Unit 2– Introduction to Probability

Homework #3 (Unit 2 – Introduction to Probability)

SOLUTIONS

1. Let A and B denote two independent genetic traits. Suppose the probability that an individual will exhibit trait A is ½ and the probability that an individual will exhibit trait B is ¾. What is the probability that an individual will exhibit

(a) Both traits?

Answer: .375

$$\Pr[\text{both traits}] = \Pr[A]\Pr[B]=[.50][.75]=.375$$

(b) Neither trait?

Answer: .125

$$\Pr[\text{neither trait}] = \Pr[\text{not A}]\Pr[\text{not B}]=[.50][.25]=.125$$

(c) trait A but not trait B?

Answer: .125

$$\Pr[\text{A and not B}] = \Pr[A]\Pr[\text{not B}]=[.50][.25]=.125$$

(d) trait B but not trait A?

Answer: .375

$$\Pr[\text{not A and B}] = \Pr[\text{not A}]\Pr[B]=[.50][.75]=.375$$

(e) exactly one trait?

Answer: .50

We sum the probabilities of the two mutually exclusive ways that yield "exactly one"

$$\begin{aligned}\Pr[\text{exactly one}] &= \Pr[(A, \text{not B}) \text{ or } (\text{not A}, b)] \\ &= \Pr[A, \text{not B}]+\Pr[\text{not A}, B] \\ &= [(.50)(.25)]+[(.50)(.75)] =.125+.375 \\ &= .50\end{aligned}$$

2. Suppose you are told that pr (right eye is blue) = 1/3 and pr(left eye is blue) = 1/3. Using the concepts and formulae in the lecture notes for Unit 2 (Introduction to Probability), confirm for yourself what you know by intuition, namely that pr(person is blue eyed = 1/3 by solving for pr(blue right eye and blue left eye).

Under the assumption that left and right eye colors are always the same, Pr[left is blue AND right is blue] is the same as Pr[left is blue] = 1/3

$$1/3 = Pr[\text{left is blue and right is blue}]$$
$$= Pr[\text{right is blue}]\,Pr[\text{left is blue} \mid \text{right is blue}]$$
$$= Pr[\text{right is blue}]\{1\}$$
$$= Pr[\text{right is blue}] \quad \surd$$

3. A physician develops a diagnostic test that is positive for 95% of the patients who have disease and is positive for 10% of the patients who do not have disease. Of patients tested, 20% actually have disease. Suppose you evaluate a patient by administering this diagnostic test and obtain a positive result. Using the information given, calculate the probability that this patient has disease.

Answer: .7037

Solution:

We want to calculate Probability (Disease | + test)

- **Probability (+ test | disease) = .95**
- **Probability (+ test | no disease = .10**

Probability (Disease) = .20
Probability (not Disease) = .80

$$Pr(\text{ disease} \mid +) = \frac{Pr(\text{disease and } +)}{Pr(+)}$$ by definition of conditional probability

$$= \frac{Pr(+ \mid \text{disease})\, Pr(\text{disease})}{Pr(+)}$$ because we can re-write the numerator this way

$$= \frac{Pr(+ \mid \text{disease})\, Pr(\text{disease})}{Pr(+ \mid \text{disease})\, Pr(\text{disease}) + Pr(+ \mid \text{no disease})\, Pr(\text{no disease})}$$

$$= \frac{(.95)\,(.20)}{(.95)\,(.20) + (.10)\,(.80)} = .7037$$