

Phil 104
Logic Handout #2

Conditionals and Classes

A *conditional* is a sentence of the form 'If A then B'. It is false only when the antecedent (the *if* clause) is true and the consequent (the *then* clause) is false; otherwise it is true. You can see why this is by imagining the conditional reads 'If it is cold, it snows'. This conditional is false. Why? Suppose it is cold, making the antecedent true. But suppose it does not snow, making the consequent false. But if it is cold and does not snow, the whole conditional is thereby false.

A *class* (or sometimes *set*) is a collection of things. The things in a class are its *members*. Some classes may contain *sub-classes* (sometimes *subsets*). A sub-class is a class all of whose members are members of the larger class. But not all the members of the larger class need be members of the sub-class.

For example: Consider the class of animals. It has as a sub-class the class of mammals. All mammals are animals, but not all animals are mammals. Some are amphibians, reptiles, etc.

We can express relationships between classes and sub-classes using conditionals.

For example: If something is a mammal, then it is an animal.

This sentence is true because all mammals are animals; every member of the class of mammals is an animal.

But this sentence is false: If something is an animal, then it is a mammal.

This is because there are some animals which are not mammals.

Co-extensionality and the Bi-conditional

All the members of a class or set are called the *extension* of the set. If an object is in a class, it is said to be in the *extension* of that class.

Two classes may be *co-extensive*. This means they have all the same members.

For example: The class containing all the people taking Phil 104 during Fall 03 at Uconn-Avery Point, and the class containing all the people sitting in Room 312 of the Undergraduate building at Uconn-Avery Point right now, are co-extensive.

Suppose two classes, A and B, are co-extensive. Then 1) there are no members of A which are not also members of B, and 2) there are no members of B which are not also members of A.

Recalling what we said about conditionals above, we can say the following about 1) and 2):

1.1) If something is a member of A, then it is a member of B (If A then B).

2.1) If something is a member of B, then it is a member of A (If B then A).

A more efficient way of saying this would be to combine 1.1) and 1.2) like this:

3) Something is a member of A *if and only if* it is a member of B (A *if and only if* B).

A sentence of the form 'A if and only if B' is called a *bi-conditional*. A bi-conditional is *symmetrical*; 'A if and only if B' is equivalent to 'B if and only if A'. Moreover, a bi-conditional expresses a relation of *logical equivalence* between A and B. That is to say, A and B are, for logical purposes, the same thing when they are related by a bi-conditional.

This is easy to see when you recall that A and B are co-extensive. If A and B have the same members, then they are the same class—they are equivalent.

Because the bi-conditional expresses logical equivalence, it is often used in definitions. For example: Something is a table if and only if it has legs and a top, is non-living, (and whatever else characterizes all and only tables).

The truth conditions for the bi-conditional are as follows:
A sentence of the form 'A if and only if B' is true if and only if A and B have the same truth-value.
If A and B are both true, the bi-conditional is true; if A and B are both false, it is also true. Otherwise, it is false.

(Do you understand why I used 'if and only if' when I gave the truth conditions?)

Necessary and Sufficient Conditions

Consider two things or events, N and P. N is a *necessary condition* for P iff (if and only if) N must happen or be present in order for P to happen or be present.
For example: *Being cold* is a necessary condition for snow because it must be cold in order for it to snow.
But note that the existence of a necessary condition N doesn't mean P *will happen*. It can be cold without snowing.

This should remind you of conditionals again. Suppose N is a necessary condition for P. Now suppose P happens or comes to be. Does N exist or not?
If N is a necessary condition for P then it must exist if P does. This is because it is necessary for the existence of P.

So we can express the relation of being a necessary condition by using a conditional. 'N is a necessary condition for P' can be expressed as 'If P then N'. This is because if we have P, then we must have N, since N is necessary for P.
But again note, the converse doesn't hold. We can't express this relation with 'If N then P'. This is because we can have a necessary condition without having that which it is a necessary condition for.

Now consider S and Q. S is a *sufficient condition* for Q iff whenever S happens, Q happens; S is sufficient to generate Q.
For example: Cheating or plagiarizing is a sufficient condition for failing this course. If you cheat or plagiarize, you will fail.

From this it is easy to see that the relation of being a sufficient condition can also be expressed by a conditional. 'S is a sufficient condition for Q' can be expressed as 'If S then Q'. If we have S, then we have Q, because S is sufficient to produce Q.

Again note that the relation is not symmetrical; 'If Q then S' will not be true if S is a sufficient condition for Q. This is because, although S is sufficient to produce Q, Q *could* come about by some other means. So just because we have Q doesn't mean we have S.

Here are two other things to notice about necessary and sufficient conditions:

1) Consider P and Q. Assume P is a necessary condition for Q. What relation, if any, does Q have to P?

'P is a necessary condition for Q' is the same as 'If Q then P'.

But this conditional can also be used to express a sufficient condition relation. Which one?

'If Q then P' is the same as 'Q is a sufficient condition for P'.

So P is a necessary condition for Q iff Q is a sufficient condition for P.

Try it with 'It is cold' and 'it snows'. We said being cold was a necessary condition for snow. So 'If it snows, then it is cold.'

But this same sentence also tells us that snow is a sufficient condition for being cold—surely snow betokens cold (notice that the relation between these need not be *causal*—one doesn't have to *cause* the other. It merely has to be the case that if one exists, so does the other).

2) The second thing to notice is this. Suppose P is both a necessary *and* sufficient condition for Q. (In other words, one must have P in order to have Q, but having P will also generate Q).

Which conditionals are true of this relation?

Nec. cond.—If Q then P.

Suff. cond.—If P then Q.

But if both of these are true of the same relation, we can express the relation by means of the bi-conditional.

So if P is both necessary and sufficient for Q, then 'P iff Q' is true.

Interestingly, if P is both necessary and sufficient for Q, P and Q are *logically equivalent*.

