UNIT 1: TRANSLATIONS IN FUNCTION LOGIC

1. Directions for all Exercise Sets in Unit 1

Using the suggested abbreviations (the capitalized words), translate each of the following into the language of function logic. Symbolize all and only capitalized words.

Important: Write down your lexicon.

In each case, clearly indicate what each predicate letter, each function letter, and each proper-noun letter symbolizes.

Examples:

$V[\alpha]$	:	α is a voter
$R[\alpha,\beta]$	:	α respects β
$S[\alpha,\beta,\gamma]$	:	α sold β to γ
$m(\alpha)$	:	the mother of α
$f(\alpha)$	:	α's father
$s(\alpha,\beta)$	:	the sum of α and β
$p(\alpha,\beta)$	:	α plus β
j	:	Jay

In writing down the lexicon, use lower case Greek letters as generic arguments (place holders); in particular, use 'α' for one-place functors, 'α', 'β' for two-place functors, and 'α','β','γ' for three-place functors. [brief note on Greek alphabet: α is alpha, β is beta, γ is gamma, δ is delta; and, of course, the word 'alphabet' comes from 'αβ']

Also, in quantified sentences, identify the universe of discourse (the domain over which the quantifiers range) . For example,

U : things (the most general domain)
U : persons
U : numbers
etc.

Alternatively, you may write:

$\forall x$: every thing
$\forall x$: every person
$\forall x$: every number
etc.

Final Note: You may use numerals ('1', '2', etc.) to symbolize themselves.

2. EXERCISE SET A

1. JAY's MOTHER is TALL.
2. The SQUARE of TWO is EVEN.
3. The square ROOT of 2 is not RATIONAL.
4. The FIRST bar of JAY's favorite SONG is not MEMORABLE.
5. JAY'S FAVORITE song is LOUDER than KAY'S FAVORITE song.
6. The SQUARE of TWO PLUS FOUR is EVEN (symbolize both readings).
7. The PRODUCT of 3 and 4 is LARGER than the SUM of 3 and 4.
8. JAY'S MOTHER is TALLER than KAY'S FATHER.
9. JAY'S MIDDLE name is the SAME as the FIRST name of the PRESIDENT of the US.
10. $(a + b) \times c = (a \times c) + (b \times c)$.
11. $a + b$ is less than $a \times b$
12. $(a + b)^2 = a^2 + (2(ab) + b^2)$.
13. Both of JAY's parents are DEMOCRATS.
14. Neither of KAY's parents is a DEMOCRAT.
15. JAY's parents aren't both POLITICIANS.
16. Exactly one of KAY's parents is a POLITICIAN.
17. At least one of JAY'S parents is an POLITICIAN
18. The sQUARE root of the SUM of 2 and 4 is not RATIONAL.
19. JAY's FATHER is neither HEAVIER nor TALLER than JAY's MOTHER.
20. CAIN did not CLAIM to be his BROTHER's KEEPER.

3. EXERCISE SET B

21. Every WOMAN RESPECTS her MOTHER.
22. Not every MAN RESPECTS his FATHER.
23. Not every BOOK has been WRITTEN by its (putative) AUTHOR.
24. No PERSON is OLDER than his/her MOTHER.
25. No one PRESENT ENJOYS JAY'S FAVORITE song.
26. At least one WOMAN is TALLER than her FATHER.
27. Everyone who LOVES him(her)self also LOVES his(her) MOTHER.
28. The SQUARE of any EVEN number is EVEN.
29. There is no ODD number whose SQUARE is EVEN.
30. The PRODUCT of any two EVEN numbers is also EVEN.
31. If one number is LARGER than a second number, then the SQUARE of the former is LARGER than the SQUARE of the latter.
32. The PRODUCT of any two WHOLE numbers is LARGER than (or equal to) both of them.
33. There is no NATURAL number that is LARGER than its SQUARE.
34. Any number ADDED to its NEGATIVE EQUALS ZERO.
35. Everyone is RATIONAL whose MOTHER and FATHER are RATIONAL.
36. No WOMAN RESPECTS her FATHER if he is a SCOUNDREL.
37. No MAN HONORS his FATHER if he is a CROOK (symbolize both readings).
38. The SQUARE of the HYPOTENUSE of a RIGHT triangle is EQUAL to the SUM of the SQUARES of its BASE and VERTICAL sides.
39. Everyone who RESPECTS his/her FATHER also RESPECTS his/her MOTHER.
40. KAY doesn't LIKE any BOOK RECOMMENDED to her by her FATHER's WIFE.

4. EXERCISE SET C

41. For any numbers x,y,z, $x+(y+z)=(x+y)+z$.

42. For any three points x,y,z, the DISTANCE between x and y ADDED to the DISTANCE between y and z is GREATER than the DISTANCE between x and z.

43. Everyone KNOWS someone's MOTHER.

44. There is someone who KNOWS everyone's MOTHER.

45. Everyone RESPECTS the MOTHER of each of his/her FRIENDS.

46. Everyone who RESPECTS the MOTHER of everyone also RESPECTS the FATHER of everyone.

47. If everyone HONORS his/her FATHER, then everyone HONORS his/her MOTHER.

48. Everyone who KNOWS everyone's MOTHER KNOWS everyone's FATHER.

49. Everyone who RESPECTS his/her MOTHER RESPECTS every FRIEND of his/her MOTHER.

50. Everyone who RESPECTS the MOTHER of every POLITICIAN also RESPECTS the FATHER of every POLITICIAN.

51. Every PERSON who KNOWS the MOTHER of every SENATOR also KNOWS the AGE of every SENATOR.

52. Everyone who RESPECTS the MOTHER of each of his/her FRIENDS also RESPECTS the FATHER of each of his/her FRIENDS.

53. KAY doesn't LIKE any BOOK RECOMMENDED to her by any FRIEND of her MOTHER.

54. Every PERSON who RESPECTS the PRESIDENT of every COUNTRY RESPECTS the PRESIDENT of his/her own COUNTRY.

55. Every PERSON who KNOWS the HEAD (of state) of every NATION is RESPECTED by every CITIZEN of his/her own NATION.

56. The FAVORITE song of the PRESIDENT of a COUNTRY IS also the FAVORITE song of all of its CITIZENS.

57. Every PRESIDENT has a NEPHEW whose MIDDLE name is the SAME.

58. For any two natural numbers, the first is LARGER than the second if and only if the SQUARE of the first is LARGER than the SQUARE of the second. (U = Natural Numbers)

59. Every MOTHER IS someone's MOTHER. (Everyone who is a MOTHER is the MOTHER of someone.)

60. A person is a MOTHER if and only if she IS someone's MOTHER.
    (...iff there is someone whose mother she is.)

5. EXERCISE SET D

61. The NEGATION of any TAUTOLOGY is a CONTRADICTION.

62. The NEGATION of any TRUE statement is FALSE, and the NEGATION of any FALSE statement is TRUE.

63. The CONJUNCTION of any two sentences IMPLIES both of them.

64. If an ARGUMENT is VALID, but its CONCLUSION is FALSE, then at least one of its PREMISES is FALSE.

65. Any ARGUMENT whose PREMISES are all TRUE but whose CONCLUSION is FALSE is *in*VALID (i.e., *not* VALID).

66. A CONDITIONAL is FALSE if and only if its ANTECEDENT is TRUE and its CONSEQUENT is FALSE. (U = sentences)

67. All TAUTOLOGIES are logically EQUIVALENT to each other.

68. The RESULT of applying any one-place FUNCTION sign to any singular TERM is a singular TERM.

69. The RESULT of applying any one-place PREDICATE sign to any singular TERM is a SENTENCE.

70. The FIRST symbol of every (officially displayed) CONJUNCTION is a PARENTHESIS.

71. The NEGATION of any FORMULA is also a FORMULA.

72. The CONJUNCTION of any two FORMULAS is also a FORMULA.

73. Every *in*VALID ARGUMENT admits a COUNTEREXAMPLE. (I.e., For every invalid argument there is a counterexample to it.)

74. If all the PREMISES of an ARGUMENT are TAUTOLOGIES, and the argument is VALID, then the CONCLUSION of that argument is also a TAUTOLOGY.

75. If the ANTECEDENT of a CONDITIONAL is TRUE, and its CONSEQUENT is FALSE, then the CONDITIONAL is FALSE; otherwise, it is TRUE.

6. Answers to Exercises

1. The Lexicon

The lexicon is as follows; note that many letters are duplicated.

Proper nouns

c	Cain
j	Jay
k	Kay
u	the US

1-place function signs

a(α)	the age of α
	the (putative) author of α
	the antecedent of α
b(α)	the brother of α
	the base side of α
c(α)	the country of α
	the conclusion of α
	the consequent of α
f(α)	the father of α
	the first bar of α
	the favorite song of α
	the first name of α
	the first symbol of α
h(α)	the hypotenuse of
	the head of state of α
k(α)	the keeper of α
m(α)	the mother of α
	the middle name of α
n(α)	the negation of α
	the nation of α
p(α)	the president of α
q(α)	the square of
r(α)	the square root of
s(α)	the square of α
f(α)	the favorite song of α
v(α)	the vertical side of α
w(α)	the wife of α
	2-place function signs
a(α,β)	α added to β
c(α,β)	the conjunction of α and β
d(α,β)	the distance between α and β
p(α,β)	the product of α and β
r(α,β)	the result of applying α to β
s(α,β)	the sum of α and β

1-place predicates

A[α]	α is an attorney
	α is an argument
B[α]	α is a book
C[α]	α is a crook
	α is a country
	α is a contradiction
	α is a conditional
	α is a conjunction
D[α]	α is a Democrat
E[α]	α is even
F[α]	α is false
	α is a one-place function sign
	α is a formula
M[α]	α is a man
	α is memorable
N[α]	α is a nation
	α is a natural number
O[α]	α is odd
P[α]	α is a politician
	α is a president
	α is a predicate
	α is a parenthesis
	α is a person
	α is present
R[α]	α is rational
	α is a right triangle
S[α]	α is a scoundrel
	α is a senator
	α is a sentence
T[α]	α is tall
	α is a tautology
	α is true
	α is a singular term
V[α]	α is valid
W[α]	α is a woman

2-place predicates

$C[\alpha,\beta]$	α claimed to be β
	α is a citizen of β
	α is a counterexample to β
$E[\alpha,\beta]$	α equals β
	α and β are logically equivalent
	α enjoys β
$F[\alpha,\beta]$	α is a friend of β
$G[\alpha,\beta]$	α is greater than β
$H[\alpha,\beta]$	α is heavier than β
$I[\alpha,\beta]$	α implies β
	α is β
$K[\alpha,\beta]$	α knows β
$L[\alpha,\beta]$	α is larger than β
	α is louder than β
	α is less than to β
	α likes β
	α loves β
$N[\alpha,\beta]$	α is a nephew of β
$O[\alpha,\beta]$	α is older than β
$P[\alpha,\beta]$	α is a premise of β
$R[\alpha,\beta]$	α respects β
$S[\alpha,\beta]$	α is the same as β
$T[\alpha,\beta]$	α is taller than
$W[\alpha,\beta]$	α wrote β

3-place predicates

$R[\alpha,\beta,\gamma]$	α recommends β to γ

2. Exercise Set A

1. $T[m(j)]$
2. $E[s(2)]$
3. $\sim R[r(2)]$
4. $\sim M[f(s(j))]$
5. $L[f(j),f(k)]$
6. $E[q(p(2,4))]$; $E[p(s(2),4)]$
7. $L[p(3,4),s(3,4)]$
8. $T[m(j),f(k)]$
9. $S[m(j),f(p(u))]$
10. $E[p(s(a,b),c), s(p(a,c),p(b,c))]$
11. $L[s(a,b),p(a,b)]$
12. $E[q(s(a,b)),s(q(a),s(p(2,p(a,b)),q(b)))]$
13. $D[f(j)]$ & $D[m(j)]$
14. $\sim D[f(k)]$ & $\sim D[m(k)]$
15. $\sim(P[f(j)]$ & $P[m(j)])$
16. $(P[f(k)]$ & $\sim P[m(k)]) \vee (P[m(k)]$ & $\sim P[f(k)])$
17. $P[f(j)] \vee P[m(j)]$
18. $\sim R[r(s(2,4)]$
19. $\sim T[f(j),m(j)]$ & $\sim H[f(j),m(j)]$
20. $\sim C[c,k(b(c))]$

3. Exercise Set B

21. $\forall x\{Wx \to R[x,m(x)]\}$
22. $\sim\forall x\{Mx \to R[x,f(x)]\}$
23. $\sim\forall x\{Bx \to W[a(x),x]\}$
24. $\sim\exists x\{Px$ & $O[x,m(x)]\}$
25. $\sim\exists x\{Px$ & $E[x,f(j)]\}$
26. $\exists x\{Wx$ & $T[x,f(x)]\}$
27. $\forall x\{Lxx \to L[x,m(x)]\}$
28. $\forall x\{Ex \to E[s(x)]\}$
29. $\sim\exists x\{Ox$ & $E[s(x)]\}$

30. $\forall x\forall y\{[Ex \& Ey] \to E[p(x,y)]\}$
31. $\forall x\forall y\{Lxy \to L[s(x),s(y)]\}$
32. $\forall x\forall y\{[Wx \& Wy] \to \{L[p(x,y),x] \& L[p(x,y),y]\}\}$
33. $\sim\exists x\{Nx \& L[x,s(x)]\}$
34. $\forall xE[s(x,n(x)),0]$
35. $\forall x\{[R[m(x)] \& R[f(x)]] \to Rx\}$
36. $\sim\exists x\{Wx \& R[x,f(x)] \& S[f(x)]\}$
37. $\sim\exists x\{Mx \& H[x,f(x)] \& Cx\}$; $\sim\exists x\{Mx \& H[x,f(x)] \& C[f(x)]\}$
38. $\forall x\{Rx \to E[q(h(x)),s(q(b(x)),q(v(x)))]\}$
39. $\forall x\{R[x,f(x)] \to R[x,m(x)]\}$
40. $\forall x\{[Bx \& R[w(f(k)),x,k]] \to \sim Lkx\}$

4. Exercise Set C

41. $\forall x\forall y\forall z\{E[s(x,s(y,z))\ s(s(x,y),z)]\}$
42. $\forall x\forall y\forall z\{G[s(d(x,y),d(y,z)),d(x,z)]\}$
43. $\forall x\exists yK[x,m(y)]$
44. $\exists x\forall yK[x,m(y)]$
45. $\forall x\forall y\{Fyx \to R[x,m(y)]\}$
46. $\forall x\{\forall yR[x,m(y)] \to \forall yR[x,f(y)]\}$
47. $\forall xH[x,f(x)] \to \forall xH[x,m(x)]$
48. $\forall x\{\forall yK[x,m(y)] \to \forall yK[x,f(y)]\}$
49. $\forall x\{R[x,m(x)] \to \forall y\{F[y,m(x)] \to Rxy\}\}$
50. $\forall x\{\forall y\{Py \to R[x,m(y)]\} \to \forall y\{Py \to R[x,f(y)]\}\}$
51. $\forall x\{\{Px \& \forall y\{Sy \to K[x,m(y)]\}\} \to \forall y\{Sy \to K[x,a(y)]\}\}$
52. $\forall x\{\forall y\{Fyx \to R[x,m(y)]\} \to \forall y\{Fyx \to R[x,f(y)]\}\}$
53. $\forall x\forall y\{\{Bx \& F[y,m(k)] \& Ryxk\} \to \sim Lkx\}$
54. $\forall x\{\{Px \& \forall y\{Cy \to R[x,p(y)]\}\} \to R[x,p(c(x))]\}$
55. $\forall x\{[Px \& \forall y\{Ny \to K[x,h(y)]\}] \to \forall y\{C[y,n(x)] \to Ryx\}\}$
56. $\forall x\{Cx \to \forall y\{Cyx \to I[f(p(x)),f(y)]\}\}$
57. $\forall x\{Px \to \exists y\{Nyx \& S[m(y),m(x)]\}\}$
58. $\forall x\forall y\{L[x,y] \leftrightarrow L[q(x),q(y)]\}$
59. $\forall x\{Mx \to \exists yI[x,m(y)]\}$
60. $\forall x\{Mx \leftrightarrow \exists yI[x,m(y)]\}$

5. Exercise Set D

61. $\forall x\{Tx \to C[n(x)]\}$
62. $\forall x\{Tx \to F[n(x)]\} \& \forall x\{Fx \to T[n(x)]\}$
63. $\forall x\forall y\{I[c(x,y),x] \& I[c(x,y),y]\}$
64. $\forall x\{\{Ax \& Vx \& F[c(x)]\} \to \exists y\{Pyx \& Fy\}\}$
65. $\forall x\{\{Ax \& \forall y\{Pyx \to Ty\} \& F[c(x)]\} \to \sim Vx\}$
66. $\forall x\{Cx \to (Fx \leftrightarrow [T[a(x)] \& F[c(x)]])\}$
67. $\forall x\forall y((T[x] \& T[y]) \to (Exy \& Eyx))$
68. $\forall x\forall y\{\{Fx \& Ty\} \to T[r(x,y)]\}$
69. $\forall x\forall y\{\{Px \& Ty\} \to S[r(x,y)]\}$
70. $\forall x\{Cx \to P[f(x)]\}$
71. $\forall x\{Fx \to F[n(x)]\}$
72. $\forall x\forall y\{\{Fx \& Fy\} \to F\{c(x,y)]\}$
73. $\forall x\{\{Ax \& \sim Vx\} \to \exists yCyx\}$
74. $\forall x\{\{Ax \& \forall y\{Pyx \to Ty\} \& Vx\} \to T[c(x)]\}$
75. $\forall x\{Cx \to ((\{T[a(x)] \& F[c(x)]\} \to Fx) \& (\sim\{T[a(x)] \& F[c(x)]\} \to Tx))\}$