# Unpublished appendix to 'Developing Country Exports of Manufactures: Moving Up the Ladder to Escape the Fallacy of Composition?'

## 1 A More General ARDL Model

While the Engel-Granger OLS estimation of a static levels regression has the properties of being economical and superconsistent, it carries a finite sample bias, suggesting that superior estimates might be obtained by accounting for the short-run dynamics. Simulations suggest that including dynamic terms has benefits in finite samples.[1] While the I(0) terms are not important asymptotically, they do assume importance in small samples. The ARDL model can be reparametrized to yield an error correction model. However, the particular form of the ECM yielded by such a reparametrization is not particularly convenient to estimate when the cointegrating parameters are unknown, given the non-linear estimation required due to the interaction between $\phi$ and $\alpha_j$.[2] A simple transformation of the ARDL model, termed the "Bewley transformation," allows asymptotically valid inference using the t-statistics on the long-run coefficients. Such a transformation offers an alternative way of estimating the cointegrating relationship which has some finite sample advantages over the Engel-Granger approach. As suggested by the simulations in Inder (1993), incorporating more information in the regression (in this case, via the Bewley transformation) is likely to yield estimators with improved characteristics.

Consider the following ARDL(p,q) model:[3]

$$Y_t = \alpha_0 + \sum_{j=0}^{q} \beta_j L^j X_t + \sum_{i=1}^{p} \gamma_i L^i Y_t + \epsilon_t \tag{1}$$

where $L$ represents the lag operator.[4] This can be reparameterized as follows:

$$\left(1 - \sum_{i=1}^{p} \gamma_i L^i\right) Y_t = \alpha_0 + \sum_{j=0}^{q} \beta_j L^j X_t + \epsilon_t \tag{2}$$

Now,

[1] See Patterson (2000), for example.

[2] See, for instance, Wickens and Breusch (1987) for a discussion.

[3] This particular derivation is based on Patterson (2000).

[4] For example, $L^0 X_t = X_t$, $L^1 X_t = X_{t-1}$, and so on.

$$\begin{aligned}
\sum_{j=0}^{q}\beta_j L^j X_t &= \sum_{j=0}^{q}\beta_j X_t - \sum_{j=1}^{q}\beta_j X_t + \sum_{j=1}^{q}\beta_j X_{t-1} \\
&- \sum_{j=2}^{q}\beta_j X_{t-1} + \sum_{j=2}^{q}\beta_j X_{t-2} - \sum_{j=3}^{q}\beta_j X_{t-2} + \ldots - \beta_q X_{t-q+1} \\
&= \sum_{j=0}^{q}\beta_j X_t - \sum_{j=1}^{q}\beta_j \Delta X_t - \sum_{j=2}^{q}\beta_j \Delta X_{t-1} - \ldots - \beta_q X_{t-q+1} \\
&= B_0 X_t - \sum_{j=0}^{q} B_j \Delta X_{t-j+1} \qquad (3)
\end{aligned}$$

where $B_0 = \sum_{j=0}^{q}\beta_j$, $B_1 = \sum_{j=1}^{q}\beta_j$, $\ldots$, $B_q = \beta_q$.
Also,

$$\begin{aligned}
\left(1 - \sum_{i=1}^{p}\gamma_i L^i\right) Y_t &= Y_t - \sum_{i=1}^{p}\gamma_i L^i Y_t \\
&= Y_t - \sum_{i=1}^{p}\gamma_i Y_t + \sum_{i=1}^{p}\gamma_i Y_t - \sum_{i=1}^{p}\gamma_i Y_{t-1} + \sum_{i=2}^{p}\gamma_i Y_{t-1} \\
&- \sum_{i=2}^{p}\gamma_i Y_{t-2} + \sum_{i=3}^{p}\gamma_i Y_{t-2} + \ldots + \gamma_p Y_{t-p} \\
&= \left(1 - \sum_{i=1}^{p}\gamma_i\right) Y_t + \sum_{i=1}^{p}\gamma_i \Delta Y_t + \sum_{i=2}^{p}\gamma_i \Delta Y_{t-1} + \ldots + \gamma_p \Delta Y_{t-p+1} \\
&= (1 - \Gamma_1) Y_t + \sum_{i=1}^{p}\Gamma_i \Delta Y_{t-j+1} \qquad (4)
\end{aligned}$$

where $\Gamma_1 = \sum_{i=1}^{p}\gamma_i$, $\Gamma_2 = \sum_{i=2}^{p}\gamma_i$, $\ldots$, $\Gamma_p = \gamma_p$.

Thus the ARDL(p,q) model of equation (2) can be rewritten as:

$$Y_t = \frac{\alpha_0}{1-\Gamma_1} + \frac{B_0}{1-\Gamma_1}X_t - \frac{(\sum_{i=1}^{q} B_j)}{1-\Gamma_1}\Delta X_{t-j+1} - \frac{(\sum_{i=1}^{p}\Gamma_i)}{1-\Gamma_1}\Delta Y_{t-j+1} + \frac{\epsilon_t}{1-\Gamma_1} \qquad (5)$$

or,

$$Y_t = \alpha_0^* + B_0^* X_t - B_j^* \Delta X_{t-j+1} - \Gamma_j^* \Delta Y_{t-j+1} + \epsilon_t^* \qquad (6)$$

Note that the long-run coefficients $\alpha_0^*$ and $B_0^* X_t$ are now isolated, and the remaining variables are stationary. Note also that the long-run solution to the model in equation (6) can be found by setting $\Delta X_t = \Delta Y_t = \Delta\epsilon_t = 0$. This transformation is particularly useful when $Y_t$ and $X_t$ are I(1).

Finally, note that when p = q = 1, $\Gamma_1 = \gamma_1$, $B_0 = \beta_0 + \beta_1$, $B_1 = \beta_1$, and equation (6) reduces to:

$$Y_t = \frac{\alpha_0}{1-\gamma_1} + \frac{\beta_0+\beta_1}{1-\gamma_1}X_t - \frac{\beta_1}{1-\gamma_1}\Delta X_t - \frac{\gamma_1}{1-\gamma_1}\Delta Y_t + \frac{\epsilon_t}{1-\gamma_1} \qquad (7)$$

Due to the presence of contemporaneous links between variables, the Bewley transformation requires the use of instrumental variables.[5] Typically $Y_{t-1}$ is used as an instrument for $\Delta Y_t$.

## References


Inder, B. (1993). Estimating long-run relationships in economics. *Journal of Econometrics*, **57**, 53–68.

Patterson, K. (2000). *An Introduction to Applied Econometrics: A Time Series Approach*. St. Martin's Press, New York, U.S.A.

Wickens, M. R. and Breusch, T. S. (1987). Dynamic specification, the long-run, and the estimation of transformed regression models. *Economic Journal*, **Conference 1988**, 189–205.


[5]Notice that the presence of a first-differenced instance of the dependent variable on the right hand side of equation (7) means that the level instance of the same variable is present on both sides.