INTRO LOGIC

DAY 22

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UNIT 4
Derivations
in
Predicate Logic

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Overview

✓	Exam 1:	Sentential Logic	Translations (+)
✓	Exam 2:	Sentential Logic	Derivations
✓	Exam 3:	Predicate Logic	Translations
+	Exam 4:	Predicate Logic	Derivations
+	Exam 5:	(finals week)	very similar to Exam 3
+	Exam 6:	(finals week)	very similar to Exam 4

Exams 5 and 6 are scheduled for
Tuesday, May 20
10:30-12:30
Mahar

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Predicate Logic Subsumes Sentential Logic

Every **rule** of Sentential Logic is also a **rule** of Predicate Logic.

- CD
- ID
- $\rightarrow$O
- $\vee$O
- etc.

Every **strategy** of Sentential Logic is also a **strategy** of Predicate Logic.

- SHOW: $\mathcal{A} \rightarrow C$
- SHOW: $\sim \mathcal{A}$
- SHOW: $\mathcal{A} \vee \mathcal{B}$
- SHOW: $\mathcal{A} \& \mathcal{B}$
- etc.

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SL-Example 1

if no one is **H**, then **k** is not **H**

(1)	SHOW: $\sim\exists xHx \rightarrow \sim Hk$	CD
(2)	$\sim\exists xHx$	As
(3)	SHOW: $\sim Hk$	$\sim$D
(4)	Hk	As
(5)	SHOW: ※	DD
(6)	???	2,$\sim\exists$O
(?)	※	???

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SL-Example 2

if everyone is un-H, then no one is H

(1)	SHOW: $\forall x\sim Hx \rightarrow \sim\exists xHx$	CD
(2)	$\forall x\sim Hx$	As
(3)	SHOW: $\sim\exists xHx$	$\sim$D
(4)	$\exists xHx$	As
(5)	SHOW: ※	DD
(6)	???	2,$\forall$O
(7)	???	4,$\exists$O
(?)	※	???

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SL-Example 3

if someone is F or H, then someone is F or someone is H		
(1)	SHOW: $\exists x(Fx \vee Hx) \rightarrow (\exists xFx \vee \exists xHx)$	CD
(2)	$\exists x(Fx \vee Hx)$	As
(3)	SHOW: $\exists xFx \vee \exists xHx$	$\vee$D (ID)
(4)	$\sim (\exists xFx \vee \exists xHx)$	As
(5)	SHOW: ※	DD
(6)	$\sim\exists xFx$	4,$\sim\vee$O
(7)	$\sim\exists xHx$	4,$\sim\vee$O
(8)	???	2,$\exists$O
(9)	???	6,$\sim\exists$O
(10)	???	7,$\sim\exists$O
(?)	※	???

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SL-Example 4

if everyone is **F** and **H**,
then everyone is **F** and everyone is **H**

(1)	SHOW: $\forall x(Fx \,\&\, Hx) \rightarrow (\forall xFx \,\&\, \forall xHx)$	CD
(2)	$\forall x(Fx \,\&\, Hx)$	As
(3)	SHOW: $\forall xFx \,\&\, \forall xHx$	&D
(4)	SHOW: $\forall xFx$	??
(?)	??	??
(?)	??	??
(?)	SHOW: $\forall xHx$	??
(?)	??	??
(?)	??	??

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Rules of Predicate Logic (overview)

Logical operators	RULES		
	IN	OUT	∼OUT
&	&I	&O	∼&O
∨	∨I	∨O	∼∨O
→	CD	→O	∼→O
∀	UD	∀O	∼∀O
∃	∃I	∃O	∼∃O

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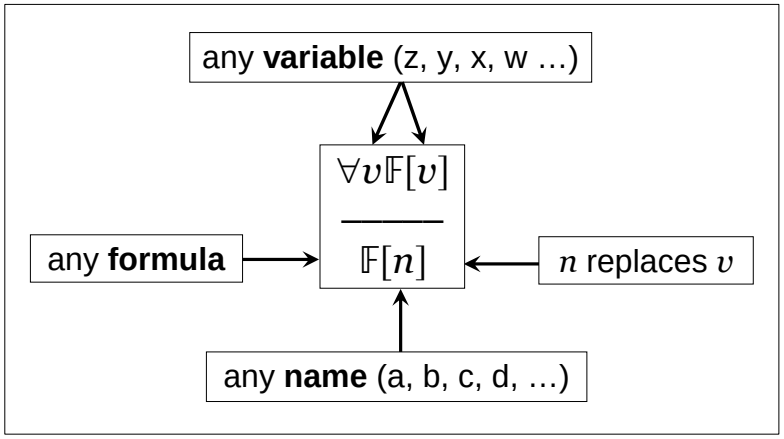
Universal-Out (∀O)



numerous restrictions (later)

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∀O – Example 1a

1. remove quantifier

2. choose name

3. substitute name for variable

∀	x	H	x
			a

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∀O – Example 1b

1. remove quantifier

2. choose name

3. substitute name for variable

∀	x	H	x
			b

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∀O – Example 1c

1. remove quantifier
2. choose name
3. substitute name for variable

∀	x	H	x
			c

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∀O – Example 2a

∀	x	(	F	a	→	H	a	)
				a			a	

1. remove quantifier
2. remove parentheses
3. choose name
4. sub name for variable

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∀O – Example 2b

∀	x	(	F	b	→	H	b	)
				b			b	

1. remove quantifier
2. remove parentheses
3. choose name
4. sub name for variable

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Derivation Example 1

every **F** is **H** ; **k** is **F** / **k** is **H**

(1)	∀x(Fx → Hx)	Pr	
(2)	Fk	Pr	
(3)	~~SHOW~~: Hk	DD	
(4)	Fk → Hk	1,	∀O
(5)	Hk	2,4,	→O

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Example 2

every **F** is un-**H** ; **k** is **F** / not every **F** is **H**			
(1)	∀x(Fx → ∼Hx)	Pr	
(2)	Fk	Pr	
(3)	SHOW: ∼∀x(Fx → Hx)	∼D	
(4)	∀x(Fx → Hx)	As	
(5)	SHOW: ※	DD	
(6)	Fk → ∼Hk	1,	∀O
(7)	Fk → Hk	4,	∀O
(8)	∼Hk	2,6,	→O
(9)	Hk	2,7,	→O
(10)	※	8,9,	※I

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Example 3

every **F** **R**'s him/herself ; **j** doesn't **R** anyone / **j** is not **F**			
(1)	∀x(Fx → Rxx)	Pr	
(2)	∀x∼Rjx	Pr	
(3)	SHOW: ∼Fj	∼D	
(4)	Fj	As	
(5)	SHOW: ※	DD	
(6)	Fj → Rjj	1,	∀O
(7)	∼Rjj	2,	∀O
(8)	Rjj	4,6,	→O
(9)	※	7,8,	※I

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Example 4

if anyone is F, then everyone is H
j is F
/ k is H

(1)	$\forall x \{ Fx \rightarrow \forall y Hy \}$	Pr	
(2)	Fj	Pr	
(3)	SHOW: Hk	DD	
(4)	$Fj \rightarrow \forall y Hy$	1,	$\forall$O
(5)	$\forall y Hy$	2,4,	$\rightarrow$O
(6)	Hk	5,	$\forall$O

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Example 5

everyone who **R**'s him(her)self is **F**
everyone who **R**'s anyone **R**'s him(her)self
j is not **F**
/ **j** does not **R k**

(1)	$\forall x \{ Rxx \rightarrow Fx \}$	Pr
(2)	$\forall x \forall y \{ Rxy \rightarrow Rxx \}$	Pr
(3)	$\sim$Fj	Pr
(4)	SHOW: $\sim$Rjk	

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(1)	∀x { Rxx → Fx }	Pr	
(2)	∀x ∀y { Rxy → Rxx }	Pr	
(3)	∼Fj	Pr	
(4)	SHOW: ∼Rjk	∼D	
(5)	Rjk	As	
(6)	SHOW: ※	DD	
(7)	∀y { Rjy → Rjj }	2,	∀O
(8)	Rjk → Rjj	7,	∀O
(9)	Rjj	5,8,	→O
(10)	Rjj → Fj	1,	∀O
(11)	Fj	9,10,	→O
(12)	※	3,11,	※I

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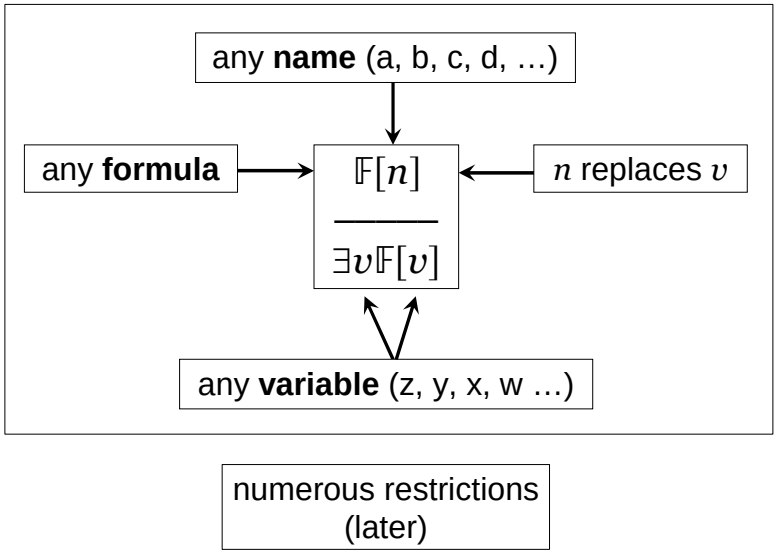
Existential-In (∃I)



numerous restrictions (later)

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∃I – Example 1

∃	x	F	x
			x

1. select name/variable
2. replace name by variable
3. restore missing parentheses (if any) — there aren't any
4. insert quantifier

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∃I – Example 2

∃	x	(	F	x	&	H	a	)
				x			x	

1. select name/variable
2. replace name by variable
3. restore missing parentheses (if any)
4. insert quantifier

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∃I – Examples 3,4,5

	Kay	respects	Kay	(herself)
therefore	someone respects herself			
therefore	someone respects Kay			
therefore	Kay respects someone			

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∃I – Example 3

∃	x	R	x	x
			x	x

1. select name/variable
2. replace name by variable
3. restore missing parentheses (if any) — there aren't any
4. insert quantifier

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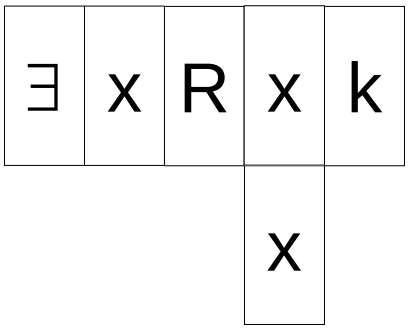
∃I – Example 4



1. select name/variable
2. replace name by variable
3. restore missing parentheses (if any) — there aren't any
4. insert quantifier

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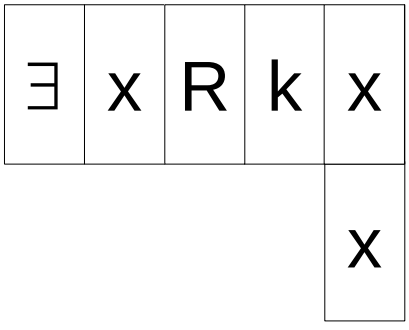
∃I – Example 5



1. select name/variable
2. replace name by variable
3. restore missing parentheses (if any) — there aren't any
4. insert quantifier

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Derivation Example 6

every **F** is **H ; k** is **F** / someone is **H**			
(1)	$\forall x (Fx \rightarrow Hx)$	Pr	
(2)	Fk	Pr	
(3)	SHOW: $\exists xHx$	DD	
(4)	$Fk \rightarrow Hk$	1,	$\forall O$
(5)	Hk	2,4,	$\rightarrow O$
(6)	$\exists xHx$	5,	$\exists I$

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Example 7

if someone is **F** then everyone is **H** **k** is **F** / **j** is **H**			
(1)	$\exists xFx \rightarrow \forall xHx$	Pr	
(2)	Fk	Pr	
(3)	SHOW: Hj	DD	
(4)	$\exists xFx$	2,	$\exists I$
(5)	$\forall xHx$	1,4,	$\rightarrow O$
(6)	Hj	5,	$\forall O$

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Example 8

every F R's him/herself
k is F / someone R's k

(1)	$\forall x(Fx \rightarrow Rxx)$	Pr	
(2)	Fk	Pr	
(3)	~~SHOW~~: $\exists xRxk$	DD	
(4)	$Fk \rightarrow Rkk$	1,	$\forall O$
(5)	Rkk	2,4,	$\rightarrow O$
(6)	$\exists xRxk$	5,	$\exists I$

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Example 9

everyone who **R**'s someone is **R**'ed by everyone
k **R**'s herself
/ j **R**'s k

(1)	$\forall x \{ \exists yRxy \rightarrow \forall yRyx \}$	Pr	
(2)	Rkk	Pr	
(3)	~~SHOW~~: Rjk	DD	
(4)	$\exists yRky \rightarrow \forall yRyk$	1,	$\forall O$
(5)	$\exists yRky$	2,	$\exists I$
(6)	$\forall yRyk$	4,5,	$\rightarrow O$
(7)	Rjk	6,	$\forall O$

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THE END

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