
| (2) | ~~SHOW~~: $\exists x\{Fx\,\&\,\forall y[Fy\rightarrow y=x]\}$ | 4,∃I |
| (3) | │ $\forall y(Fy\leftrightarrow y=a)$ | 1,∃O |
| (4) | │ ~~SHOW~~: $Fa\,\&\,\forall y[Fy\rightarrow y=a]$ | 5,9,&I |
| (5) | │ │ ~~SHOW~~: $Fa$ | DD |
| (6) | │ │ │ $Fa\leftrightarrow a=a$ | 3,∀O |
| (7) | │ │ │ $a=a$ | REF= |
| (8) | │ │ │ $Fa$ | 6,7,SL |
| (9) | │ │ ~~SHOW~~: $\forall y[Fy\rightarrow y=a]$ | UD |
| (10) | │ │ │ ~~SHOW~~: $Fb\rightarrow b=a$ | CD |
| (11) | │ │ │ │ $Fb$ | As |
| (12) | │ │ │ │ ~~SHOW~~: $b=a$ | DD |
| (13) | │ │ │ │ │ $Fb\leftrightarrow b=a$ | 3,∀O |
| (14) | │ │ │ │ │ $b=a$ | 11,13,SL |

**#25b:**

| | | |
|---|---|---|
| (1) | $\exists x\{Fx \,\&\, \forall y[Fy \rightarrow y{=}x]\}$ | Pr |
| (2) | ~~SHOW~~: $\exists x \forall y(Fy \leftrightarrow y{=}x)$ | 4,∃I |
| (3) | │ $Fa \,\&\, \forall y[Fy \rightarrow y{=}a]$ | 1,∃O |
| (4) | │ ~~SHOW~~: $\forall y(Fy \leftrightarrow y{=}a)$ | UD |
| (5) | │ │ ~~SHOW~~: $Fb \leftrightarrow b{=}a$ | 6,8↔I |
| (6) | │ │ │ ~~SHOW~~: $Fb \rightarrow b{=}a$ | DD |
| (7) | │ │ │ │ $Fb \rightarrow b{=}a$ | 3b,∀O |
| (8) | │ │ │ ~~SHOW~~: $b{=}a \rightarrow Fb$ | CD |
| (9) | │ │ │ │ $b{=}a$ | As |
| (10) | │ │ │ │ ~~SHOW~~: $Fb$ | DD |
| (11) | │ │ │ │ │ $Fb$ | 3a,9,LL |

**#26:**

| | | |
|---|---|---|
| (1) | $\exists x Fx$ | Pr |
| (2) | $\forall x \forall y\{(Fx \,\&\, Fy) \rightarrow x{=}y\}$ | Pr |
| (3) | ~~SHOW~~: $\exists x \forall y\{Fy \leftrightarrow y{=}x\}$ | 3,∃I |
| (4) | │ $Fa$ | 1,∃O |
| (5) | │ ~~SHOW~~: $\forall y\{Fy \leftrightarrow y{=}a\}$ | UD |
| (6) | │ │ ~~SHOW~~: $Fb \leftrightarrow b{=}a$ | 7,12,↔I |
| (7) | │ │ │ ~~SHOW~~: $Fb \rightarrow b{=}a$ | CD |
| (8) | │ │ │ │ $Fb$ | As |
| (9) | │ │ │ │ ~~SHOW~~: $b{=}a$ | DD |
| (10) | │ │ │ │ │ $(Fb \,\&\, Fa) \rightarrow b{=}a$ | 2,∀O2 |
| (11) | │ │ │ │ │ $b{=}a$ | 4,8,10,SL |
| (12) | │ │ │ ~~SHOW~~: $b{=}a \rightarrow Fb$ | CD |
| (13) | │ │ │ │ $b{=}a$ | As |
| (14) | │ │ │ │ ~~SHOW~~: $Fb$ | DD |
| (15) | │ │ │ │ │ $Fb$ | 4,13,LL |

**#27:**

| | | |
|---|---|---|
| (1) | $\exists x Fx$ | Pr |
| (2) | $\sim\exists x \exists y(x{\neq}y \,\&\, [Fx \,\&\, Fy])$ | Pr |
| (3) | ~~SHOW~~: $\exists x[Fx \,\&\, \sim\exists y(y{\neq}x \,\&\, Fy)]$ | 5,∃I |
| (4) | │ $Fa$ | 1,∃O |
| (5) | │ ~~SHOW~~: $Fa \,\&\, \sim\exists y(y{\neq}a \,\&\, Fy)$ | DD |
| (6) | │ │ ~~SHOW~~: $\sim\exists y(y{\neq}a \,\&\, Fy)$ | ID |
| (7) | │ │ │ $\exists y(y{\neq}a \,\&\, Fy)$ | As |
| (8) | │ │ │ ~~SHOW~~: ⋇ | DD |
| (9) | │ │ │ │ $b{\neq}a \,\&\, Fb$ | 7,∃O |
| (10) | │ │ │ │ $\forall x\sim\exists y(x{\neq}y \,\&\, [Fx \,\&\, Fy])$ | 2,~∃O |
| (11) | │ │ │ │ $\sim\exists y(b{\neq}y \,\&\, [Fb \,\&\, Fy])$ | 10,∀O |
| (12) | │ │ │ │ $\forall y\sim(b{\neq}y \,\&\, [Fb \,\&\, Fy])$ | 11,~∃O |
| (13) | │ │ │ │ $\sim(b{\neq}a \,\&\, [Fb \,\&\, Fa])$ | 12,∀O |
| (14) | │ │ │ │ $\sim(Fb \,\&\, Fa)$ | 9a,13,SL |
| (15) | │ │ │ │ $\sim Fa$ | 9b,14,SL |
| (16) | │ │ │ │ ⋇ | 4,15,SL |
| (17) | │ │ $Fa \,\&\, \sim\exists y(y{\neq}a \,\&\, Fy)$ | 4,6,SL |

**#28:**

| | | |
|---|---|---|
| (1) | $\exists x Fx$ | Pr |
| (2) | $\sim\exists x\exists y(x{\neq}y \,\&\, [Fx \,\&\, Fy])$ | Pr |
| (3) | ~~SHOW~~: $\exists x\forall y(Fy \leftrightarrow y{=}x)$ | 5,∃I |
| (4) | $Fa$ | 1,∃O |
| (5) | ~~SHOW~~: $\forall y(Fy \leftrightarrow y{=}a)$ | UD |
| (6) | ~~SHOW~~: $Fb \leftrightarrow b{=}a$ | 11,19↔I |
| (7) | $\forall x{\sim}\exists y(x{\neq}y \,\&\, [Fx \,\&\, Fy])$ | 2,~∃O |
| (8) | $\sim\exists y(b{\neq}y \,\&\, [Fb \,\&\, Fy])$ | 7,∀O |
| (9) | $\forall y{\sim}(b{\neq}y \,\&\, [Fb \,\&\, Fy])$ | 8,~∃O |
| (10) | $\sim(b{\neq}a \,\&\, [Fb \,\&\, Fa])$ | 9,∀O |
| (11) | ~~SHOW~~: $Fb \rightarrow b{=}a$ | CD |
| (12) | $Fb$ | As |
| (13) | ~~SHOW~~: $b{=}a$ | ID |
| (14) | $b{\neq}a$ | As |
| (15) | ~~SHOW~~: ※ | DD |
| (16) | $\sim(Fb \,\&\, Fa)$ | 10,14,SL |
| (17) | $\sim Fa$ | 12,16,SL |
| (18) | ※ | 4,17,SL |
| (19) | ~~SHOW~~: $b{=}a \rightarrow Fb$ | CD |
| (20) | $b{=}a$ | As |
| (21) | ~~SHOW~~: $Fb$ | DD |
| (22) | $\sim Fb$ | As |
| (23) | ~~SHOW~~: ※ | DD |
| (24) | $Fb$ | 4,20,LL |
| (25) | ※ | 22,24,SL |

**#29a:**

| | | |
|---|---|---|
| (1) | $\sim\exists x\forall y\{Fy \leftrightarrow y{=}x\}$ | Pr |
| (2) | ~~SHOW~~: $\exists x Fx \rightarrow \exists x\exists y(x{\neq}y \,\&\, Fx \,\&\, Fy)$ | CD |
| (3) | $\exists x Fx$ | As |
| (4) | ~~SHOW~~: $\exists x\exists y(x{\neq}y \,\&\, Fx \,\&\, Fy)$ | 18,∃I |
| (5) | $Fa$ | 3,∃O |
| (6) | $\forall x{\sim}\forall y(Fy \leftrightarrow y{=}x)$ | 1,~∃O |
| (7) | $\sim\forall y(Fy \leftrightarrow y{=}a)$ | 6,∀O |
| (8) | $\exists y{\sim}(Fy \leftrightarrow y{=}a)$ | 7,~∀O |
| (9) | $\sim(Fb \leftrightarrow b{=}a)$ | 8,∃O |
| (10) | $\sim Fb \leftrightarrow b{=}a$ | 9,SL |
| (11) | ~~SHOW~~: $Fb$ | ID |
| (12) | $\sim Fb$ | As |
| (13) | ~~SHOW~~: ※ | DD |
| (14) | $b{=}a$ | 10,12,SL |
| (15) | $Fb$ | 5,14,LL |
| (16) | ※ | 12,15,SL |
| (17) | $b{\neq}a$ | 10,11,SL |
| (18) | $b{\neq}a \,\&\, Fb \,\&\, Fa$ | 5,11,17,SL |

**#29b:**

| | | |
|---|---|---|
| (1) | $\exists xFx \rightarrow \exists x\exists y(x{\neq}y \,\&\, Fx \,\&\, Fy)$ | Pr |
| (2) | ~~SHOW~~: $\sim\exists x\forall y(Fy \leftrightarrow y{=}x)$ | ID |
| (3) | $\exists x\forall y(Fy \leftrightarrow y{=}x)$ | As |
| (4) | ~~SHOW~~: ※ | DD |
| (5) | $\forall y(Fy \leftrightarrow y{=}a)$ | 3,∃O |
| (6) | $Fa \leftrightarrow a{=}a$ | 5,∀O |
| (7) | $a{=}a$ | REF= |
| (8) | $Fa$ | 6,7,SL |
| (9) | $\exists xFx$ | 8,∃I |
| (10) | $\exists x\exists y(x{\neq}y \,\&\, Fx \,\&\, Fy)$ | 1,9,SL |
| (11) | $b{\neq}c \,\&\, Fb \,\&\, Fc$ | 10,∃O2 |
| (12) | $Fb \leftrightarrow b{=}a$ | 5,∀O |
| (13) | $b{=}a$ | 11b,12,SL |
| (14) | $Fc \leftrightarrow c{=}a$ | 5,∀O |
| (15) | $c{=}a$ | 11c,14,SL |
| (16) | $b{=}c$ | 13,15,LL |
| (17) | ※ | 11a,16,SL |

**#30:**

| | | |
|---|---|---|
| (1) | $\exists x\forall y(Fy \leftrightarrow y{=}x)$ | Pr |
| (2) | ~~SHOW~~: $\forall x(Fx \rightarrow Gx) \leftrightarrow \exists x(Fx \,\&\, Gx)$ | 3,13,↔I |
| (3) | ~~SHOW~~: $\forall x(Fx \rightarrow Gx) \rightarrow \exists x(Fx \,\&\, Gx)$ | CD |
| (4) | $\forall x(Fx \rightarrow Gx)$ | As |
| (5) | ~~SHOW~~: $\exists x(Fx \,\&\, Gx)$ | ∃D |
| (6) | $\forall y(Fy \leftrightarrow y{=}a)$ | 1,∃O |
| (7) | $Fa \leftrightarrow a{=}a$ | 6,∀O |
| (8) | $a{=}a$ | REF= |
| (9) | $Fa$ | 7,8,SL |
| (10) | $Fa \rightarrow Ga$ | 4,∀O |
| (11) | $Ga$ | 9,10,SL |
| (12) | $Fa \,\&\, Ga$ | 9,11,SL |
| (13) | ~~SHOW~~: $\exists x(Fx \,\&\, Gx) \rightarrow \forall x(Fx \rightarrow Gx)$ | CD |
| (14) | $\exists x(Fx \,\&\, Gx)$ | As |
| (15) | ~~SHOW~~: $\forall x(Fx \rightarrow Gx)$ | UD |
| (16) | ~~SHOW~~: $Fa \rightarrow Ga$ | CD |
| (17) | $Fa$ | As |
| (18) | ~~SHOW~~: $Ga$ | DD |
| (19) | $Fb \,\&\, Gb$ | 14,∃O |
| (20) | $\forall y(Fy \leftrightarrow y{=}c)$ | 1,∃O |
| (21) | $Fb \leftrightarrow b{=}c$ | 20,∀O |
| (22) | $b{=}c$ | 19a,21,SL |
| (23) | $Fa \leftrightarrow a{=}c$ | 20,∀O |
| (24) | $a{=}c$ | 17,23,SL |
| (25) | $a{=}b$ | 22,24,LL |
| (26) | $Ga$ | 19b,25,LL |

**#31:**

| | | |
|---|---|---|
| (1) | $\forall x(Fx \to Gx)$ | Pr |
| (2) | $\exists x\forall y(Gy \leftrightarrow y=x)$ | Pr |
| (3) | ~~SHOW~~: $\exists xFx \to \exists x\forall y(Fy \leftrightarrow y=x)$ | CD |
| (4) | $\exists xFx$ | As |
| (5) | ~~SHOW~~: $\exists x\forall y(Fy \leftrightarrow y=x)$ | 7,∃I |
| (6) | $Fa$ | 4,∃O |
| (7) | ~~SHOW~~: $\forall y(Fy \leftrightarrow y=a)$ | UD |
| (8) | ~~SHOW~~: $Fb \leftrightarrow b=a$ | 9,22↔I |
| (9) | ~~SHOW~~: $Fb \to b=a$ | CD |
| (10) | $Fb$ | As |
| (11) | ~~SHOW~~: $b=a$ | DD |
| (12) | $\forall y(Gy \leftrightarrow y=c)$ | 2,∃O |
| (13) | $Fa \to Ga$ | 1,∀O |
| (14) | $Ga$ | 6,13,SL |
| (15) | $Ga \leftrightarrow a=c$ | 12,∀O |
| (16) | $a=c$ | 14,15,SL |
| (17) | $Fb \to Gb$ | 1,∀O |
| (18) | $Gb$ | 10,17,SL |
| (19) | $Gb \leftrightarrow b=c$ | 12,∀O |
| (20) | $b=c$ | 18,19,SL |
| (21) | $b=a$ | 16,20,LL |
| (22) | ~~SHOW~~: $b=a \to Fb$ | CD |
| (23) | $b=a$ | As |
| (24) | ~~SHOW~~: $Fb$ | DD |
| (25) | $Fb$ | 6,23,LL |

**#32a:**

| | | |
|---|---|---|
| (1) | $\forall x(Fx \to x=a)$ | Pr |
| (2) | ~~SHOW~~: $\exists xFx \to \forall x(Fx \leftrightarrow x=a)$ | CD |
| (3) | $\exists xFx$ | As |
| (4) | ~~SHOW~~: $\forall x(Fx \leftrightarrow x=a)$ | UD |
| (5) | ~~SHOW~~: $Fb \leftrightarrow b=a$ | 6,8↔I |
| (6) | ~~SHOW~~: $Fb \to b=a$ | CD |
| (7) | $Fb \to b=a$ | 1,∀O |
| (8) | ~~SHOW~~: $b=a \to Fb$ | CD |
| (9) | $b=a$ | As |
| (10) | ~~SHOW~~: $Fb$ | DD |
| (11) | $Fc$ | 3,∃O |
| (12) | $Fc \to c=a$ | 1,∀O |
| (13) | $c=a$ | 11,12,SL |
| (14) | $Fa$ | 11,13,LL |
| (15) | $Fb$ | 9,14,LL |

**#32b:**

| | | |
|---|---|---|
| (1) | $\exists xFx \to \forall x(Fx \leftrightarrow x=a)$ | Pr |
| (2) | ~~SHOW~~: $\forall x(Fx \to x=a)$ | UD |
| (3) | ~~SHOW~~: $Fb \to b=a$ | CD |
| (4) | $Fb$ | As |
| (5) | ~~SHOW~~: $b=a$ | DD |
| (6) | $\exists xFx$ | 4,∃I |
| (7) | $\forall x(Fx \leftrightarrow x=a)$ | 1,6,SL |
| (8) | $Fb \leftrightarrow b=a$ | 7,∀O |
| (9) | $b=a$ | 4,8,SL |

**#33a:**

[done using separation of cases]

| | | |
|---|---|---|
| (1) | $\forall x \forall y\{[Fx \,\&\, Fy] \rightarrow x=y\}$ | Pr |
| (2) | ~~SHOW~~: $\exists x \forall y\{Fy \rightarrow y=x\}$ | 3,SC |
| (3) | │ $\exists xFx \vee {\sim}\exists xFx$ | SL |
| (4) | │ │ $c1 : \exists xFx$ | As |
| (5) | │ │ ~~SHOW~~: $\exists x \forall y\{Fy \rightarrow y=x\}$ | 7,∃I |
| (6) | │ │ │ $Fa$ | 4,∃O |
| (7) | │ │ │ ~~SHOW~~: $\forall y\{Fy \rightarrow y=a\}$ | UCD |
| (8) | │ │ │ │ $Fb$ | As |
| (9) | │ │ │ │ ~~SHOW~~: $b=a$ | DD |
| (10) | │ │ │ │ │ $[Fb \,\&\, Fa] \rightarrow b=a$ | 1,∀O |
| (11) | │ │ │ │ │ $b=a$ | 6,9,11,SL |
| (12) | │ │ $c2$: ${\sim}\exists xFx$ | As |
| (13) | │ │ ~~SHOW~~: $\exists x \forall y\{Fy \rightarrow y=x\}$ | ID |
| (14) | │ │ │ ${\sim}\exists x \forall y\{Fy \rightarrow y=x\}$ | As |
| (15) | │ │ │ ~~SHOW~~: ※ | 18,19,SL |
| (16) | │ │ │ │ ${\sim}\forall y\{Fy \rightarrow y=a\}$ | 14,~∃O |
| (17) | │ │ │ │ ${\sim}\{Fc \rightarrow c=a\}$ | 16,~∀O |
| (18) | │ │ │ │ $Fc$ | 17,SL |
| (19) | │ │ │ │ ${\sim}Fc$ | 12,~∃O |

**#33a:**

[done using simple ID]

| | | |
|---|---|---|
| (1) | $\forall x \forall y\{[Fx \,\&\, Fy] \rightarrow x=y\}$ | Pr |
| (2) | ~~SHOW~~: $\exists x \forall y\{Fy \rightarrow y=x\}$ | ID |
| (3) | │ ${\sim}\exists x \forall y\{Fy \rightarrow y=x\}$ | As |
| (4) | │ ~~SHOW~~: ※ | 10b,12 |
| (5) | │ │ ${\sim}\forall y\{Fy \rightarrow y=a\}$ | 3,~∃O |
| (6) | │ │ ${\sim}\{Fb \rightarrow b=a\}$ | 5,~∀O |
| (7) | │ │ $Fb \,\&\, b\neq a$ | 6,SL |
| (8) | │ │ ${\sim}\forall y\{Fy \rightarrow y=b\}$ | 3,~∃O |
| (9) | │ │ ${\sim}\{Fc \rightarrow c=b\}$ | 8,~∀O |
| (10) | │ │ $Fc \,\&\, c\neq b$ | 9,SL |
| (11) | │ │ $[Fc \,\&\, Fb] \rightarrow c=b$ | 1,∀O |
| (12) | │ │ $c=b$ | 7a,10a,11,SL |

**#33b:**

| | | |
|---|---|---|
| (1) | $\exists x \forall y\{Fy \rightarrow y=x\}$ | Pr |
| (2) | ~~SHOW~~: $\forall x \forall y\{[Fx \,\&\, Fy] \rightarrow x=y\}$ | UD2 |
| (3) | │ ~~SHOW~~: $[Fa \,\&\, Fb] \rightarrow a=b$ | CD |
| (4) | │ │ $Fa \,\&\, Fb$ | As |
| (5) | │ │ ~~SHOW~~: $a=b$ | DD |
| (6) | │ │ $\forall y\{Fy \rightarrow y=c\}$ | 1,∃O |
| (7) | │ │ $Fa \rightarrow a=c$ | 6,∀O |
| (8) | │ │ $Fb \rightarrow b=c$ | 6,∀O |
| (9) | │ │ $a=c$ | 4,7,SL |
| (10) | │ │ $b=c$ | 4,8,SL |
| (11) | │ │ $a=b$ | 9,10,LL |

**#34:**

| | | |
|---|---|---|
| (1) | $\exists xFx$ | Pr |
| (2) | $\sim\exists x\exists y(x{\neq}y\ \&\ [Fx\ \&\ Fy])$ | Pr |
| (3) | ~~SHOW~~: $\exists x\forall y\{Fy \leftrightarrow y{=}x\}$ | 5,∃I |
| (4) | $Fa$ | 1,∃O |
| (5) | ~~SHOW~~: $\forall y(Fy \leftrightarrow y{=}a)$ | UD |
| (6) | ~~SHOW~~: $Fb \leftrightarrow b{=}a$ | 11,14↔I |
| (7) | $\forall x{\sim}\exists y(x{\neq}y\ \&\ [Fx\ \&\ Fy])$ | 2,QN |
| (8) | $\sim\exists y(a{\neq}y\ \&\ [Fa\ \&\ Fy])$ | 7,∀O |
| (9) | $\forall y{\sim}(a{\neq}y\ \&\ [Fa\ \&\ Fy])$ | 8,QN |
| (10) | $\sim(a{\neq}b\ \&\ [Fa\ \&\ Fb])$ | 9,∀O |
| (11) | ~~SHOW~~: $Fb \rightarrow b{=}a$ | CD |
| (12) | $Fb$ | As |
| (13) | $b{=}a$ | 4,10,12,SL |
| (14) | ~~SHOW~~: $b{=}a \rightarrow Fb$ | CD |
| (15) | $b{=}a$ | As |
| (16) | $Fb$ | 4,15,LL |

**#35:**

| | | |
|---|---|---|
| (1) | $\exists x\forall y\{Fy \leftrightarrow y{=}x\}$ | Pr |
| (2) | ~~SHOW~~: $\exists x\{Fx\ \&\ {\sim}\exists y[y{\neq}x\ \&\ Fy]\}$ | 16,∃I |
| (3) | $\forall y(Fy \leftrightarrow y{=}a)$ | 1,∀O |
| (4) | $Fa \leftrightarrow a{=}a$ | 3,∀O |
| (5) | $a{=}a$ | REF= |
| (6) | $Fa$ | 4,5,SL |
| (7) | ~~SHOW~~: $\sim\exists y[y{\neq}a\ \&\ Fy]$ | ID |
| (8) | $\exists y[y{\neq}a\ \&\ Fy]$ | As |
| (9) | ~~SHOW~~: ※ | DD |
| (10) | $b{\neq}a\ \&\ Fb$ | 8,∃O |
| (11) | $b{\neq}a$ | 10,SL |
| (12) | $Fb$ | 10,SL |
| (13) | $Fb \leftrightarrow b{=}a$ | 3,∀O |
| (14) | $b{=}a$ | 12,13,SL |
| (15) | ※ | 11,14,SL |
| (16) | $Fa\ \&\ {\sim}\exists y[y{\neq}a\ \&\ Fy]$ | 6,7,SL |

**#36:**

| | | |
|---|---|---|
| (1) | $\exists x\{\forall y(Gy \leftrightarrow y{=}x)\ \&\ \forall y(Fy \rightarrow Ryx)\}$ | Pr |
| (2) | ~~SHOW~~: $\forall x\forall y([Fx\ \&\ Gy] \rightarrow Rxy)$ | UD2 |
| (3) | ~~SHOW~~: $[Fa\ \&\ Gb] \rightarrow Rab$ | CD |
| (4) | $Fa\ \&\ Gb$ | As |
| (5) | ~~SHOW~~: $Rab$ | DD |
| (6) | $\forall y\{Gy \leftrightarrow y{=}c\}\ \&\ \forall y\{Fy \rightarrow Ryc\}$ | 1,∃O |
| (7) | $\forall y\{Gy \leftrightarrow y{=}c\}$ | 6,SL |
| (8) | $\forall y\{Fy \rightarrow Ryc\}$ | 6,SL |
| (9) | $Gb \leftrightarrow b{=}c$ | 7,∀O |
| (10) | $b{=}c$ | 4,9,SL |
| (11) | $Fa \rightarrow Rac$ | 8,∀O |
| (12) | $Rac$ | 4,11,SL |
| (13) | $Rab$ | 10,12,LL |

**#37:**

| | | |
|---|---|---|
| (1) | $\exists x \forall y(Fy \leftrightarrow y{=}x)$ | Pr |
| (2) | $\forall x(Fx \rightarrow Gx)$ | Pr |
| (3) | ~~SHOW~~: $\exists x \forall y([Fy \,\&\, Gy] \leftrightarrow y{=}x)$ | 5,∃I |
| (4) | $\forall y(Fy \leftrightarrow y{=}a)$ | 1,∃O |
| (6) | ~~SHOW~~: $\forall y([Fy \,\&\, Gy] \leftrightarrow y{=}a)$ | UD |
| (7) | ~~SHOW~~: $(Fb \,\&\, Gb) \leftrightarrow b{=}a$ | 8,13,↔I |
| (8) | ~~SHOW~~: $(Fb \,\&\, Gb) \rightarrow b{=}a$ | CD |
| (9) | $Fb \,\&\, Gb$ | As |
| (10) | ~~SHOW~~: $b{=}a$ | DD |
| (11) | $Fb \leftrightarrow b{=}a$ | 4,∀O |
| (12) | $b{=}a$ | 9a,11,SL |
| (13) | ~~SHOW~~: $b{=}a \rightarrow (Fb \,\&\, Gb)$ | CD |
| (14) | $b{=}a$ | As |
| (15) | ~~SHOW~~: $Fb \,\&\, Gb$ | DD |
| (16) | $Fb \leftrightarrow b{=}a$ | 4,∀O |
| (17) | $Fb$ | 14,16,SL |
| (18) | $Fb \rightarrow Gb$ | 2,∀O |
| (19) | $Gb$ | 17,18,SL |
| (20) | $Fb \,\&\, Gb$ | 17,19,SL |

**#38a:**

| | | |
|---|---|---|
| (1) | $\exists xFx \,\&\, \forall x \forall y([Fx \,\&\, Fy]) \rightarrow x{=}y)$ | Pr |
| (2) | ~~SHOW~~: $\exists x \forall y(Fy \leftrightarrow y{=}x)$ | 4,∃I |
| (3) | $Fa$ | 1a,∃O |
| (4) | ~~SHOW~~: $\forall y(Fy \leftrightarrow y{=}a)$ | UD |
| (5) | ~~SHOW~~: $Fb \leftrightarrow b{=}a$ | 6,11↔I |
| (6) | ~~SHOW~~: $Fb \rightarrow b{=}a$ | CD |
| (7) | $Fb$ | As |
| (8) | ~~SHOW~~: $b{=}a$ | DD |
| (9) | $(Fb \,\&\, Fa) \rightarrow b{=}a$ | 1b,∀O2 |
| (10) | $b{=}a$ | 3,7,9,SL |
| (11) | ~~SHOW~~: $b{=}a \rightarrow Fb$ | CD |
| (12) | $b{=}a$ | As |
| (13) | ~~SHOW~~: $Fb$ | DD |
| (14) | $Fb$ | 3,12,LL |

**#38b:**

| | | |
|---|---|---|
| (1) | $\exists x \forall y(Fy \leftrightarrow y{=}x)$ | Pr |
| (2) | ~~SHOW~~: $\exists xFx \,\&\, \forall x \forall y\{[Fx \,\&\, Fy] \rightarrow x{=}y\}$ | 2,8,&I |
| (3) | ~~SHOW~~: $\exists xFx$ | 7,∃I |
| (4) | $\forall y(Fy \leftrightarrow y{=}a)$ | 1,∃O |
| (5) | $Fa \leftrightarrow a{=}a$ | 4,∀O |
| (6) | $a{=}a$ | REF= |
| (7) | $Fa$ | 5,6,SL |
| (8) | ~~SHOW~~: $\forall x \forall y\{[Fx \,\&\, Fy] \rightarrow x{=}y\}$ | UD2 |
| (9) | ~~SHOW~~: $[Fa \,\&\, Fb] \rightarrow a{=}b$ | CD |
| (10) | $Fa \,\&\, Fb$ | As |
| (11) | ~~SHOW~~: $a{=}b$ | DD |
| (12) | $\forall y(Fy \leftrightarrow y{=}c)$ | 1,∃O |
| (13) | $Fa \leftrightarrow a{=}c$ | 12,∀O |
| (14) | $Fb \leftrightarrow b{=}c$ | 12,∀O |
| (15) | $a{=}c$ | 10,13,SL |
| (16) | $b{=}c$ | 10,14,SL |
| (17) | $a{=}b$ | 15,16,LL |

**#39:**

| | | |
|---|---|---|
| (1) | $\forall x \exists y(Fx \leftrightarrow y{=}x)$ | Pr |
| (2) | ~~SHOW~~: $\exists x \exists y[x{\neq}y] \vee \forall x Fx$ | ID |
| (3) | $\sim(\exists x \exists y[x{\neq}y] \vee \forall x Fx)$ | As |
| (4) | ~~SHOW~~: ※ | DD |
| (5) | $\sim\exists x \exists y[x{\neq}y]\ \&\ \sim\forall x Fx$ | 3,SL |
| (6) | $\exists x{\sim}Fx$ | 5b,QN |
| (7) | $\sim Fa$ | 6,∃O |
| (8) | $\exists y(Fa \leftrightarrow y{=}a)$ | 1,∀O |
| (9) | $Fa \leftrightarrow b{=}a$ | 8,∀O |
| (10) | $\sim[b{=}a]$ | 7,9,SL |
| (11) | $\forall x{\sim}\exists y[x{\neq}y]$ | 5a,QN |
| (12) | $\sim\exists y[b{\neq}y]$ | 11,∀O |
| (13) | $\forall y{\sim}[b{\neq}y]$ | 12,QN |
| (14) | $\sim[b{\neq}a]$ | 13,∀O |
| (15) | ※ | 10,14,SL |

**#40:**

| | | |
|---|---|---|
| (1) | $\exists x(\forall y[Fy \leftrightarrow y{=}x]\ \&\ Gx)$ | Pr |
| (2) | ~~SHOW~~: $\exists x \forall y([Fy\ \&\ Gy] \leftrightarrow y{=}x)$ | ∃D |
| (3) | $\forall y(Fy \leftrightarrow y{=}a)\ \&\ Ga$ | 1,∃O |
| (4) | ~~SHOW~~: $\forall y([Fy\ \&\ Gy] \leftrightarrow y{=}a)$ | UD |
| (5) | ~~SHOW~~: $(Fb\ \&\ Gb) \leftrightarrow b{=}a$ | 6,11,↔I |
| (6) | ~~SHOW~~: $(Fb\ \&\ Gb) \rightarrow b{=}a$ | CD |
| (7) | $Fb\ \&\ Gb$ | As |
| (8) | ~~SHOW~~: $b{=}a$ | DD |
| (9) | $Fb \leftrightarrow b{=}a$ | 3a,∀O |
| (10) | $b{=}a$ | 7a,9,SL |
| (11) | ~~SHOW~~: $b{=}a \rightarrow (Fb\ \&\ Gb)$ | CD |
| (12) | $b{=}a$ | As |
| (13) | ~~SHOW~~: $Fb\ \&\ Gb$ | DD |
| (14) | $Gb$ | 3b,12,LL |
| (15) | $Fb \leftrightarrow b{=}a$ | 3a,∀O |
| (16) | $Fb$ | 12,15,SL |
| (17) | $Gb\ \&\ Fb$ | 14,16,SL |

**#41:**

| | | |
|---|---|---|
| (1) | $\exists x \forall y\{Fy \leftrightarrow y=x\}$ | Pr |
| (2) | $\forall x\{Fx \rightarrow Gx\}$ | Pr |
| (3) | ~~SHOW~~: $\exists x \forall y\{[Fy \,\&\, Gy] \leftrightarrow y=x\}$ | 4,∃I |
| (4) | │ $\forall y\{Fy \leftrightarrow y=a\}$ | 1,∃O |
| (5) | │ $Fa \leftrightarrow a=a$ | 4,∀O |
| (6) | │ $a=a$ | REF= |
| (7) | │ $Fa$ | 5,6,SL |
| (8) | │ $Fa \rightarrow Ga$ | 2,∀O |
| (9) | │ ~~SHOW~~: $\forall y\{[Fy \,\&\, Gy] \leftrightarrow y=a\}$ | UD |
| (10) | │ │ ~~SHOW~~: $[Fb \,\&\, Gb] \leftrightarrow b=a$ | 11,16,↔I |
| (11) | │ │ │ ~~SHOW~~: $[Fb \,\&\, Gb] \rightarrow b=a$ | CD |
| (12) | │ │ │ │ $Fb \,\&\, Gb$ | As |
| (13) | │ │ │ │ ~~SHOW~~: $b=a$ | DD |
| (14) | │ │ │ │ │ $Fb \leftrightarrow b=a$ | 4,∀O |
| (15) | │ │ │ │ │ $b=a$ | 12,14,SL |
| (16) | │ │ │ ~~SHOW~~: $b=a \rightarrow [Fb \,\&\, Gb]$ | CD |
| (17) | │ │ │ │ $b=a$ | As |
| (18) | │ │ │ │ ~~SHOW~~: $Fb \,\&\, Gb$ | DD |
| (19) | │ │ │ │ │ $Fb$ | 7,17,LL |
| (20) | │ │ │ │ │ $Fb \rightarrow Gb$ | 2,∀O |
| (21) | │ │ │ │ │ $Fb \,\&\, Gb$ | 19,20,SL |

**#42:**

| | | |
|---|---|---|
| (1) | $\exists x \forall y([Gy \,\&\, Hy] \leftrightarrow y=x)$ | Pr |
| (2) | $\forall x(Fx \rightarrow Gx)$ | Pr |
| (3) | ~~SHOW~~: $\exists x \forall y([Fy \,\&\, Hy] \rightarrow y=x)$ | 4,∃I |
| (4) | │ $\forall y([Gy \,\&\, Hy] \leftrightarrow y=a)$ | 1,∃O |
| (5) | │ ~~SHOW~~: $\forall y([Fy \,\&\, Hy] \rightarrow y=a)$ | UD |
| (6) | │ │ ~~SHOW~~: $(Fb \,\&\, Hb) \rightarrow b=a$ | CD |
| (7) | │ │ │ $Fb \,\&\, Hb$ | As |
| (8) | │ │ │ ~~SHOW~~: $b=a$ | DD |
| (9) | │ │ │ │ $Fb \rightarrow Gb$ | 2,∀O |
| (10) | │ │ │ │ $Gb$ | 7a,9,SL |
| (11) | │ │ │ │ $Gb \,\&\, Hb$ | 7b,10,SL |
| (12) | │ │ │ │ $(Gb \,\&\, Hb) \leftrightarrow b=a$ | 4,∀O |
| (13) | │ │ │ │ $b=a$ | 11,12,SL |

**#43:**

| | | |
|---|---|---|
| (1) | $\exists x \forall y(Fy \leftrightarrow y=x)$ | Pr |
| (2) | ~~SHOW~~: $\sim\exists x \exists y\{x \neq y \,\&\, Fx \,\&\, Fy\}$ | ID |
| (3) | │ $\exists x \exists y\{x \neq y \,\&\, Fx \,\&\, Fy\}$ | As |
| (4) | │ ~~SHOW~~: ※ | DD |
| (5) | │ │ $a \neq b \,\&\, Fa \,\&\, Fb$ | 3 ∃O2 |
| (6) | │ │ $\forall y(Fy \leftrightarrow y=c)$ | 1,∃O |
| (7) | │ │ $Fa \leftrightarrow a=c$ | 6,∀O |
| (8) | │ │ $Fb \leftrightarrow b=c$ | 6,∀O |
| (9) | │ │ $a=c$ | 5,7,SL |
| (10) | │ │ $b=c$ | 5,8,SL |
| (11) | │ │ $a=b$ | 9,10,LL |
| (12) | │ │ $a \neq b$ | 5,SL |
| (13) | │ │ ※ | 11,12,SL |

**#44:**

| | | |
|---|---|---|
| (1) | $\exists x\{\forall y(Fy \leftrightarrow y{=}x) \,\&\, \forall y(Gy \rightarrow Rxy)\}$ | Pr |
| (2) | ~~SHOW~~: $\forall x(Gx \rightarrow \exists y\forall z\{[Fz \,\&\, Rzx] \leftrightarrow z{=}y\})$ | UD |
| (3) | ~~SHOW~~: $Ga \rightarrow \exists y\forall z\{[Fz \,\&\, Rza] \leftrightarrow z{=}y\}$ | CD |
| (4) | $Ga$ | As |
| (5) | ~~SHOW~~: $\exists y\forall z\{[Fz \,\&\, Rza] \leftrightarrow z{=}y\}$ | ∃D |
| (6) | $\forall y(Fy \leftrightarrow y{=}b) \,\&\, \forall y(Gy \rightarrow Rby)$ | 1,∃O |
| (7) | $\forall y(Fy \leftrightarrow y{=}b)$ | 6,SL |
| (8) | $\forall y(Gy \rightarrow Rby)$ | 6,SL |
| (9) | ~~SHOW~~: $\forall z\{[Fz \,\&\, Rza] \leftrightarrow z{=}b\}$ | UD |
| (10) | ~~SHOW~~: $[Fc \,\&\, Rca] \leftrightarrow c{=}b$ | 11,16,↔I |
| (11) | ~~SHOW~~: $[Fc \,\&\, Rca] \rightarrow c{=}b$ | CD |
| (12) | $Fc \,\&\, Rca$ | As |
| (13) | ~~SHOW~~: $c{=}b$ | DD |
| (14) | $Fc \leftrightarrow c{=}b$ | 7,∀O |
| (15) | $c{=}b$ | 12,14,SL |
| (16) | ~~SHOW~~: $c{=}b \rightarrow [Fc \,\&\, Rca]$ | CD |
| (17) | $c{=}b$ | As |
| (18) | ~~SHOW~~: $Fc \,\&\, Rca$ | DD |
| (19) | $Ga \rightarrow Rba$ | 8,∀O |
| (20) | $Rba$ | 4,19,SL |
| (21) | $Rca1$ | 7,20,LL |
| (22) | $Fc \leftrightarrow c{=}b$ | 7,∀O |
| (23) | $Fc$ | 17,22,SL |
| (24) | $Fc \,\&\, Rca$ | 21,23,SL |

**#45a:**

| | | |
|---|---|---|
| (1) | $\exists x\forall y(Fy \leftrightarrow y{=}x)$ | Pr |
| (2) | ~~SHOW~~: $\exists x(Fx \,\&\, {\sim}\exists y[y{\neq}x \,\&\, Fy])$ | 15,∃I |
| (3) | $\forall y(Fy \leftrightarrow y{=}a)$ | 1,∃O |
| (4) | ~~SHOW~~: $Fa \,\&\, {\sim}\exists y[y{\neq}a \,\&\, Fy]$ | 5,14,&I |
| (5) | ~~SHOW~~: ${\sim}\exists y[y{\neq}a \,\&\, Fy]$ | ID |
| (6) | $\exists y[y{\neq}a \,\&\, Fy]$ | As |
| (7) | ~~SHOW~~: ※ | DD |
| (8) | $b{\neq}a \,\&\, Fb$ | 6,∃O |
| (9) | $Fb \leftrightarrow b{=}a$ | 3,∀O |
| (10) | $b{=}a$ | 8b,9,SL |
| (11) | ※ | 8a,10,SL |
| (12) | $Fa \leftrightarrow a{=}a$ | 3,∀O |
| (13) | $a{=}a$ | REF= |
| (14) | $Fa$ | 12,13,SL |

**#45b:**

| | | |
|---|---|---|
| (1) | $\exists x(Fx \,\&\, {\sim}\exists y[y{\neq}x \,\&\, Fy])$ | Pr |
| (2) | ~~SHOW~~: $\exists x \forall y(Fy \leftrightarrow y{=}x)$ | 4,∃I |
| (3) | $Fa \,\&\, {\sim}\exists y(y{\neq}a \,\&\, Fy)$ | 1,∃O |
| (4) | ~~SHOW~~: $\forall y(Fy \leftrightarrow y{=}a)$ | UD |
| (5) | ~~SHOW~~: $Fb \leftrightarrow b{=}a$ | 6,12,↔I |
| (6) | ~~SHOW~~: $Fb \rightarrow b{=}a$ | CD |
| (7) | $Fb$ | As |
| (8) | ~~SHOW~~: $b{=}a$ | DD |
| (9) | $\forall y{\sim}(y{\neq}a \,\&\, Fy)$ | 3b,~∃O |
| (10) | ${\sim}(b{\neq}a \,\&\, Fb)$ | 9,∀O |
| (11) | $b{=}a$ | 7,10,SL |
| (12) | ~~SHOW~~: $b{=}a \rightarrow Fb$ | CD |
| (13) | $b{=}a$ | As |
| (14) | ~~SHOW~~: $Fb$ | DD |
| (15) | $\forall y{\sim}(y{\neq}a \,\&\, Fy)$ | 3b,~∃O |
| (16) | ${\sim}(b{\neq}a \,\&\, Fb)$ | 15,∀O |
| (17) | $Fb$ | 13,16,SL |

**#46:**

| | | |
|---|---|---|
| (1) | $\exists x \exists y\{x{\neq}y \,\&\, \forall z(Fz \leftrightarrow [z{=}x \vee z{=}y])\}$ | Pr |
| (2) | ~~SHOW~~: ${\sim}\exists x \exists y \exists z\{x{\neq}y \,\&\, x{\neq}z \,\&\, y{\neq}z \,\&\, Fx \,\&\, Fy \,\&\, Fz\}$ | ID |
| (3) | $\exists x \exists y \exists z\{x{\neq}y \,\&\, x{\neq}z \,\&\, y{\neq}z \,\&\, Fx \,\&\, Fy \,\&\, Fz\}$ | As |
| (4) | ~~SHOW~~: ※ | SC |
| (5) | $a{\neq}b \,\&\, a{\neq}c \,\&\, b{\neq}c \,\&\, Fa \,\&\, Fb \,\&\, Fc$ | 3,∃O3 |
| (6) | $p{\neq}q \,\&\, \forall z(Fz \leftrightarrow [z{=}p \vee z{=}q])$ | 1,∃O2 |
| (7) | $Fa \leftrightarrow [a{=}p \vee a{=}q]$ | 6b,∀O |
| (8) | $Fb \leftrightarrow [b{=}p \vee b{=}q]$ | 6b,∀O |
| (9) | $Fc \leftrightarrow [c{=}p \vee c{=}q]$ | 6b,∀O |
| (10) | $a{=}p \vee a{=}q$ | 5d,7,SL |
| (11) | $b{=}p \vee b{=}q$ | 5e,8,SL |
| (12) | $c{=}p \vee c{=}q$ | 5f.9,SL |
| (13) | $c1$: | |

this is be beginning of a long and tedious case-by-case argument,
based on the three disjunctions 10-12. There are 8(!) cases to consider.
Try finishing this one for a challenge.
Don't worry; it won't be on the exam.

**#47a:**

| | | |
|---|---|---|
| (1) | $\exists x \exists y(x{\neq}y \& Fx \& Fy) \& \forall x \forall y \forall z([Fx \& Fy \& Fz] \rightarrow [x{=}y \vee x{=}z \vee y{=}z])$ | Pr |
| (2) | ~~SHOW~~: $\exists x \exists y\{x{\neq}y \;\&\; \forall z[Fz \leftrightarrow (z{=}x \vee z{=}y)]\}$ | 4, ∃I2 |
| (3) | $a{\neq}b \;\&\; Fa \;\&\; Fb$ | 1a,∃O2 |
| (4) | ~~SHOW~~: $a{\neq}b \;\&\; \forall z[Fz \leftrightarrow (z{=}a \vee z{=}b)]$ | 3a,5,SL |
| (5) | ~~SHOW~~: $\forall z[Fz \leftrightarrow (z{=}a \vee z{=}b)]$ | UD |
| (6) | ~~SHOW~~: $Fc \leftrightarrow (c{=}a \vee c{=}b)$ | 7,21,↔I |
| (7) | ~~SHOW~~: $Fc \rightarrow (c{=}a \vee c{=}b)$ | CD |
| (8) | $Fc$ | As |
| (9) | ~~SHOW~~: $c{=}a \vee c{=}b$ | ID |
| (10) | $\sim(c{=}a \vee c{=}b)$ | As |
| (11) | ~~SHOW~~: ※ | DD |
| (12) | $[Fa \;\&\; Fb \;\&\; Fc] \rightarrow [a{=}b \vee a{=}c \vee b{=}c]$ | 1b,∀O3 |
| (13) | $a{=}b \vee a{=}c \vee b{=}c$ | 3b,3c,8,12,SL |
| (14) | $c{\neq}a \;\&\; c{\neq}b$ | 10,SL |
| (15) | $c{=}a \leftrightarrow a{=}c$ | SL |
| (16) | $a{\neq}c$ | 14a,15,SL |
| (17) | $a{=}c \vee b{=}c$ | 3a,13,SL |
| (18) | $b{=}c$ | 16,17,SL |
| (19) | $c{=}b$ | 18,SYM= |
| (20) | ※ | 14b,19,SL |
| (21) | ~~SHOW~~: $(c{=}a \vee c{=}b) \rightarrow Fc$ | CD |
| (22) | $c{=}a \vee c{=}b$ | As |
| (23) | ~~SHOW~~: $Fc$ | SC |
| (24) | $c1: c{=}a$ | As |
| (25) | ~~SHOW~~: $Fc$ | DD |
| (26) | $Fc$ | 3b,24,LL |
| (27) | $c2: c{=}b$ | As |
| (28) | ~~SHOW~~: $Fc$ | DD |
| (29) | $Fc$ | 3c,27,LL |

**#47b:**

| | | |
|---|---|---|
| (1) | $\exists x \exists y\{x \neq y \,\&\, \forall z[Fz \leftrightarrow (z=x \vee z=y)]\}$ | Pr |
| (2) | ~~SHOW~~: $\exists x \exists y(x \neq y \,\&\, Fx \,\&\, Fy) \,\&$ $\forall x \forall y \forall z([Fx \,\&\, Fy \,\&\, Fz] \to [x=y \vee x=z \vee y=z])$ | (4,21)&,∃I2 |
| (3) | $a \neq b \,\&\, \forall z[Fz \leftrightarrow (z=a \vee z=b)]$ | 1,∃O2 |
| (4) | ~~SHOW~~: $\exists x \exists y(x \neq y \,\&\, Fx \,\&\, Fy)$ | 5,∃I2 |
| (5) | ~~SHOW~~: $a \neq b \,\&\, Fa \,\&\, Fb$ | DD |
| (6) | ~~SHOW~~: $Fa$ | ID |
| (7) | $\sim Fa$ | As |
| (8) | ~~SHOW~~: ※ | DD |
| (9) | $Fa \leftrightarrow (a=a \vee a=b)$ | 3b,∀O |
| (10) | $a \neq a \,\&\, a \neq b$ | 7,9,SL |
| (11) | $a=a$ | REF= |
| (12) | ※ | 10a,11,SL |
| (13) | ~~SHOW~~: $Fb$ | ID |
| (14) | $\sim Fb$ | As |
| (15) | ~~SHOW~~: ※ | DD |
| (16) | $Fb \leftrightarrow (b=a \vee b=b)$ | 3b,∀O |
| (17) | $b \neq a \,\&\, b \neq b$ | 14,16,SL |
| (18) | $b=b$ | REF= |
| (19) | ※ | 17b,18,SL |
| (20) | $a \neq b \,\&\, Fa \,\&\, Fb$ | 3a,6,13,SL |
| (21) | ~~SHOW~~: $\forall x \forall y \forall z\{[Fx \,\&\, Fy \,\&\, Fz] \to [x=y \vee x=z \vee y=z]\}$ | UD3 |
| (22) | ~~SHOW~~: $(Fc \,\&\, Fd \,\&\, Fe) \to (c=d \vee c=e \vee d=e)$ | CD |
| (23) | $Fc \,\&\, Fd \,\&\, Fe$ | As |
| (24) | ~~SHOW~~: $c=d \vee c=e \vee d=e$ | ∨D |
| (25) | $c \neq d \,\&\, c \neq e \,\&\, d \neq e$ | As |
| (26) | ~~SHOW~~: ※ | 27-29,SC |
| (27) | $c=a \vee c=b$ | 3b,23a,∀↔O |
| (28) | $d=a \vee d=b$ | 3b,23b,∀↔O |
| (29) | $e=a \vee e=b$ | 3b,23c,∀↔O |
| (30) | $c1: c=a \,\&\, d=a \,\&\, e=a$ | As |
| (31) | ~~SHOW~~: ※ | |
| (32) | ... | |
| (33) | $c2: c=a \,\&\, d=a \,\&\, e=b$ | As |
| (34) | ~~SHOW~~: ※ | |
| (35) | ... | |
| (36) | $c3: c=a \,\&\, d=b \,\&\, e=a$ | As |
| (37) | ~~SHOW~~: ※ | |
| (38) | | |
| (37) | ... | |

Once again, we are in a tedious 8-CASE argument; see previous problem.