

# UNIT 2: DERIVATIONS IN FUNCTION LOGIC

## 1. EXERCISE SET A (Review of Predicate Logic)

For each of the following arguments, construct a formal derivation of the conclusion (indicated by "/") from the premises.

1.  $\forall x(Fx \rightarrow Gx) / \forall xFx \rightarrow \forall xGx$
2.  $\forall x(Fx \rightarrow Gx); \sim \exists x(Gx \& Hx) / \sim \exists x(Fx \& Hx)$
3.  $\exists xFx \rightarrow \exists x \sim Gx / \forall x[Fx \rightarrow \sim \forall xGx]$
4.  $\forall x[Fx \rightarrow \sim \forall xGx] / \exists xFx \rightarrow \exists x \sim Gx$
5.  $\exists x \sim \exists y(Fy \& Ryx) / \forall x(Fx \rightarrow \exists y \sim Rxy)$
6.  $\forall x \exists y Rxy; \forall x \forall y [Rxy \rightarrow Rxx]; \forall x [Rxx \rightarrow \forall y Ryx] / \forall x \forall y Rxy$
7.  $\forall x \exists y Rxy; \forall x [\exists y Rxy \rightarrow \exists y Ryx]; \forall x \forall y [Ryx \rightarrow \forall y Ryx] / \forall x \forall y Rxy$

## 2. EXERCISE SET B

- 1.A. For each of the following three formulas, list the formula that is obtained by a single application of the rule Universal Elimination ( $\forall O$ ), using the singular term ' $f(a)$ '.
- 1.B. For each of the following three formulas, list the formula that is obtained by successive applications of the rule Universal Elimination ( $\forall O$ ), substituting ' $f(a)$ ' for ' $x$ ' and ' $s(a,b)$ ' for ' $y$ '.
  - (1)  $\forall x \forall y (Rxy \rightarrow Rxx)$
  - (2)  $\forall x \forall y \{ (Nx \& Ny) \rightarrow N[p(x,y)] \}$
  - (3)  $\forall x \forall y (R[x,m(y)] \rightarrow R[m(y),x])$
- 2.A. For each of the following three formulas, list every (non-trivial) formula that can be obtained by a single application of the rule Existential Introduction ( $\exists I$ ).
  - (1)  $R[a, f(a)]$
  - (2)  $R[f(a), f(a)]$
  - (3)  $R[s(a), p(a,b)]$
- 2.B. For each formula obtained in 2.A, list every (non-trivial) formula that can be obtained by one further application of  $\exists I$ .

### 3. EXERCISE SET C

**Directions For All Derivation Exercises.** For each of the following, construct a formal derivation of the conclusion (indicated by "/") from the premises (if any). In cases in which two formulas are separated by "//", construct a derivation of each formula from the other.

1.  $P[f(b)] / \exists xP[f(x)]$
2.  $P[f(b)] / \exists xPx$
3.  $\exists xF[f(x)] / \exists xFx$
4.  $\forall x(Fx \rightarrow Hx); F[p(a,b)] / H[p(a,b)]$
5.  $\forall x(Fx \rightarrow Hx); F[p(a,b)] / \exists xHx$
6.  $\forall x(Fx \rightarrow Hx); \sim H[f(a)] / \sim F[f(a)]$
7.  $\forall x(Fx \rightarrow Hx); \sim H[f(a)] / \sim \forall xFx$
8.  $\forall x(Fx \rightarrow Gx); \forall x[(Fx \& Gx) \rightarrow \sim Hx]; H[p(a,b)] / \exists x(Hx \& \sim Fx)$
9.  $\forall x(Fx \rightarrow Gx); \forall x(Gx \rightarrow Hx); F[m(a)] / \exists x(Gx \& Hx)$
10.  $\forall x(Fx \rightarrow Gx); \forall x(Gx \rightarrow Hx); F[m(a)] / \exists xG[m(x)] \& \exists xH[m(x)]$
11.  $\forall x\{R[x,s(a)] \rightarrow \sim R[x,p(a,b)]\}; R[s(a),s(a)] / \exists x\sim R[x,p(a,b)]$
12.  $\exists xR[x,f(n)] \rightarrow \forall xR[x,f(n)]; \sim R[g(n),f(n)] / \sim R[f(n),f(n)]$
13.  $\forall x(Fx \rightarrow Rxx); F[p(c,d)] / \exists xR[x,p(c,d)]$
14.  $\forall x(Fx \rightarrow Rxx); \forall x\sim R[m(a),x] / \sim F[m(a)]$
15.  $\forall x\{Fx \rightarrow \forall yRxy\}; F[g(a)] / R[g(a),g(a)]$
16.  $\exists xR[f(a),x] \rightarrow \forall xR[x,f(a)]; \sim R[g(b),f(a)] / \sim R[f(a),f(a)]$

**4. EXERCISE SET D**

17.  $\forall x[\exists yRxy \rightarrow \forall yRyx]; R[f(a,b),f(a,b)] / R[g(a,b),f(a,b)]$
18.  $\forall x(Rxx \rightarrow Fx); \forall x\forall y(Rxy \rightarrow Rxx); \sim F[g(a,b)] / \sim R[g(a,b),b]$
19.  $\forall x(Fx \rightarrow Gx); \forall xM[m(x),x] / \forall x\{Fx \rightarrow \exists y\{Gy \& M[m(x),y]\}\}$
20.  $\forall x\forall y\{(Nx \& Ny) \rightarrow N[p(x,y)]\}; \exists x\{Nx \& \forall y\{Ny \rightarrow E[p(x,y)]\}\} / \exists x(Nx \& Ex)$
21.  $\forall x\{R[x,f(x)] \rightarrow \sim Rxx\} / \sim \exists x\forall yR[f(x),y]$
22.  $\forall x\forall yP[p(x,y)] / \forall x\exists yP[p(c(y),p(y,x))]$
23.  $\forall x(Fx \rightarrow Gx); \forall x\{Gx \rightarrow F[f(x)]\}; \exists xF[g(x)] / \exists xG[f(x)]$
24.  $Fa; \forall x\{Fx \rightarrow F[m(x)]\}; \forall xR[x,m(x)]; \forall x\forall y\forall z\{(Rxy \& Ryz) \rightarrow Rxz\} / \exists x\{Fx \& \{Rax \& R[m(a),x]\}\}$
25.  $\forall x\{Fx \rightarrow R[x,m(x)]\}; \forall xH[m(x)] / \forall x\{Fx \rightarrow \exists y(Hy \& Rxy)\}$
26.  $\forall x\{D[f(x)] \rightarrow D[m(x)]\}; \sim \exists x\{Hx \& D[m(x)]\} / \forall x\{D[f(x)] \rightarrow \sim Hx\}$
27.  $\forall x(\exists yR[x,f(y)] \rightarrow Hx); \forall x(Fx \rightarrow \exists yRxy); \forall x\forall y\{Rxy \rightarrow R[x,f(y)]\} / \forall x(Fx \rightarrow Hx)$

**5. EXERCISE SET E**

28.  $\forall x(Fx \rightarrow Rxx); \exists xF[m(x)] / \exists x\exists yR[x,m(y)]$
29.  $\forall x(Fx \rightarrow Rxx); \exists xF[m(x)] / \exists x\exists yR[m(x),y]$
30.  $\forall x\{F[g(x)] \rightarrow F[m(x)]\}; \sim \exists x\{Fx \& F[m(x)]\} / \forall x\{Fx \rightarrow \sim F[g(x)]\}$
31.  $\forall x\{F[g(x)] \rightarrow F[m(x)]\}; \forall x\{Fx \rightarrow \sim F[m(x)]\} / \sim \exists x\{Fx \& F[g(x)]\}$
32.  $\forall x\forall y\{Rxy \rightarrow Ryx\}; \forall x\forall y\{R[x,s(y)] \rightarrow Ryx\} / \forall x\forall y\{R[s(x),y] \rightarrow Ryx\}$
33.  $\forall x\exists yK[x,s(y)]; \forall x\forall y\{K[x,y] \rightarrow R[x,s(y)]\} / \forall x\exists yR[s(x),s(y)]$
34.  $\forall x\forall y\forall z\{[Rxy \& Ryz] \rightarrow Rxz\}; \forall x\forall y\{Rxy \rightarrow R[f(y),f(x)]\}; \forall xR[x,f(f(x))]$   
 $/ \forall x\forall y\{R[x,f(y)] \rightarrow R[y,f(x)]\}$
35.  $\forall x\forall y\{Rxy \rightarrow Ryx\}; \forall x\exists yR[x,s(y)] / \forall x\exists yRyx$
36.  $\forall x(Fx \leftrightarrow \sim F[f(x)]) / \exists x\{Fx \& \sim F[f(x)]\}$
37.  $\forall xF[m(x)] / \exists x\{Fx \& F[m(x)]\}$
38.  $/ \forall x\{R[x,f(x)] \leftrightarrow \exists y\{\forall z(R[z,y] \rightarrow R[z,f(x)]) \& R[x,y]\}\}$

**1. ANSWERS TO UNIT 2 EXERCISES****1. EXERCISE SET A****#1:**

(1)	$\forall x(Fx \rightarrow Gx)$	Pr
(2)	SHOW: $\forall xFx \rightarrow \forall xGx$	UD
(3)	$\forall xFx$	As
(4)	SHOW: $\forall xGx$	UD
(5)	SHOW: $Ga$	DD
(6)	$Fa \rightarrow Ga$	1, $\forall O$
(7)	$Fa$	3, $\forall O$
(8)	$Ga$	6,7,SL

**#2:**

(1)	$\forall x(Fx \rightarrow Gx)$	Pr
(2)	$\sim \exists x(Gx \& Hx)$	Pr
(3)	SHOW: $\sim \exists x(Fx \& Hx)$	ID
(4)	$\exists x(Fx \& Hx)$	As
(5)	SHOW: $\times$	DD
(6)	$Fa \& Ha$	4, $\exists O$
(7)	$Fa$	6,SL
(8)	$Ha$	6,SL
(9)	$Fa \rightarrow Ga$	1, $\forall O$
(10)	$Ga$	7,9,SL
(11)	$Ga \& Ha$	8,10,SL
(12)	$\exists x(Gx \& Hx)$	11, $\exists I$
(13)	$\times$	2,12,SL

**#3:**

(1)	$\exists xFx \rightarrow \exists x \sim Gx$	Pr
(2)	SHOW: $\forall x[Fx \rightarrow \sim \forall xGx]$	UD
(3)	SHOW: $Fa \rightarrow \sim \forall xGx$	CD
(4)	$Fa$	As
(5)	SHOW: $\sim \forall xGx$	ID
(6)	$\forall xGx$	As
(7)	SHOW: $\times$	DD
(8)	$\exists xFx$	4, $\exists I$
(9)	$\exists x \sim Gx$	1,8,SL
(10)	$\sim Gb$	9, $\exists O$
(11)	$Gb$	6, $\forall O$
(12)	$\times$	10,11,SL

**#4:**

(1)	$\forall x[Fx \rightarrow \sim \forall xGx]$	Pr
(2)	SHOW: $\exists xFx \rightarrow \exists x \sim Gx$	CD
(3)	$\exists xFx$	As
(4)	SHOW: $\exists x \sim Gx$	DD
(5)	$Fa$	3, $\exists O$
(6)	$Fa \rightarrow \sim \forall xGx$	1, $\forall O$
(7)	$\sim \forall xGx$	5,6,SL
(8)	$\exists x \sim Gx$	7, $Q_N$

**#5:**

(1)	$\exists x \sim \exists y (Fy \ \& \ Ryx)$	Pr
(2)	SHOW: $\forall x (Fx \rightarrow \exists y \sim Rxy)$	UD
(3)	SHOW: $Fa \rightarrow \exists y \sim Ray$	CD
(4)	$Fa$	As
(5)	SHOW: $\exists y \sim Ray$	ID
(6)	$\sim \exists y \sim Ray$	As
(7)	SHOW: $\times$	DD
(8)	$\forall y \sim \sim Ray$	6,QN
(9)	$\sim \exists y (Fy \ \& \ Ryb)$	1, $\exists$ O
(10)	$\forall y \sim (Fy \ \& \ Ryb)$	9,QN
(11)	$\sim (Fa \ \& \ Rab)$	10, $\forall$ O
(12)	$\sim \sim Rab$	8, $\forall$ O
(13)	$\sim Fa \vee \sim Rab$	11,SL
(14)	$\sim Fa$	12,13,SL
(15)	$\times$	4,14,SL

**#6:**

(1)	$\forall x \exists y Rxy$	Pr
(2)	$\forall x \forall y [Rxy \rightarrow Rxx]$	Pr
(3)	$\forall x [Rxx \rightarrow \forall y Ryx]$	Pr
(4)	SHOW: $\forall x \forall y Rxy$	UD
(5)	SHOW: $\forall y Ray$	UD
(6)	SHOW: $Rab$	DD
(7)	$\exists y Rby$	1, $\forall$ O
(8)	$Rbc$	7, $\exists$ O
(9)	$\forall y [Rby \rightarrow Rbb]$	2, $\forall$ O
(10)	$Rbc \rightarrow Rbb$	9, $\forall$ O
(11)	$Rbb$	8,9,SL
(12)	$Rbb \rightarrow \forall y Ryb$	3, $\forall$ O
(13)	$\forall y Ryb$	11,12,SL
(14)	$Rab$	13, $\forall$ O

**#7:**

(1)	$\forall x \exists y Rxy$	Pr
(2)	$\forall x [\exists y Rxy \rightarrow \exists y Ryx]$	Pr
(3)	$\forall x \forall y [Ryx \rightarrow \forall y Ryx]$	Pr
(4)	SHOW: $\forall x \forall y Rxy$	UD
(5)	SHOW: $\forall y Ray$	UD
(6)	SHOW: $Rab$	DD
(7)	$\exists y Rby$	1, $\forall$ O
(8)	$\exists y Rby \rightarrow \exists y Ryb$	2, $\forall$ O
(9)	$\exists y Ryb$	7,8,SL
(10)	$Rcb$	9, $\exists$ O
(11)	$\forall y (Ryb \rightarrow \forall y Ryb)$	3, $\forall$ O
(12)	$Rcb \rightarrow \forall y Ryb$	11, $\forall$ O
(13)	$\forall y Ryb$	10,12,SL
(14)	$Rab$	13, $\forall$ O

**2. EXERCISE SET B**

1A1:  $\forall y(R[f(a),y] \rightarrow R[f(a),f(a)])$

1A2:  $\forall y\{(N[f(a) \& Ny] \rightarrow N[p(f(a),y)])\}$

1A3:  $\forall y(R[f(a),m(y)] \rightarrow R[m(y),f(a)])$

1B1:  $R[f(a),s(a,b)] \rightarrow R[f(a),f(a)]$

1B2:  $(N[f(a)] \& N[s(a,b)]) \rightarrow N[p(f(a),s(a,b))]$

1B3:  $R[f(a),m(s(a,b))] \rightarrow R[m(s(a,b)),f(a)]$

2A1: (a)  $\exists xR[x,f(x)]$

(b)  $\exists xR[x,f(a)]$

(c)  $\exists xR[a,f(x)]$

(d)  $\exists xRax$

2A2: (a)  $\exists xR[x,x]$

(b)  $\exists xR[x,f(a)]$

(c)  $\exists xR[f(a),x]$

(d)  $\exists xR[f(x),f(x)]$

(e)  $\exists xR[f(x),f(a)]$

(f)  $\exists xR[f(a),f(x)]$

2A3: (a)  $\exists xR[s(x),p(x,b)]$

(b)  $\exists xR[s(a),p(x,b)]$

(c)  $\exists xR[x,p(a,b)]$

(d)  $\exists xR[s(a),x]$

(e)  $\exists xR[s(a),p(a,x)]$

(f)  $\exists xR[s(x),p(a,b)]$

2B1: (a) none

(b)  $\exists y\exists xR[x,y]$

$\exists y\exists xR[x,f(y)]$

(c)  $\exists y\exists xR[y,f(x)]$

(d)  $\exists y\exists xRyx$

2B2: (a) none

(b)  $\exists y\exists xRxy$

$\exists y\exists xR[x,f(y)]$

(c)  $\exists y\exists xRyx$

$\exists y\exists xR[f(y),x]$

(d) none

(e)  $\exists y\exists xR[f(x),y]$

$\exists y\exists xR[f(x),f(y)]$

(f)  $\exists y\exists xR[y,f(x)]$

$\exists y\exists xR[f(y),f(x)]$

2B3: (a)  $\exists y\exists xR[s(x),p(x,y)]$

(b)  $\exists y\exists xR[y,p(x,b)]$

$\exists y\exists xR[s(a),p(x,y)]$

$\exists y\exists xR[s(y),p(x,b)]$

(c)  $\exists y\exists xRxy$

$\exists y\exists xR[x,p(y,b)]$

$\exists y\exists xR[x,p(a,y)]$

(d)  $\exists y\exists xRyx$

$\exists y\exists xR[s(y),x]$

(e)  $\exists y\exists xR[s(a),p(y,x)]$

$\exists y\exists xR[s(y),p(y,x)]$

$\exists y\exists xR[y,p(a,x)]$

$\exists y\exists xR[s(y),p(a,x)]$

(f)  $\exists y\exists xR[s(x),y]$

$\exists y\exists xR[s(x),p(a,y)]$

$\exists y\exists xR[s(x),p(y,b)]$

**3. EXERCISE SETS C-E****1:**

(1)	$P[f(b)]$	Pr
(2)	SHOW: $\exists xP[f(x)]$	DD
(3)	$\exists xP[f(x)]$	1, $\exists$ I

**#2:**

(1)	$P[f(b)]$	Pr
(2)	SHOW: $\exists xPx$	DD
(3)	$\exists xPx$	1, $\exists$ I

**#3:**

(1)	$\exists xF[f(x)]$	Pr
(2)	SHOW: $\exists xFx$	DD
(3)	$F[f(a)]$	1, $\exists$ O
(4)	$\exists xFx$	3, $\exists$ I

**4:**

(1)	$\forall x(Fx \rightarrow Hx)$	Pr
(2)	$F[p(a,b)]$	Pr
(3)	SHOW: $H[p(a,b)]$	DD
(4)	$F[p(a,b)] \rightarrow H[p(a,b)]$	1, $\forall$ O
(5)	$H[p(a,b)]$	2,4,SL

**#5:**

(1)	$\forall x(Fx \rightarrow Hx)$	Pr
(2)	$F[p(a,b)]$	Pr
(3)	SHOW: $\exists xHx$	DD
(4)	$F[p(a,b)] \rightarrow H[p(a,b)]$	1, $\forall$ O
(5)	$H[p(a,b)]$	2,4,SL
(6)	$\exists xHx$	5, $\exists$ I

**#6:**

(1)	$\forall x(Fx \rightarrow Hx)$	Pr
(2)	$\sim H[f(a)]$	Pr
(3)	SHOW: $\sim F[f(a)]$	DD
(4)	$F[f(a)] \rightarrow H[f(a)]$	1, $\forall$ O
(5)	$\sim F[f(a)]$	2,4,SL

**#7:**

(1)	$\forall x(Fx \rightarrow Hx)$	Pr
(2)	$\sim H[f(a)]$	Pr
(3)	SHOW: $\sim \forall xFx$	ID
(4)	$\forall xFx$	As
(5)	SHOW: $\times$	7,8,SL
(6)	$F[f(a)] \rightarrow H[f(a)]$	1, $\forall$ O
(7)	$\sim F[f(a)]$	2,6,SL
(8)	$F[f(a)]$	4, $\forall$ O

**#8:**

(1)	$\forall x(Fx \rightarrow Gx)$	Pr
(2)	$\forall x[(Fx \& Gx) \rightarrow \sim Hx]$	Pr
(3)	$H[p(a,b)]$	Pr
(4)	SHOW: $\exists x(Hx \& \sim Fx)$	10, $\exists I$
(5)	$(F[p(a,b)] \& G[p(a,b)]) \rightarrow \sim H[p(a,b)]$	2, $\forall O$
(6)	$\sim(F[p(a,b)] \& G[p(a,b)])$	3,5,SL
(7)	$F[p(a,b)] \rightarrow \sim G[p(a,b)]$	6,SL
(8)	$F[p(a,b)] \rightarrow G[p(a,b)]$	1, $\forall O$
(9)	$\sim F[p(a,b)]$	7,8,SL
(10)	$H[p(a,b)] \& \sim F[p(a,b)]$	3,9,SL

**#9:**

(1)	$\forall x(Fx \rightarrow Gx)$	Pr
(2)	$\forall x(Gx \rightarrow Hx)$	Pr
(3)	$F[m(a)]$	Pr
(4)	SHOW: $\exists x(Gx \& Hx)$	9, $\exists I$
(5)	$F[m(a)] \rightarrow G[m(a)]$	2, $\forall O$
(6)	$G[m(a)]$	3,5,SL
(7)	$G[m(a)] \rightarrow H[m(a)]$	2, $\forall O$
(8)	$H[m(a)]$	6,7,SL
(9)	$G[m(a)] \& H[m(a)]$	6,8,SL

**#10:**

(1)	$\forall x(Fx \rightarrow Gx)$	Pr
(2)	$\forall x(Gx \rightarrow Hx)$	Pr
(3)	$F[m(a)]$	Pr
(4)	SHOW: $\exists xG[m(x)] \& \exists xH[m(x)]$	7,8, $\&I$
(5)	$G[m(a)]$	1,3, $\forall \rightarrow O$
(6)	$H[m(a)]$	2,5, $\forall \rightarrow O$
(7)	$\exists xG[m(x)]$	5, $\exists I$
(8)	$\exists xH[m(x)]$	6, $\exists I$

**#11:**

(1)	$\forall x\{R[x,s(a)] \rightarrow \sim R[x,p(a,b)]\}$	Pr
(2)	$R[s(a),s(a)]$	Pr
(3)	SHOW: $\exists x \sim R[x,p(a,b)]$	5, $\exists I$
(4)	$R[s(a),s(a)] \rightarrow \sim R[s(a),p(a,b)]$	1, $\forall O$
(5)	$\sim R[s(a),p(a,b)]$	2,4,SL

**#12:**

(1)	$\exists xR[x,f(n)] \rightarrow \forall xR[x,f(n)]$	Pr
(2)	$\sim R[g(n),f(n)]$	Pr
(3)	SHOW: $\sim R[f(n),f(n)]$	ID
(4)	$R[f(n),f(n)]$	As
(5)	SHOW: $\times$	DD
(6)	$\exists xR[x,f(n)]$	4, $\exists I$
(7)	$\forall xR[x,f(n)]$	1,6,SL
(8)	$R[g(n),f(n)]$	7, $\forall O$
(9)	$\times$	2,8,SL

**#13:**

(1)	$\forall x(Fx \rightarrow Rxx)$	Pr
(2)	$F[p(c,d)]$	Pr
(3)	SHOW: $\exists xR[x,p(c,d)]$	5, $\exists$ I
(4)	$F[p(c,d)] \rightarrow R[(p(c,d)),p(c,d)]$	1, $\forall$ O
(5)	$R[(p(c,d)),p(c,d)]$	2,4,SL

**#14:**

(1)	$\forall x(Fx \rightarrow Rxx)$	Pr
(2)	$\forall x \sim R[m(a),x]$	Pr
(3)	SHOW: $\sim F[m(a)]$	ID
(4)	$F[m(a)]$	As
(5)	SHOW: $\times$	7,8,SL
(6)	$F[m(a)] \rightarrow R[m(a),m(a)]$	1, $\forall$ O
(7)	$R[m(a),m(a)]$	4,6,SL
(8)	$\sim R[m(a),m(a)]$	2, $\forall$ O

**#15:**

(1)	$\forall x(Fx \rightarrow \forall yRxy)$	Pr
(2)	$F[g(a)]$	Pr
(3)	SHOW: $R[g(a),g(a)]$	DD
(4)	$F[g(a)] \rightarrow \forall yR[g(a),y]$	1, $\forall$ O
(5)	$\forall yR[g(a),y]$	2,4,SL
(6)	$R[g(a),g(a)]$	5, $\forall$ O

**#16:**

(1)	$\exists xR[f(a),x] \rightarrow \forall xR[x,f(a)]$	Pr
(2)	$\sim R[g(b),f(a)]$	Pr
(3)	SHOW: $\sim R[f(a),f(a)]$	ID
(4)	$R[f(a),f(a)]$	As
(5)	SHOW: $\times$	DD
(6)	$\exists xR[f(a),x]$	4, $\exists$ I
(7)	$\forall xR[x,f(a)]$	1,6, $\exists$ I
(8)	$R[g(b),f(a)]$	7, $\forall$ O
(9)	$\times$	2,8,SL

**#17:**

(1)	$\forall x[\exists yRxy \rightarrow \forall yRyx]$	Pr
(2)	$R[f(a,b),f(a,b)]$	Pr
(3)	SHOW: $R[g(a,b),f(a,b)]$	DD
(4)	$\exists yR[f(a,b),y] \rightarrow \forall yR[y,f(a,b)]$	1, $\forall$ O
(5)	$\exists yR[f(a,b),y]$	2, $\exists$ I
(6)	$\forall yR[y,f(a,b)]$	4,5,SL
(7)	$R[g(a,b),f(a,b)]$	6, $\forall$ O

**#18:**

(1)	$\forall x(Rxx \rightarrow Fx)$	Pr
(2)	$\forall x\forall y(Rxy \rightarrow Rxx)$	Pr
(3)	$\sim F[g(a,b)]$	Pr
(4)	SHOW: $\sim R[g(a,b),b]$	ID
(5)	$R[g(a,b),b]$	As
(6)	SHOW: $\times$	DD
(7)	$R[g(a,b),b] \rightarrow R[g(a,b),g(a,b)]$	2, $\forall O2$
(8)	$R[g(a,b),g(a,b)]$	5,7,SL
(9)	$R[g(a,b),g(a,b)] \rightarrow F[g(a,b)]$	1, $\forall O$
(10)	$F[g(a,b)]$	8,9,SL
(11)	$\times$	3,10,SL

**#19:**

(1)	$\forall x(Fx \rightarrow Gx)$	Pr
(2)	$\forall xM[m(x),x]$	Pr
(3)	SHOW: $\forall x\{Fx \rightarrow \exists y\{Gy \& M[m(x),y]\}\}$	UD
(4)	SHOW: $Fa \rightarrow \exists y\{Gy \& M[m(a),y]\}$	CD
(5)	$Fa$	As
(6)	SHOW: $\exists y\{Gy \& M[m(a),y]\}$	10, $\exists I$
(7)	$Fa \rightarrow Ga$	1, $\forall O$
(8)	$Ga$	5,7,SL
(9)	$M[m(a),a]$	2, $\forall O$
(10)	$Ga \& M[m(a),a]$	8,9,SL

**#20:**

(1)	$\forall x\forall y\{(Nx \& Ny) \rightarrow N[p(x,y)]\}$	Pr
(2)	$\exists x\{Nx \& \forall y\{Ny \rightarrow E[p(x,y)]\}\}$	Pr
(3)	SHOW: $\exists x(Nx \& Ex)$	11, $\exists I$
(4)	$Na \& \forall y\{Ny \rightarrow E[p(a,y)]\}$	2, $\exists O$
(5)	$Na$	4,SL
(6)	$\forall y\{Ny \rightarrow E[p(a,y)]\}$	4,SL
(7)	$Na \rightarrow E[p(a,a)]$	6, $\forall O$
(8)	$E[p(a,a)]$	5,7,SL
(9)	$(Na \& Na) \rightarrow N[p(a,a)]$	1, $\forall O2$
(10)	$N[p(a,a)]$	5,9,SL
(11)	$N[p(a,a)] \& E[p(a,a)]$	8,10,SL

**#21:**

(1)	$\forall x\{R[x,f(x)] \rightarrow \sim Rxx\}$	Pr
(2)	SHOW: $\sim \exists x\forall yR[f(x),y]$	ID
(3)	$\exists x\forall yR[f(x),y]$	As
(4)	SHOW: $\times$	DD
(5)	$\forall yR[f(a),y]$	3, $\exists O$
(6)	$R[f(a),f(f(a))]$	5, $\forall O$
(7)	$R[f(a),f(f(a))] \rightarrow \sim R[f(a),f(a)]$	1, $\forall O$
(8)	$\sim R[f(a),f(a)]$	6,7,SL
(9)	$R[f(a),f(a)]$	5, $\forall O$
(10)	$\times$	8,9,SL

**#22:**

(1)	$\forall x\forall yP[p(x,y)]$	Pr
(2)	<b>SHOW:</b> $\forall x\exists yP[p(c(y),p(y,x))]$	UD
(3)	<b>SHOW:</b> $\exists yP[p(c(y),p(y,a))]$	4, $\exists$ I
(4)	$P[p(c(b),p(b,a))]$	1, $\forall$ O2

**#23:**

(1)	$\forall x(Fx \rightarrow Gx)$	Pr
(2)	$\forall x\{Gx \rightarrow F[f(x)]\}$	Pr
(3)	$\exists xF[g(x)]$	Pr
(4)	<b>SHOW:</b> $\exists xG[f(x)]$	11, $\exists$ I
(5)	$F[g(a)]$	3, $\exists$ O
(6)	$F[g(a)] \rightarrow G[g(a)]$	1, $\forall$ O
(7)	$G[g(a)]$	5,6,SL
(8)	$G[g(a)] \rightarrow F[f(g(a))]$	2, $\forall$ O
(9)	$F[f(g(a))]$	7,8,SL
(10)	$F[f(g(a))] \rightarrow G[f(g(a))]$	1, $\forall$ O
(11)	$G[f(g(a))]$	9,10,SL

**#24:**

(1)	$Fa$	Pr
(2)	$\forall x\{Fx \rightarrow F[m(x)]\}$	Pr
(3)	$\forall xR[x,m(x)]$	Pr
(4)	$\forall x\forall y\forall z\{(Rxy \ \& \ Ryz) \rightarrow Rxz\}$	Pr
(5)	<b>SHOW:</b> $\exists x\{Fx \ \& \ \{Rax \ \& \ R[m(a),x]\}\}$	12, $\exists$ I
(6)	$F[m(a)]$	1,2, $\forall$ $\rightarrow$ O
(7)	$F[m(m(a))]$	2,6, $\forall$ $\rightarrow$ O
(8)	$R[a,m(a)]$	3, $\forall$ O
(9)	$R[m(a),m(m(a))]$	3, $\forall$ O
(10)	$\{R[a,m(a)] \ \& \ R[m(a),m(m(a))]\} \rightarrow R[a,m(m(a))]$	4, $\forall$ O3
(11)	$R[a,m(m(a))]$	8,9,10,SL
(12)	$F[m(m(a))] \ \& \ \{R[a,m(m(a))] \ \& \ R[m(a),m(m(a))]\}$	8,9,11,SL

**#25:**

(1)	$\forall x\{Fx \rightarrow R[x,m(x)]\}$	Pr
(2)	$\forall xH[m(x)]$	Pr
(3)	<b>SHOW:</b> $\forall x\{Fx \rightarrow \exists y(Hy \ \& \ Rxy)\}$	UD
(4)	<b>SHOW:</b> $Fa \rightarrow \exists y(Hy \ \& \ Ray)$	CD
(5)	$Fa$	As
(6)	<b>SHOW:</b> $\exists y(Hy \ \& \ Ray)$	10, $\exists$ I
(7)	$Fa \rightarrow R[a,m(a)]$	1, $\forall$ O
(8)	$R[a,m(a)]$	5,7,SL
(9)	$H[m(a)]$	2, $\forall$ O
(10)	$H[m(a)] \ \& \ R[a,m(a)]$	8,9,SL

**#26:**

(1)	$\forall x\{D[f(x)] \rightarrow D[m(x)]\}$	Pr
(2)	$\sim\exists x\{Hx \& D[m(x)]\}$	Pr
(3)	SHOW: $\forall x\{D[f(x)] \rightarrow \sim Hx\}$	UD
(4)	SHOW: $D[f(a)] \rightarrow \sim Ha$	CD
(5)	$D[f(a)]$	As
(6)	SHOW: $\sim Ha$	ID
(7)	$Ha$	As
(8)	SHOW: $\times$	DD
(9)	$D[f(a)] \rightarrow D[m(a)]$	1, $\forall O$
(10)	$D[m(a)]$	5,9,SL
(11)	$Ha \& D[m(a)]$	7,10,SL
(12)	$\sim(Ha \& D[m(a)])$	2, $\sim\exists O$
(13)	$\times$	2,12,SL

**#27:**

(1)	$\forall x(\exists yR[x,f(y)] \rightarrow Hx)$	Pr
(2)	$\forall x(Fx \rightarrow \exists yRxy)$	Pr
(3)	$\forall x\forall y(Rxy \rightarrow R[x,f(y)])$	Pr
(4)	SHOW: $\forall x(Fx \rightarrow Hx)$	UD
(5)	SHOW: $Fa \rightarrow Ha$	CD
(6)	$Fa$	As
(7)	SHOW: $Ha$	DD
(8)	$Fa \rightarrow \exists yRay$	2, $\forall O$
(9)	$\exists yRay$	6,8,SL
(10)	$Rab$	9, $\exists O$
(11)	$Rab \rightarrow R[a,f(b)]$	3, $\forall O2$
(12)	$R[a,f(b)]$	10,11,SL
(13)	$\exists yR[a,f(y)] \rightarrow Ha$	1, $\forall O$
(14)	$\exists yR[a,f(y)]$	12, $\exists I$
(15)	$Ha$	13,14,SL

**#28:**

(1)	$\forall x(Fx \rightarrow Rxx)$	Pr
(2)	$\exists xF[m(x)]$	Pr
(3)	SHOW: $\exists x\exists yR[x,m(y)]$	DD
(4)	$F[m(a)]$	2, $\exists O$
(5)	$F[m(a)] \rightarrow R[m(a),m(a)]$	1, $\forall O$
(6)	$R[m(a),m(a)]$	4,5,SL
(7)	$\exists yR[m(a),m(y)]$	6, $\exists I$
(8)	$\exists x\exists yR[x,m(y)]$	7, $\exists I$

**#29:**

(1)	$\forall x(Fx \rightarrow Rxx)$	Pr
(2)	$\exists xF[m(x)]$	Pr
(3)	SHOW: $\exists x\exists yR[m(x),y]$	DD
(4)	$F[m(a)]$	2, $\exists O$
(5)	$F[m(a)] \rightarrow R[m(a),m(a)]$	1, $\forall O$
(6)	$R[m(a),m(a)]$	4,5,SL
(7)	$\exists yR[m(a),y]$	6, $\exists I$
(8)	$\exists x\exists yR[m(x),y]$	7, $\exists I$

**#30:**

(1)	$\forall x\{F[g(x)] \rightarrow F[m(x)]\}$	Pr
(2)	$\sim\exists x\{Fx \& F[m(x)]\}$	Pr
(3)	SHOW: $\forall x\{Fx \rightarrow \sim F[g(x)]\}$	UD
(4)	SHOW: $Fa \rightarrow \sim F[g(a)]$	CD
(5)	$Fa$	As
(6)	SHOW: $\sim F[g(a)]$	DD
(7)	$\forall x\sim\{Fx \& F[m(x)]\}$	2,QN
(8)	$\sim\{Fa \& F[m(a)]\}$	7, $\forall$ O
(9)	$\sim F[m(a)]$	5,8,SL
(10)	$F[g(a)] \rightarrow F[m(a)]$	1, $\forall$ O
(11)	$\sim F[g(a)]$	9,10,SL

**#31:**

(1)	$\forall x\{F[g(x)] \rightarrow F[m(x)]\}$	Pr
(2)	$\forall x\{Fx \rightarrow \sim F[m(x)]\}$	Pr
(3)	SHOW: $\sim\exists x\{Fx \& F[g(x)]\}$	ID
(4)	$\exists x\{Fx \& F[g(x)]\}$	As
(5)	SHOW: $\times$	DD
(6)	$Fa \& F[g(a)]$	4, $\exists$ O
(7)	$Fa \rightarrow \sim F[m(a)]$	2, $\forall$ O
(8)	$F[g(a)] \rightarrow F[m(a)]$	1, $\forall$ O
(9)	$\times$	6,7,8,SL

**#32:**

(1)	$\forall x\forall y\{Rxy \rightarrow Ryx\}$	Pr
(2)	$\forall x\forall y\{R[x,s(y)] \rightarrow Ryx\}$	Pr
(3)	SHOW: $\forall x\forall y\{R[s(x),y] \rightarrow Ryx\}$	UD
(4)	SHOW: $R[s(a),b] \rightarrow Rba$	CD
(5)	$R[s(a),b]$	As
(6)	SHOW: $Rba$	DD
(7)	$R[s(a),b] \rightarrow R[b,s(a)]$	1, $\forall$ O
(8)	$R[b,s(a)]$	5,7,SL
(9)	$R[b,s(a)] \rightarrow Rab$	2, $\forall$ O
(10)	$Rab$	8,9,SL
(11)	$Rab \rightarrow Rba$	1, $\forall$ O
(12)	$Rba$	10,11,SL

**#33:**

(1)	$\forall x\exists yK[x,s(y)]$	Pr
(2)	$\forall x\forall y\{K[x,y] \rightarrow R[x,s(y)]\}$	Pr
(3)	SHOW: $\forall x\exists yR[s(x),s(y)]$	UD
(4)	SHOW: $\exists yR[s(a),s(y)]$	DD
(5)	$\exists yK[s(a),s(y)]$	1, $\forall$ O
(6)	$K[s(a),s(b)]$	5, $\exists$ O
(7)	$K[s(a),s(b)] \rightarrow R[s(a),s(s(b))]$	2, $\forall$ O2
(8)	$R[s(a),s(s(b))]$	6,7,SL
(9)	$\exists yR[s(a),s(y)]$	8, $\exists$ I

**#34:**

(1)	$\forall x\forall y\forall z\{Rxy \ \& \ Ryz\} \rightarrow Rxz$	Pr
(2)	$\forall x\forall y\{Rxy \rightarrow R[f(y),f(x)]\}$	Pr
(3)	$\forall xR[x,f(f(x))]$	Pr
(4)	SHOW: $\forall x\forall y\{R[x,f(y)] \rightarrow R[y,f(x)]\}$	UD2
(5)	SHOW: $R[a,f(b)] \rightarrow R[b,f(a)]$	CD
(6)	$R[a,f(b)]$	As
(7)	SHOW: $R[b,f(a)]$	DD
(8)	$R[a,f(b)] \rightarrow R[f(f(b)),f(a)]$	2, $\forall$ O2
(9)	$R[f(f(b)),f(a)]$	6,7,SL
(10)	$R[b,f(f(b))]$	3, $\forall$ O
(11)	$[R[b,f(f(b))] \ \& \ R[f(f(b)),f(a)]] \rightarrow R[b,f(a)]$	1, $\forall$ O3
(12)	$R[b,f(a)]$	9,10,11,SL

**#35:**

(1)	$\forall x\forall y[Rxy \rightarrow Ryx]$	Pr
(2)	$\forall x\exists yR[x,s(y)]$	Pr
(3)	SHOW: $\forall x\exists yRyx$	UD
(4)	SHOW: $\exists yRya$	DD
(5)	$\exists yR[a,s(y)]$	2, $\forall$ O
(6)	$R[a,s(b)]$	5, $\exists$ O
(7)	$R[a,s(b)] \rightarrow R[s(b),a]$	1, $\forall$ O2
(8)	$R[s(b),a]$	6,7,SL
(9)	$\exists yRya$	8, $\exists$ I

**#36:**

(1)	$\forall x\{Fx \leftrightarrow \sim F[f(x)]\}$	Pr
(2)	SHOW: $\exists x\{Fx \ \& \ \sim F[f(x)]\}$	SC
(3)	$Fa \vee \sim Fa$	SL
(4)	c1: $Fa$	As
(5)	SHOW: $\exists x\{Fx \ \& \ \sim F[f(x)]\}$	DD
(6)	$Fa \leftrightarrow \sim F[f(a)]$	1, $\forall$ O
(7)	$Fa \ \& \ \sim F[f(a)]$	4,6,SL
(8)	$\exists x\{Fx \ \& \ \sim F[f(x)]\}$	7, $\exists$ I
(9)	c2: $\sim Fa$	As
(10)	SHOW: $\exists x\{Fx \ \& \ \sim F[f(x)]\}$	DD
(11)	$Fa \leftrightarrow \sim F[f(a)]$	1, $\forall$ O
(12)	$F[f(a)]$	9,11,SL
(13)	$F[f(a)] \leftrightarrow \sim F[f(f(a))]$	1, $\forall$ O
(14)	$\sim F[f(f(a))]$	12,13,SL
(15)	$F[f(a)] \ \& \ \sim F[f(f(a))]$	12,14,SL
(16)	$\exists x\{Fx \ \& \ \sim F[f(x)]\}$	15, $\exists$ I

**#37:**

(1)	$\forall xF[m(x)]$	Pr
(2)	SHOW: $\exists x\{Fx \ \& \ F[m(x)]\}$	DD
(3)	$F[m(a)]$	1, $\forall$ O
(4)	$F[m(m(a))]$	1, $\forall$ O
(5)	$F[m(a)] \ \& \ F[m(m(a))]$	3,4,SL
(6)	$\exists x\{Fx \ \& \ F[m(x)]\}$	5, $\exists$ I

**#38:**

(1)	SHOW: $\forall x\{R[x,f(x)] \leftrightarrow \exists y\{\forall z(R[z,y] \rightarrow R[z,f(x)]) \& R[x,y]\}\}$	UD
(2)	SHOW: $R[a,f(a)] \leftrightarrow \exists y\{\forall z(R[z,y] \rightarrow R[z,f(a)]) \& R[a,y]\}$	3,10, $\leftrightarrow$ I
(3)	SHOW: $R[a,f(a)] \rightarrow \exists y\{\forall z(R[z,y] \rightarrow R[z,f(a)]) \& R[a,y]\}$	CD
(4)	$R[a,f(a)]$	As
(5)	SHOW: $\exists y\{\forall z(R[z,y] \rightarrow R[z,f(a)]) \& R[a,y]\}$	6, $\exists$ I
(6)	SHOW: $\forall z(R[z,f(a)] \rightarrow R[z,f(a)]) \& R[a,f(a)]$	DD
(7)	SHOW: $\forall z(R[z,f(a)] \rightarrow R[z,f(a)])$	UD
(8)	SHOW: $R[b,f(a)] \rightarrow R[b,f(a)]$	SL
(9)	$\forall z(R[z,f(a)] \rightarrow R[z,f(a)]) \& R[a,f(a)]$	4,7,SL
(10)	SHOW: $\exists y\{\forall z(R[z,y] \rightarrow R[z,f(a)]) \& R[a,y]\} \rightarrow R[a,f(a)]$	CD
(11)	$\exists y\{\forall z(R[z,y] \rightarrow R[z,f(a)]) \& R[a,y]\}$	As
(12)	SHOW: $R[a,f(a)]$	DD
(13)	$\forall z(R[z,b] \rightarrow R[z,f(a)]) \& R[a,b]$	11, $\exists$ O
(14)	$R[a,b] \rightarrow R[a,f(a)]$	13a, $\forall$ O
(15)	$R[a,f(a)]$	13b,14,SL