Unit 5 – The Normal Distribution
Homework #8 (Unit 5 – Normal part 2 of 2)

SOLUTIONS

1. **This exercise gives you additional practice in calculating probabilities under normal curves with non-zero mean and non-unit variance.**

Suppose that, in a certain population, the distribution of GRE scores is normal with mean $\mu=600$ and standard deviation $\sigma=80$.

a. What is the probability of a score less than 450 or greater than 750?

Answer: **.0608**

Solution:

Define the random variable X = GRE score.

Thus, X is distributed normal with mean $\mu=600$ and standard deviation $\sigma=80$.

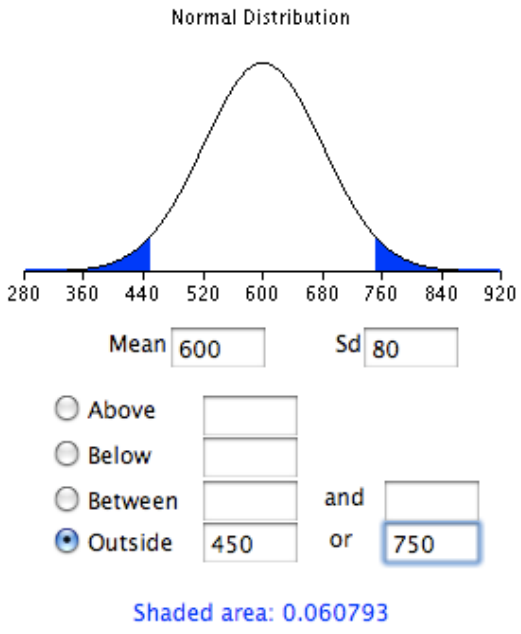
We write this more compactly as X ~ Normal ($\mu=600$, $\sigma=80$). →

Probability { score < 450 OR score > 750 }

$$= \text{pr}[X < 450] + \text{pr}[X > 750]$$

$$= .0608$$

Use the calculator at http://davidlane.com/hyperstat/z_table.html with the following selections: (1) Click on button for "area from value" (2) OUTSIDE (3) values 450 and 750 (4) mean = 600 and (5) SD=80.



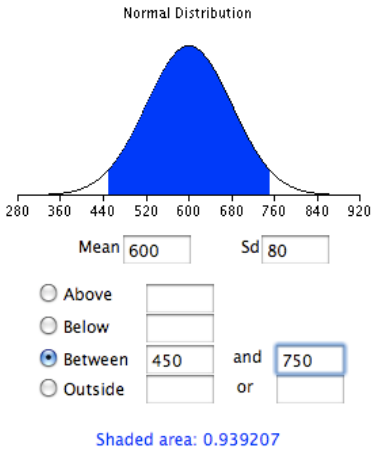
b. What proportion of students has scores between 450 and 750?

Answer: **.9392**

Solution: "Proportion" of students with scores between 450 and 750 → we want:

$$= \text{pr}[450 < X < 750]$$
$$= .9392$$

Use the calculator at http://davidlane.com/hyperstat/z_table.html with the following selections: (1) Click on button for "area from value" (2) BETWEEN (3) values 450 and 750 (4) mean = 600 and (5) SD=80.



c. What score is equal to the 95th percentile?

Answer: **731.2**

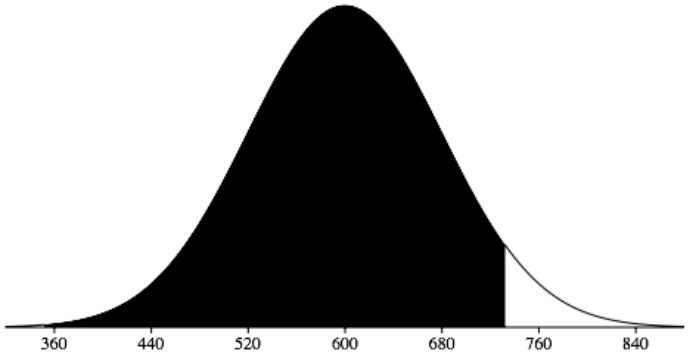
There are at least two solutions to this question:

Solution I – Simple "plug in" variety

Solution II – 2 step solution that re-enforces the concepts..
Step 1: Obtain the 95th percentile for Z~Normal(0,1). Call this $Z_{.95}$
Step 2: Use $Z_{.95}$ and the formula on page 19 of the course notes to obtain $X_{.95}$

Solution I:

Use the calculator at http://davidlane.com/hyperstat/z_table.html with the following selections: (1) Click on button for 'value from an area" (2) area = .95 (3) mean = 600 and (4) SD=80. Don't forget to click RECALCULATE.



○ Area from a value (Use to compute p from Z)
◉ Value from an area (Use to compute Z for confidence intervals)

Specify Parameters:

Area	.95
Mean	600
SD	80

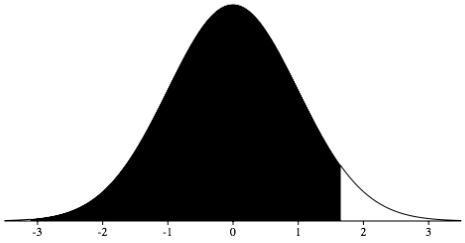
Results:

Recalculate

○ Above
◉ Below 731.617
○ Between
○ Outside

Solution II *Step 1*:

Use the calculator at http://davidlane.com/hyperstat/z_table.html with the following selections: (1) Click on button for "value from an area" (2) BELOW, (3) area = .95 (4) mean = 0 and (5) SD=1.



○ Area from a value (Use to compute p from Z)
◉ Value from an area (Use to compute Z for confidence intervals)

Specify Parameters:

Area .95

Mean 0

SD 1

Results:

Recalculate

○ Above

◉ Below 1.645

○ Between

○ Outside

Solution II *Step 2*:

Use the formula on page 19 of the unit 5 notes with the following inputs: (1) $Z_{.95} = 1.645$ (2) $\mu = 600$ and $\sigma = 80$

$$\begin{aligned} X_{.95} &= \sigma Z_{.95} + \mu \\ &= (80)[1.645] + 600 \\ &= 731.6 \end{aligned}$$

2. The Chapin Social Insight Test evaluates how accurately the subject appraises other people. In the reference population used to develop the test, Chapin Social Insight Test scores are distributed normal with mean $\mu=25$ and standard deviation $\sigma=5$.

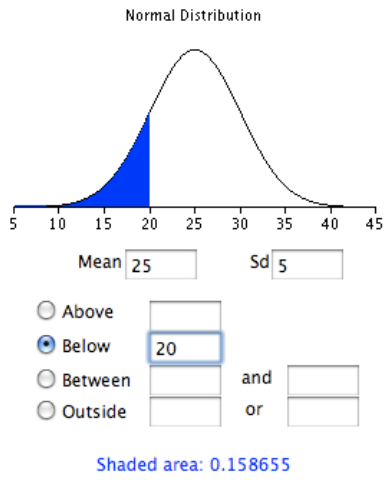
a. What proportion of the population has scores below 20 on the Chapin test?

Answer: **.1587**

Solution:
The solution for the "proportion of the population" is a probability calculation.

Define the random variable X = Chapin Social Insight Test Score.
X is distributed Normal ($\mu=25$, $\sigma=5$).
Want: $pr(X < 20) = .1587$

Use the calculator at http://davidlane.com/hyperstat/z_table.html with the following selections: (1) Click on button for "area from a value" (2) BELOW (3) value =20 (4) mean = 25 and (4) SD=5.



b. What proportion has scores below 10?

Answer: **.0014**

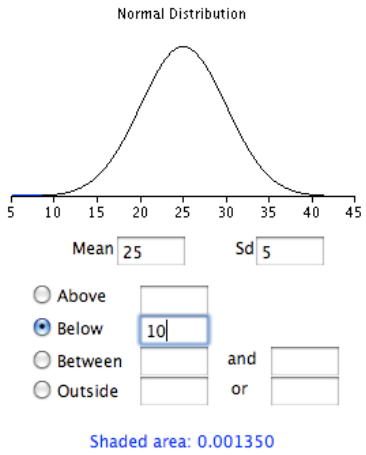
Solution:

This is similar to "a". →

The solution for the "proportion of the population" is a probability calculation.

X is distributed Normal (μ=25, σ=5).

Want: $pr(X < 10) = .0014$



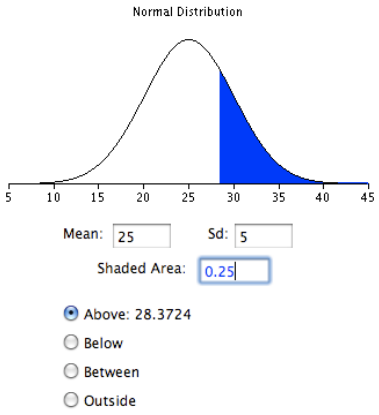
url used: http://davidmlane.com/hyperstat/z_table.html

c. How high a score must you have in order to be in the top quarter of the population in social insight?

Answer: **28.35**

Solution:

Use the calculator at http://davidlane.com/hyperstat/z_table.html with the following selections: (1) Click on button for 'value from an area" (2) ABOVE (3) area = .25 (4) mean = 25 and (5) SD=5.



3. A normal distribution has mean μ=100 and standard deviation σ=15 (for example, IQ). Give limits, symmetric about the mean, within which 95% of the population would lie:

Solution:
This exercise is asking you to work with the following characteristic of the Normal distribution:

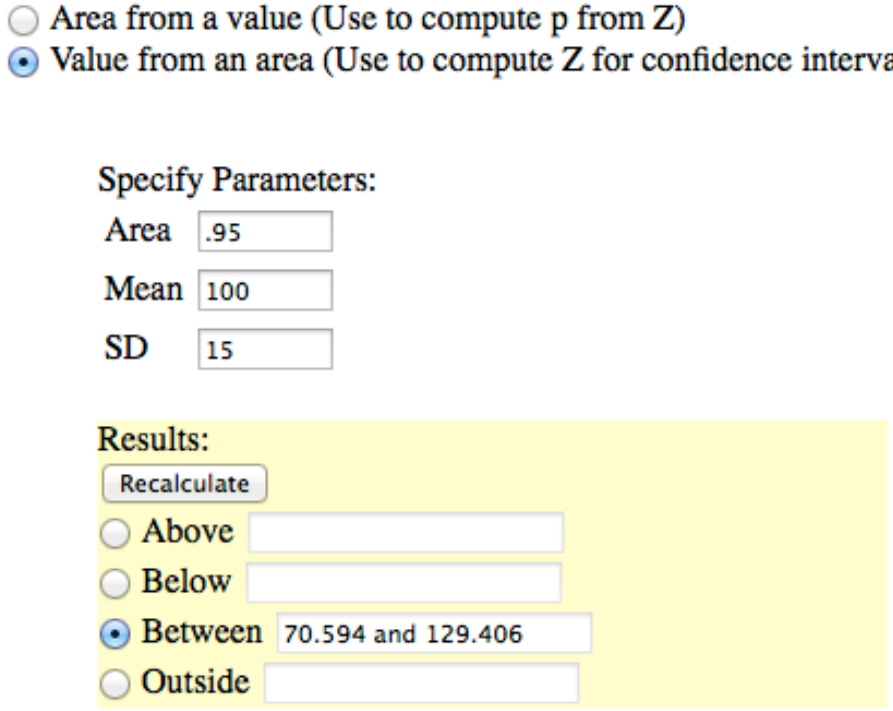
If $X_1, X_2, \ldots X_n$ are a simple random sample, each distributed Normal(μ, σ^2)
Then the sample mean of n observations is distributed Normal(($\mu, \sigma^2/n$)

Tip!
The "David Lane" calculator does not have a special box labeled SE
So, to use the "David Lane" calculator for the normal distribution of a sample mean, it is necessary to input the value of $\sqrt{\sigma^2 / n}$ in the box "SD"

a) Individual observations

Answer: 70.6, 129.4

"Individual observations" → want to set the "David Lane" boxes as follows:
"MEAN" = μ=100
"SD" = σ = 15.



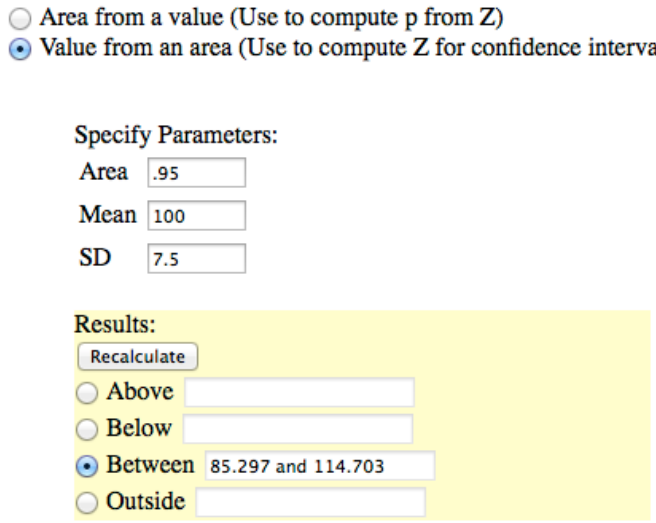
b) Means of 4 observations

Answer: 85.3, 114.7

"Means of 4 observations" → want to set the "David Lane" boxes as follows:

"MEAN" = μ=100

"SD" = SE = $\sqrt{(\sigma^2/n)} = \sigma/\sqrt{4} = 15/2 = 7.5$



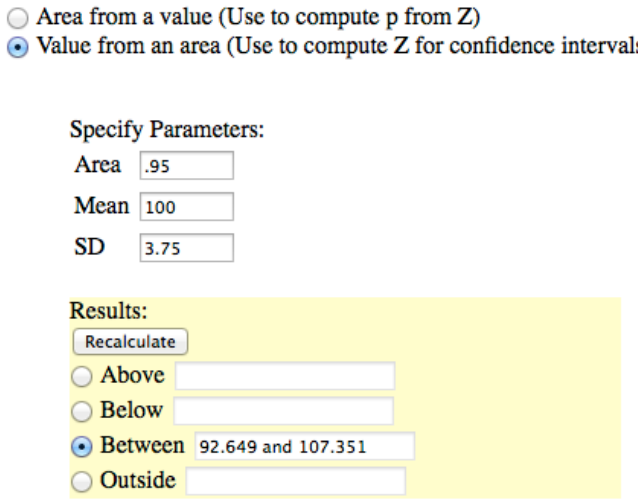
c) Means of 16 observations

Answer: 92.65, 107.35

"Means of 16 observations" → want to set the "David Lane" boxes as follows:

"MEAN" = μ=100

"SD" = SE = $\sqrt{(\sigma^2/n)} = \sigma/\sqrt{16} = 15/4 = 3.75$



d) Means of 100 observations

Answer: 97.06, 102.94

"Means of 100 observations" → want to set the "David Lane" boxes as follows:

"MEAN" = μ=100

"SD" = SE = $\sqrt{(\sigma^2/n)} = \sigma/\sqrt{100} = 15/10 = 1.5$